

# Robot Accuracy Off-line Compensation Method based on Laser Tracker

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## Abstract

The repeated positioning accuracy of industrial robots is relatively high, but the absolute positioning accuracy is low, which restricts the application of robots in some scenarios with high positioning accuracy requirements, such as robot hole making. Focus on the low absolute positioning accuracy of Kuka KR 210 R2700 extra articulated arm industrial robot, a laser tracker based robot calibration and off-line accuracy compensation method is proposed in this paper. A more accurate robot calibration model is constructed by introducing MDH modeling method, Considering the influence of coordinate transformation, kinematic parameters and end-effector, the error model is constructed; Through the calibration of laser tracker, a unified coordinate system is established to obtain the exact actual position value. By combining the actual position and the nominal position, the modified Gauss-Newton method is used to carry out finite iterative approximation, calculate each deviation in the error model and complete the parameter compensation; Finally, the end positioning accuracy of the robot is tested by experiment, and the results illustrate that the maximum error is improved by 56% compared with before calibration, Arithmetic mean error improved by 61%, The RMS error is improved by 62%, which proves that this calibration method can significantly reduce the end position error of KUKA industrial robot, that is, the absolute positioning accuracy of the robot is greatly improved.

## Keywords

Industrial Robot; Positioning Accuracy; Precision Compensation; Laser Tracker; Improved Gauss-newton Method.

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## 1. Introduction

With the proposal of "Industry 4.0" and "Made in China 2025", China takes intelligent manufacturing as an important national research direction, continuously strengthens the basic industrial capabilities, optimizes the dynamic allocation of resources, promotes the deep integration of information technology and manufacturing technology, and promotes the innovative development of advanced technology and equipment in manufacturing, so as to realize the revolutionary leap from a manufacturing power to a manufacturing power [1]. As the core equipment of intelligent manufacturing, industrial robots have been widely favored by the manufacturing industry in recent years due to their significant advantages such as high task flexibility, strong human-computer interaction and collaboration ability, low space requirements, and low manufacturing and maintenance costs [2], In assembly, chemical cleaning, arc welding, drilling, handling, painting, polishing and mold forming, industrial robots can be used instead of manual work [3]. However, the

repeated positioning accuracy of industrial robots is relatively high, and the absolute positioning accuracy is low, which restricts the application of robots in some scenes with high positioning accuracy requirements, such as: collision damage between end assembly parts and precision parts in the process of precision assembly; In applications such as aircraft drilling and riveting, the absolute positioning accuracy of robots can not meet the high requirements of hole position accuracy. In the welding process, the absolute positioning accuracy of the robot cannot meet the operation requirements, which will directly affect the welding quality [4]. In addition, structural wear, aging of materials and widening of gear backlash during operation of industrial robots will further reduce the absolute positioning accuracy of industrial robots, thus affecting the processing and manufacturing accuracy of products. Therefore, adopting error compensation method to improve the absolute positioning accuracy of industrial robots is an effective method [5].

Among the sources of positioning errors at the end of industrial robots, almost 90% of the robot positioning errors are caused by its own kinematic parameter errors [5-6]. Using the method based on kinematics model to modify kinematics parameters and calibrate and compensate the robot system can effectively improve the positioning accuracy of the robot. Therefore, many scholars have carried out relevant research. Yongjie Ren et al. established the robot end positioning error equation based on the D-H model, solved it by the least square method, and realized the robot position calibration [7]. Wei Zhou et al. proposed a precision compensation method based on spatial interpolation and verified it [8]. On the basis of identifying the joint Angle compensation value in the error model by the least square method, Fei Qi et al used the circular method to compensate the robot's secondary error [9]. Most of the above model-based calibration methods are based on D-H kinematic model, and the error model is not perfect due to incomplete consideration of influencing factors. When the least square method is used to solve the problem, when the adjacent joints are parallel, singularity and ill-conditioned matrix may be generated, resulting in the unidentifiable parameters. To solve the above problems, this paper proposes a method of position accuracy compensation for industrial robots based on laser tracker. The robot calibration model is established based on the improved MDH model, and the improved Gauss-Newton method is used to iteratively complete parameter identification and compensation. Finally, the effectiveness and accuracy of the algorithm are verified by experiments with laser tracker.

## 2. Robot Motion Model and Compensation Strategy

### 2.1 Rigid Body Pose Description and Coordinate Transformation

In order to describe the kinematic relationship between each connecting rod of the robot itself and between the robot and the environment (operating objects and obstacles), they are usually regarded as rigid bodies, and the kinematic relationship of each rigid body is studied. In order to fully characterize the state of the rigid body in space, the position of the reference point and the orientation of the rigid body need to be described [10].

#### 2.1.1 Description of Position (Position Vector)

For object B in the space of cartesian coordinate system {A}, in order to fully describe the position and attitude of rigid body B in space, it is usually solid connected with a coordinate system {B}. The coordinate origin of {B} is generally chosen at the characteristic point of the object B (center of mass or center of symmetry). Therefore, relative to the reference frame {A}, the 3×1 column vector  ${}^A p_{B0}$  (called the position vector) can be used.

$${}^A p_{B0} = [p_x \quad p_y \quad p_z]^T \quad (1)$$

Describes the origin position of the coordinate system {B}, where  $p_x$ ,  $p_y$ , and  $p_z$  are the three coordinate components of the point in the coordinate system {A}. The superscript A represents the reference coordinate system {A}.

For special cases, if the coordinate system {B} is A translation of the coordinate system {A}, then the position vector of any point p on the rigid body or any point P in its extended space with respect to the coordinate system {A} can be expressed as:

$${}^A p = {}^B p + {}^A p_{B0} \tag{2}$$

There,  ${}^A p$  is the position vector of point p in the coordinate system {A} and  ${}^B p$  is the position vector of point p in the coordinate system {B}.

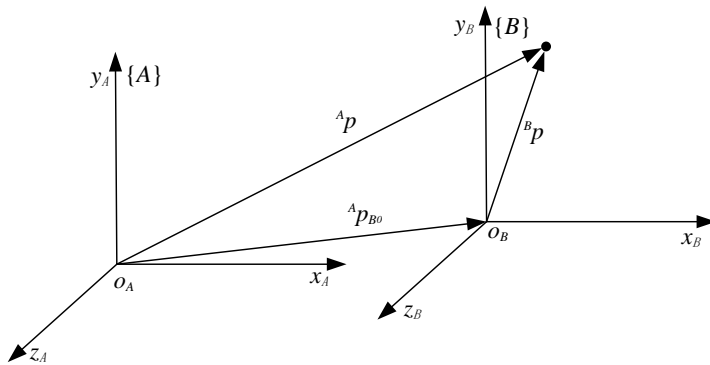


Fig. 1 Coordinate translation

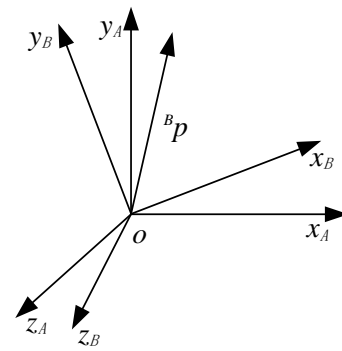


Fig. 2 Coordinate rotation

### 2.1.2 Description of Orientation (Rotation Matrix)

The orientation of the axis of the coordinate system {B} can be represented by A 3×3 matrix consisting of the cosine of the three unit principal vectors x<sub>B</sub>, y<sub>B</sub>, z<sub>B</sub> with respect to the reference frame {A}.

$${}^A_B R = \begin{bmatrix} {}^A x_B & {}^A y_B & {}^A z_B \end{bmatrix} \tag{3}$$

${}^A_B R$  is called the rotation matrix, the superscript A represents the parameter coordinate system {A}, the subscript B represents the described coordinate system {B},  ${}^A_B R$  has 9 elements, of which only 3 are independent, because the three column vectors  ${}^A x_B$ ,  ${}^A y_B$ ,  ${}^A z_B$  are all unit principal vectors, and the pair is perpendicular to each other. Therefore, the rotation matrix of the  $\theta$  Angle around the x, y, and z axes is:

$$R(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}; \quad R(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}; \quad R(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

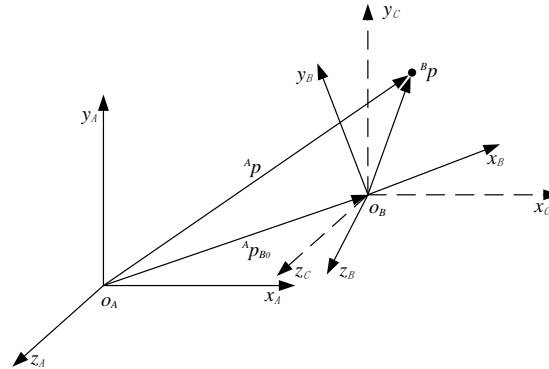
For special cases, let the coordinate system {B} be obtained by the rotation of the coordinate system {A} about each axis, and use the rotation matrix  ${}^A_B R$  to describe the orientation of {B} with respect to {A}. Then the description  ${}^A p$  and  ${}^B p$  of the same point p in the two coordinate systems {A} and {B} have the following transformation relation:

$${}^A p = {}^A_B R {}^B p \tag{4}$$

### 2.1.3 General Transformation

For the most general case: the coordinate system {B} is translated and rotated by {A}. As shown in Fig. 3, the position vector  ${}^A p_{B0}$  is used to describe the position of the coordinate origin of {B}

relative to {A}; A rotation matrix is used to describe the orientation of {B} with respect to {A}. At the same time, A transitional coordinate system {C} is specified, where the origin of the coordinates of {C} coincides with that of {B}, and the orientation of {C} is the same as that of {A}.



**Fig. 3** Composite transformation

Then there is:

$${}^C p = {}^C R {}^B p = {}^A R {}^B p \quad (5)$$

Combined with the translation formula, the composite transformation formula is obtained:

$${}^A p = {}^C p + {}^A p_{CO} = {}^A R {}^B p + {}^A p_{BO} \quad (6)$$

The composite transformation is non-homogeneous for point Bp, but it can be expressed as an equivalent homogeneous transformation form:

$$\begin{bmatrix} {}^A p \\ 1 \end{bmatrix} = \begin{bmatrix} {}^A R & {}^A p_{Bo} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^B p \\ 1 \end{bmatrix} \quad (7)$$

Where the  $4 \times 1$  column vector represents a point in three-dimensional space, which is called the homogeneous coordinate of the point and is still denoted as  ${}^A p$  or  ${}^B p$ . The above formula can be written as:

$${}^A p = {}^A T {}^B p \quad (8)$$

In the formula, the homogeneous coordinates  ${}^A p$  and  ${}^B p$  are  $4 \times 1$  column vectors, and the homogeneous transformation matrix  ${}^A T$  is a  $4 \times 4$  square matrix, which comprehensively represents the translation and rotation transformations.

## 2.2 Robot D-H Model

The articulated arm industrial robot of the Kuka KR 210 R2700 extra can be seen as an open motion chain formed by 6 links connected by 6 rotating joints, and each link is treated as a rigid body. The base is called link 0 and is not included in the 6 links. Connecting rod 1 and the base are connected by joint 1; Link 2 and link 1 are connected through joint 2, and so on.

The function of the connecting rod is to maintain a fixed geometric relationship between the joint axes at both ends, As shown in Fig. 4, the link  $i-1$  is defined by the common normal length  $a_{i-1}$  and

the Angle  $\alpha_{i-1}$  of the joint axes  $i-1$  and  $i$ .  $a_{i-1}$  and  $\alpha_{i-1}$  are respectively referred to as the length and twist Angle of link  $i-1$ ; Two adjacent links  $i$  and  $i-1$  are connected by joint  $i$ , so the joint axis  $i$  has two common normal lines perpendicular to it, each common normal line represents a link, and the distance  $d_i$  between the two common normal lines  $a_{i-1}$  and  $a_i$  is called the bias between the two links; The Angle  $\theta_i$  between  $a_{i-1}$  and  $a_i$  is called the joint Angle between the two connecting rods; Each link is described by four parameters,  $a_{i-1}$ ,  $\alpha_{i-1}$ ,  $d_i$ ,  $\theta_i$ , the first two describe the characteristics of the link itself, the last two describe the connection with the link; For rotary joint  $i$ , only  $\theta_i$  is the joint variable and the other parameters are fixed; for mobile joint  $i$ , only  $d_i$  is the joint variable and the other parameters are fixed; This method of describing mechanism motion was first proposed by Denavit and Hartenberg and is called D-H method [11].

In order to determine the relative motion relationship between the robot links, a coordinate system is fixed on each link. The coordinate system connected to the base is denoted  $\{0\}$ , and the coordinate system connected to the link  $i$  is denoted  $\{i\}$ . And determine the link coordinate system according to the following principles:

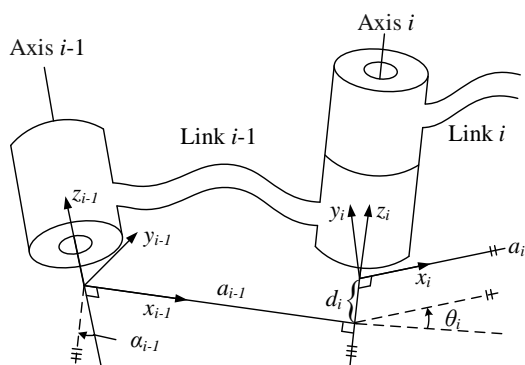


Fig. 4 Linkage description

- 1) The  $z$ -axis  $z_{i-1}$  of the coordinate system  $\{i-1\}$  is collinear with the joint axis and points arbitrarily;
- 2) The  $x$ -axis  $x_{i-1}$  of the coordinate system  $\{i-1\}$  coincides with the common perpendicular of link  $i-1$ , pointing from joint  $i-1$  to joint  $i$ . When  $x_{i-1}=0$ , take  $x_{i-1}=\pm z_i \times z_{i-1}$ ;
- 3) The  $y$ -axis  $y_{i-1}$  of the coordinate system  $\{i-1\}$  is defined by the right-hand rule, that is,  $y_{i-1}=\pm z_{i-1} \times x_{i-1}$ ;
- 4) The origin  $o_{i-1}$  of the coordinate system  $\{i-1\}$  is taken at the intersection of  $x_{i-1}$  and  $z_{i-1}$ , and when  $z_i$  and  $z_{i-1}$  are parallel, the origin is taken at the point where  $d_i=0$ ; When  $z_i$  and  $z_{i-1}$  intersect, the origin is taken at the intersection of the two axes.

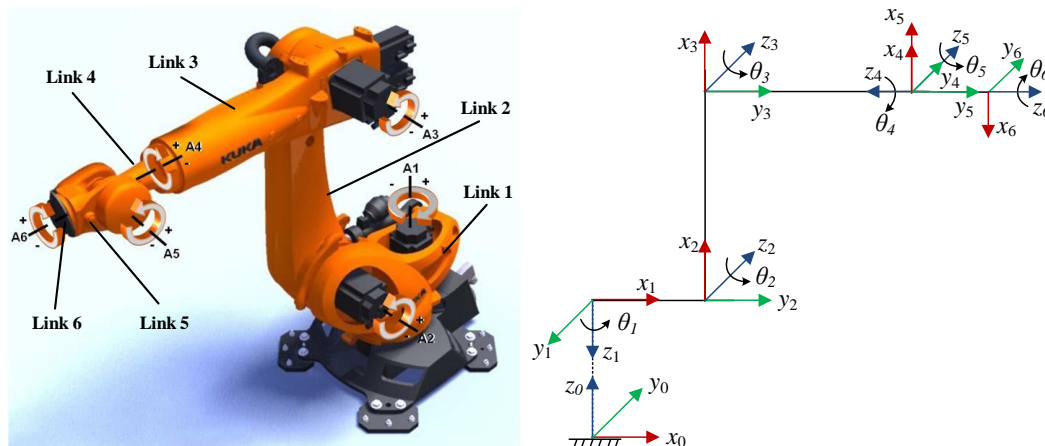


Fig. 5 Robot links coordinate system

**Table 1.** Three Scheme comparing

Numble	$a_{i-1}$ (mm)	$\alpha_{i-1}$ (°)	$d_i$ (mm)	$\theta_i$ (°)	Joint variable
1	0	180	675	0	$\theta_1$ (0°)
2	350	90	0	0	$\theta_2$ (-90°)
3	1150	0	0	0	$\theta_3$ (90°)
4	41	90	1200	0	$\theta_4$ (0°)
5	0	90	0	0	$\theta_5$ (0°)
6	0	90	215	0	$\theta_6$ (0°)

According to the above principles, the robot link coordinate system can be established, as shown in Fig. 5. So, each member of the robot has 4 parameters,  $a_{i-1}$ ,  $\alpha_{i-1}$ ,  $d_i$ , and  $\theta_i$ . They determine the kinematic configuration of each member of the robot arm. The nominal values of the links parameters of the robot studied in this paper are shown in Table 1.

According to the coordinate transformation described in Section 2.1, the homogeneous transformation matrix  ${}^i_{i-1}T$  of the coordinate system  $\{i\}$  with respect to  $\{i-1\}$  is obtained, and the transformation  ${}^i_{i-1}T$  is decomposed into four basic sub-transformation problems:

- 1) Angle  $\alpha_{i-1}$  about the  $x_{i-1}$  axis;
- 2) Move  $a_{i-1}$  along the  $x_{i-1}$  axis;
- 3) Rotate  $\theta_i$  Angle about  $z_i$  axis;
- 4) Move  $d_i$  along the  $z_i$  axis.

Therefore, the homogeneous transformation matrix  ${}^i_{i-1}T$  can be expressed as:

$$\begin{aligned}
 {}^i_{i-1}T &= \text{Rot}(x, \alpha_{i-1}) \text{Trans}(x, a_{i-1}) \text{Rot}(z, \theta_i) \text{Trans}(z, d_i) \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}
 \end{aligned}$$

The position and attitude  ${}^0_6T$  of the robot's end flange coordinate system relative to the base coordinate system are expressed as follows:

$${}^0_6T = \prod_{i=1}^6 {}^i_{i-1}T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \tag{10}$$

### 2.3 Motion Parameter Calibration and Compensation Strategy

In order to establish the error model, the parameter error is usually divided into two categories: a. joint variable error, b. Fixed parameter error; According to Denavit-Hartenberg's representation, the motion relationship between the coordinate systems of two adjacent links can be fully described by the 4 motion parameters  $a_{i-1}$ ,  $\alpha_{i-1}$ ,  $d_i$ , and  $\theta_i$ . Obviously, the error of these parameters will cause the

motion error of the robot and affect the positioning accuracy. However, when the adjacent joint axes are parallel or approximate parallel, The slight pose change of the end could not be expressed by four parameters. Therefore, Veitschegger et al. built an error model by adding a new error parameter  $\beta_i$  on the basis of the kinematics model of D-H.  $\beta_i$  is the rotation term around the y-axis, and this model is also called the MDH model [12].

$${}^{i-1}T = \text{Rot}(x, \alpha_{i-1})\text{Trans}(x, a_{i-1})\text{Rot}(z, \theta_i)\text{Trans}(z, d_i)\text{Rot}(y, \beta_i)$$

$$= \begin{bmatrix} c\theta_i c\beta_i & -s\theta_i & c\theta_i s\beta_i & a_{i-1} \\ s\theta_i c\alpha_{i-1} c\beta_i + s\alpha_{i-1} s\beta_i & c\theta_i c\alpha_{i-1} & s\theta_i c\alpha_{i-1} s\beta_i - s\alpha_{i-1} c\beta_i & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} c\beta_i - c\alpha_{i-1} s\beta_i & c\theta_i s\alpha_{i-1} & s\theta_i s\alpha_{i-1} s\beta_i + c\alpha_{i-1} c\beta_i & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

By referring to equation (10), the differential of transformation matrix T is modified, and the position and orientation are written out. The position error  $\Delta P$  and orientation error  $\Delta R$  of the end of the robot in the operation space can be obtained as follows:

$$\Delta P = \sum_{i=1}^6 \frac{\partial p}{\partial \theta_i} \Delta \theta_i + \sum_{i=1}^5 \frac{\partial p}{\partial \alpha_i} \Delta \alpha_i + \sum_{i=1}^6 \frac{\partial p}{\partial d_i} \Delta d_i + \sum_{i=1}^5 \frac{\partial p}{\partial a_i} \Delta a_i + \sum_{i=1}^6 \frac{\partial p}{\partial \beta_i} \Delta \beta_i \quad (12)$$

$$\Delta R = \sum_{i=1}^6 \frac{\partial R}{\partial \theta_i} \Delta \theta_i + \sum_{i=1}^5 \frac{\partial R}{\partial \alpha_i} \Delta \alpha_i + \sum_{i=1}^6 \frac{\partial R}{\partial \beta_i} \Delta \beta_i \quad (13)$$

Taking the KR 210 R2700 extra industrial robot for robotic hole making as an example, The requirement for position is higher than the attitude, so the observation equation of parameter deviation is established for position error. Considering that only the second and third axes of this robot are parallel, only the rotation term  $\beta_2$  around the y axis is taken into account, and the robot calibration error model can be obtained:

$$\Delta P = \sum_{i=1}^6 \frac{\partial p}{\partial \theta_i} \Delta \theta_i + \sum_{i=0}^5 \frac{\partial p}{\partial \alpha_i} \Delta \alpha_i + \sum_{i=1}^6 \frac{\partial p}{\partial d_i} \Delta d_i + \sum_{i=0}^5 \frac{\partial p}{\partial a_i} \Delta a_i + \frac{\partial p}{\partial \beta_2} \Delta \beta_2 \quad (14)$$

Written in matrix form:

$$\Delta P = J_\varepsilon \Delta \varepsilon = p^n - p^m \quad (15)$$

Where,  $\Delta P = [\Delta p_x, \Delta p_y, \Delta p_z]^T$ ,  $p^n$  is the nominal position coordinate of the end of the robot, that is, the nominal coordinate value calculated by the robot kinematics equation;  $p^m$  is expressed as the measured position coordinates at the end of the robot;  $J_\varepsilon$  is a  $3 \times 25$  error coefficient matrix, i.e.:

$$J_\varepsilon = \begin{bmatrix} \frac{\partial p_x}{\partial a_0} & \dots & \frac{\partial p_x}{\partial a_5} & \frac{\partial p_x}{\partial \alpha_0} & \dots & \frac{\partial p_x}{\partial \alpha_5} & \frac{\partial p_x}{\partial d_1} & \dots & \frac{\partial p_x}{\partial d_6} & \frac{\partial p_x}{\partial \theta_1} & \dots & \frac{\partial p_x}{\partial \theta_6} & \frac{\partial p_x}{\partial \beta_2} \\ \frac{\partial p_y}{\partial a_0} & \dots & \frac{\partial p_y}{\partial a_5} & \frac{\partial p_y}{\partial \alpha_0} & \dots & \frac{\partial p_y}{\partial \alpha_5} & \frac{\partial p_y}{\partial d_1} & \dots & \frac{\partial p_y}{\partial d_6} & \frac{\partial p_y}{\partial \theta_1} & \dots & \frac{\partial p_y}{\partial \theta_6} & \frac{\partial p_y}{\partial \beta_2} \\ \frac{\partial p_z}{\partial a_0} & \dots & \frac{\partial p_z}{\partial a_5} & \frac{\partial p_z}{\partial \alpha_0} & \dots & \frac{\partial p_z}{\partial \alpha_5} & \frac{\partial p_z}{\partial d_1} & \dots & \frac{\partial p_z}{\partial d_6} & \frac{\partial p_z}{\partial \theta_1} & \dots & \frac{\partial p_z}{\partial \theta_6} & \frac{\partial p_z}{\partial \beta_2} \end{bmatrix} \quad (16)$$

$\Delta\varepsilon$  is a  $1 \times 25$  geometric error parameter vector, namely:

$$\Delta\varepsilon = [\Delta a_0 \cdots \Delta a_5, \Delta a_0 \cdots \Delta a_5, \Delta d_1 \cdots \Delta d_6, \Delta \theta_1 \cdots \Delta \theta_6, \Delta \beta_2]^T \quad (17)$$

When the column vector of  $J\varepsilon$  is linearly independent, the motion parameter error to be calibrated can be solved by Gauss-Newton method, and the iterative equation is as follows:

$$\Delta\hat{\varepsilon} = (J_\varepsilon^T J_\varepsilon)^{-1} J_\varepsilon^T \Delta p \quad (18)$$

In the calculation process, the approximation step is determined by the inverse matrix of the matrix  $G = J_\varepsilon^T J_\varepsilon$ . If  $G$  is not positive definite, then its inverse matrix may not exist, or even if there is an inverse matrix, there will be a deviation in the approximation direction, affecting the approximation direction. Therefore, it is considered to add a multiple of the identity matrix  $I$  to the  $G$  matrix to ensure positive qualitative. After optimization:

$$\Delta\hat{\varepsilon} = (J_\varepsilon^T J_\varepsilon + uI)^{-1} J_\varepsilon^T \Delta p \quad (19)$$

In the formula,  $u$  is the weight coefficient, the initial value is 0.001, and  $\Delta\varepsilon$  is the approximate solution of the motion parameter error. According to this, the matrix  $J\varepsilon$  is modified, and the value of the weight coefficient is constantly adjusted, and the  $\Delta\varepsilon$  is obtained by finite iterations. Ignoring the measurement error, the true value of  $J\varepsilon$  will be approximated. Therefore, the parameter error calibration algorithm is as follows:

- 1) Initialize, adjust the link parameter value to the nominal value;
- 2) The differential displacement  $\Delta P$  and the corresponding matrix  $J\varepsilon$  are calculated, and the observed equations are obtained by using the nominal value and the moving position data measured by the laser tracker;
- 3) The motion parameter error estimate  $\Delta\varepsilon$  is obtained by solving the observed equations with the optimized Gauss-Newton method;
- 4) According to each component of the error vector  $\Delta\varepsilon$ , the parameters of the links are modified, and the weight coefficient  $u$  is adjusted;
- 5) Repeat 2) - 4) until the set number of iterations  $k$  is reached or each component of the resulting  $\Delta\varepsilon$  is less than the set required value;
- 6) The parameter error of each connecting rod of robot is corrected to improve the positioning accuracy of robot.

### 3. Experimental Research on Robot Accuracy Compensation

#### 3.1 Experimental Platform

As shown in Fig 6, the positioning error measurement system of industrial robots consists of measuring equipment and software, measuring targets and industrial robots. In the figure,  $P$  represents the target center to be measured,  $Base$  is the robot base coordinate system,  $Tool0$  is the robot end effector coordinate system, and  $Measure$  is the measuring device coordinate system.



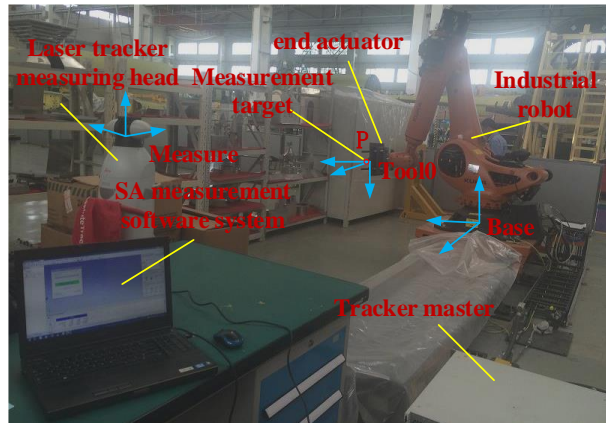


Fig. 6 Experimental platform

In this industrial robot positioning error measurement system, the measuring equipment can directly Measure the coordinates of the target center P in the measure coordinate system. On the other hand, the robot itself can also be used as a measuring device to obtain the coordinates of the target center P held by the robot in the robot Base coordinate system, and the coordinate transformation between the two can be unified in the same coordinate system, and then the position error of each measurement point in the working space of the industrial robot can be obtained by comparison.

### 3.2 Establishment and Unification of Coordinate System

#### 3.2.1 Calibration of Tool Coordinate System in Measurement System

In order to measure the positioning accuracy of the robot, it is necessary to establish the tool coordinate system, and in order to facilitate the measurement with the laser tracker, the center P of the target ball can be defined as the tool TCP point. In the robot error measurement system, the coordinate  $\mathbf{P}_B^i$  of the measurement target center P in the robot Base coordinate system can be expressed as:

$$\mathbf{P}_B^i = {}^0\mathbf{T}^i \mathbf{P}_{T0} \quad (i=1 \dots N) \quad (20)$$

Where,  ${}^0\mathbf{T}^i$  is the transformation matrix of the robot's end flange coordinate system relative to the base coordinate system, and is the function of Angle  $\theta$  of each joint.  $\mathbf{P}_{T0}$  is the position of the tool coordinate point relative to the flange coordinate system;  $N$  is the number of measurement positions in the robot workspace. Take two different measurement positions  $m$  and  $n$  of the robot, and the deviation of measurement coordinates is:

$$\Delta \mathbf{P}_B^{mn} = {}^0\mathbf{T}^m \mathbf{P}_{T0} - {}^0\mathbf{T}^n \mathbf{P}_{T0} \quad (21)$$

At the same time, the coordinate deviation of the target center P under the Measure coordinate is:

$$\Delta \mathbf{P}_M^{mn} = \mathbf{P}_M^m - \mathbf{P}_M^n \quad (22)$$

As we all know, for any two different positions of the robot in space, although their coordinate values in the robot coordinate system and the measurement coordinate system are different, the distance length of the two positions in the two coordinates is the same, that is:

$$\|\Delta \mathbf{P}_B^{mn}\|_2 = \|\Delta \mathbf{P}_M^{mn}\|_2 \quad (23)$$

Thus there is:

$$\| {}^0\mathbf{T}^m\mathbf{P}_{T0} - {}^0\mathbf{T}^n\mathbf{P}_{T0} \|_2 = \| \mathbf{P}_M^m - \mathbf{P}_M^n \|_2 \quad (24)$$

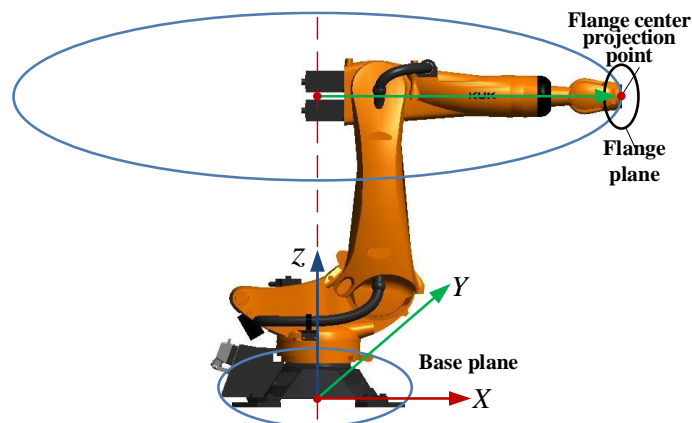
Since  ${}^0\mathbf{T}^m$ ,  ${}^0\mathbf{T}^n$  can be calculated according to  $\theta$  values in the robot teaching device, and  $\mathbf{P}_M^m$ ,  $\mathbf{P}_M^n$  can be measured from the measuring equipment, the coordinates of the target center P in the robot Tool0 coordinate system can be calculated by using the above formula, and the tool coordinate system is parallel to the flange coordinates. The position of the target center P at the end of the robot can be calibrated, so that the calibration of the tool coordinate system in the positioning error measurement system of industrial robots can be achieved, and more accurate calibration results can be obtained in this way.

### 3.2.2 Calibration of Base Coordinate System in Measurement System

The robot coordinate system is located at the center of the base of the robot frame and is the reference coordinate system of the robot mechanical structure. Due to the limitations of the size of the target ball and the installation method of the robot, the robot coordinate system cannot be directly measured by the laser tracker, so a series of feature points need to be measured to construct the robot coordinate system. The specific establishment process is as follows:

Step 1: Control the robot so that it is in the mechanical zero position, that is, the configuration of 6 joint angles is  $(A_1 0, A_2 -90, A_3 90, A_4 0, A_5 0, A_6 0)$ ;

Step 2: Determine the base plane. The base plane is the installation plane located at the bottom of the robot. As many points as possible are measured on the installation plane along the arc base of the robot, and a plane is obtained by fitting these measured points. The base plane can be obtained by offsetting the plane downward to the distance of the target sphere radius.



**Fig. 7** Robot base coordinate system establishment diagram

Step 3: Determine the origin and z-axis orientation. The target ball of the laser tracker is placed on the magnet base fixed on the end flange, the joint Angle of axes  $A_2$  to  $A_6$  is kept unchanged, and enough points are measured while rotating axis  $A_1$  to fit a circle, and the normal direction of the plane where the fitted circle is located is taken as the z-axis direction of the robot coordinate system. The intersection point between the normal line passing the center of the circle and the base plane determined in Step 2 is taken as the robot coordinate system origin.

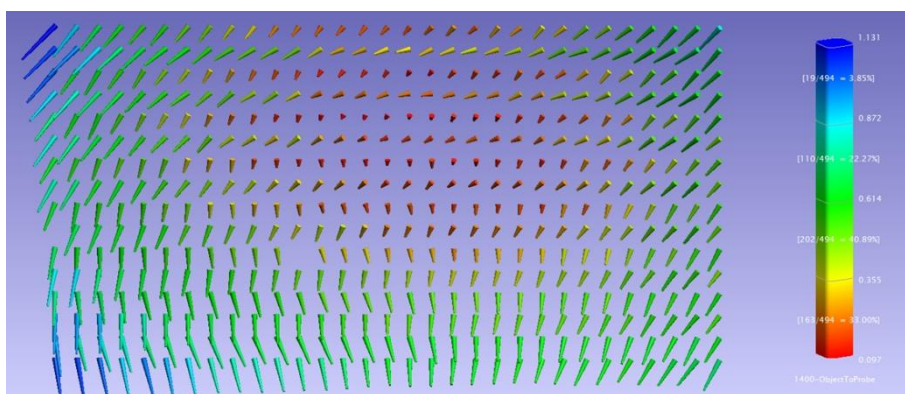
Step 4: Determine the x-axis orientation. The robot is controlled to return to the zero position of the machine, measure the position of the target ball at the six positioning holes of the end flange, and use these six points to fit a circle, and project the center of the circle to the plane where the circle is

located in step 3, then the line between the projection point and the center of the circle in Step 3 is the negative direction of the x-axis;

Step 5: Establish the robot coordinate system. With the help of SA software, the robot coordinate system can be established according to the origin, x-axis and z-axis directions.

### 3.3 Data Acquisition and Compensation Verification

After the coordinates of the laser tracker and the robot are unified, the measurement space is divided into 100mm equidistant intervals within X: 1200mm~1600mm, Y: -1500mm~1500mm, Z: 800mm~2300mm in the front hole making area of the robot base coordinate system. The analysis deviation is shown in the following figure. Then analyze and study according to the measurement data, find the optimal working space and carry out off-line error compensation.



**Fig. 8** Robot spatial error distribution

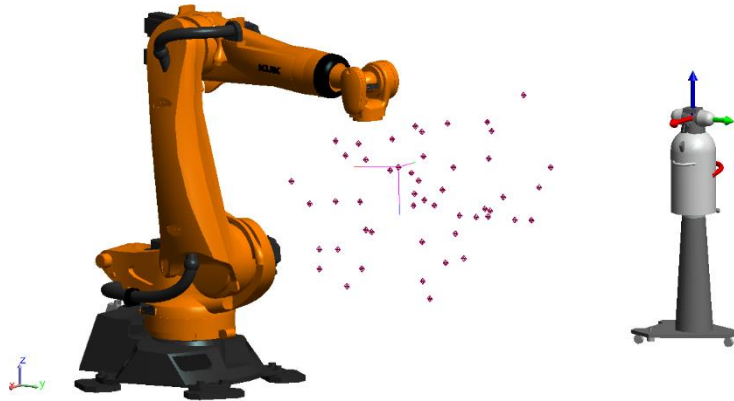
The base mark in the measuring system can be obtained by establishing a fitting according to the method in Section 3.2.2. In order to ensure the unity of the position in the measurement system and the position in the robot demonstrator, according to the method in Section 3.2.1, in order to calibrate the tool coordinate system of the measurement target (take the flange coordinate to be parallel to the tool coordinate), at least three groups of points in the robot working space need to be measured. A large number of previous relative distance error measurement results show that, When the spacing of two points in a group is 100mm, the spacing error can be guaranteed to be less than 0.1mm, so the three groups of points collected arbitrarily in the robot working space are a group of two points, and the spacing is 100mm. After calibration, the position of the target in the robot flange coordinate system and the pose of the robot in the coordinate system of the measuring equipment can be determined.

**Table 2.** Coordinate system calibration result

Calibration item	Calibration result		
	X	Y	Z
Target position	-14.17	-12.10	166.39
Base coordinate system direction	-1.21	-1.19	89.33
Base coordinate position	3745.03	-943.83	-903.71

For the identification of motion parameters, the measurement number  $k/3$  of the position must make  $k$  greater than the number of unknowns. According to the error model, there are 25 unknowns to be estimated, so at least 9 measuring points need to be collected. In order to fully represent the

characteristics of the robot, a total of 45 measuring points are collected in the borehole making working space of the robot. The distribution of data acquisition and measurement points is shown in the figure.



**Fig. 9** Sampling distribution map of measuring points

SA software is used to collect all the actual position data needed, and the position and nominal values of each parameter are sorted into the error model. After repeated iterations of the error parameters in MATLAB, the error values are obtained and corrected. The comparison of kinematic parameters before and after correction is shown in Table 3.

**Table 3.** Comparison of kinematic parameters before and after modification

Link		$a_{i-1}$ (mm)	$a_{i-1}$ ( $^{\circ}$ )	$d_i$ (mm)	$\theta_i$ ( $^{\circ}$ )	$\beta_2$ ( $^{\circ}$ )
1	Nominal value	180	0	-675.0	0	0
	deviation	-0.00072	2.66348	-3.95906	0.06025	
2	Nominal value	90	350	0	0	
	deviation	0.00157	-1.33184	-2.40728	-0.00011	
3	Nominal value	0	1150	0	0	
	deviation	0.00598	0.03828	-2.40728	0.04246	
4	Nominal value	90	-41	-1200	0	0.03
	deviation	0.17831	1.96836	-0.00794	-0.01297	
5	Nominal value	-90	0	0	0	
	deviation	-0.17439	-1.22473	1.11507	-0.04882	
6	Nominal value	90	0	215	0	
	deviation	-0.01463	1.35536	-0.05619	0.67934	

In order to verify whether the absolute positioning accuracy of the robot is improved, another 8 points in the working space of the robot are randomly measured after the calibration. It can be seen from the experimental data that the position error of the robot has been significantly reduced before and after calibration. The maximum error value, average error value and root-mean-square error of the position error of each of the 8 measurement points in the experiment are respectively solved, and these error terms are used as evaluation indexes of the absolute positioning accuracy of the robot. The results are shown in Table 4.

**Table 4.** Precision comparison after compensation

Measuring point	1	2	3	4	5	6	7	8
Nominal position (mm)	3195.65	2791.07	2789.12	3173.99	3280.88	3286.50	3503.56	3556.28
	176.32	362.58	557.48	554.57	623.81	714.19	744.90	625.49
	300.87	390.90	599.59	514.35	339.50	138.63	70.00	-158.25
Actual position (mm)	3195.68	2791.00	2789.13	3174.02	3280.93	3286.42	3503.56	3556.18
	176.21	362.57	557.62	554.59	623.80	714.02	744.81	625.41
	300.69	390.68	599.53	514.15	339.27	138.66	69.97	-158.13
$\Delta x$	-0.03	0.07	-0.01	-0.04	-0.06	0.08	0.00	0.10
$\Delta y$	0.11	0.01	-0.14	-0.01	0.02	0.17	0.09	0.08
$\Delta z$	0.18	0.22	0.07	0.19	0.23	-0.02	0.03	-0.12
$\Delta xyz$	0.21	0.23	0.16	0.20	0.24	0.19	0.10	0.17
Data comparison	pre-calibration				After calibration			
Maximum error	0.54				0.23			
Average error	0.50				0.19			
Root mean square error	0.51				0.19			

It can be seen from the table that the maximum error after calibration of the method in this paper is improved by 56% compared with that before calibration; Arithmetic mean error improved by 61%; The RMS error is improved by 62%, which proves that this calibration method can significantly reduce the end position error of KUKA industrial robot, that is, the absolute positioning accuracy of the robot is greatly improved.

#### 4. Conclusion

Focus on the low absolute positioning accuracy of Kuka KR 210 R2700 extra articulated arm industrial robot, a laser tracker based robot calibration and off-line accuracy compensation method is proposed in this paper. A more accurate robot calibration model is constructed by introducing MDH modeling method, Considering the influence of coordinate transformation, kinematic parameters and end-effector, the error model is constructed; Through the calibration of laser tracker, a unified coordinate system is established to obtain the exact actual position value. By combining the actual position and the nominal position, the modified Gauss-Newton method is used to carry out finite iterative approximation, calculate each deviation in the error model and complete the parameter compensation; Finally, the end positioning accuracy of the robot is tested by experiment, and the results illustrate that this calibration method can significantly reduce the end position error of KUKA industrial robot, and improve positioning accuracy to 0.2mm, Meet the requirements of most robot automation applications.

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School-Enterprise Cooperative Education Based on Digital Twin Technology -- Taking Industrial Robot Integrated Application Technology Course as an Example" (Project No. : 2023SCG215), Curriculum Ideological and political demonstration course of Wenzhou Polytechnic: Industrial robot workstation integrated application technology(Project No. : WZYSZKC2101).

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