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Analysis of the Coupling Vibration Characteristics of the Helical Gear Bearing Bending Torsional Shaft

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Abstract

In order to study the dynamic characteristics of helical gear transmission systems, while considering factors such as constant driving torque excitation, constant load torque excitation, gear backlash, support bearings, support damping, and gravity, an 8-degree of freedom nonlinear dynamic model of helical gear bearing bending torsion axis coupling was established using Lagrange's theorem. The fixed step Runge kutta method was used to numerically solve the model, and the vibration response sweep frequency map and vibration response time domain curve of the helical gear system considering time-varying center distance under velocity excitation were obtained. The research results indicate that time-varying center distance increases the vibration amplitude in the x direction and has a significant impact, while the vibration amplitude in the y and z directions decreases. As the velocity excitation increases, the vibration response amplitude of the system will increase.

Keywords

Helical Gear; Bending Torsional Shaft Coupling; Time-Varying Center Distance; Initial Backlash; Velocity Excitation; Vibration Response.

1. Introduction

Gear systems are widely used in various machines and mechanical equipment, and are important motion and power transmission devices. At present, many scholars at home and abroad have conducted extensive research on the dynamic characteristics of gear systems and found that the dynamic characteristics of helical gear systems are very complex. Therefore, it is still necessary to further explore the influence of various factors on their dynamic characteristics.

Due to the structure of the helical gear itself, this system has very rich nonlinear dynamic characteristics. Due to the generation of axial force, in addition to bending vibration, Torsional vibration will also produce axial vibration, and even bending torsional shaft coupling vibration, which has a very important impact on the gear system [1, 2]. In the study of dynamic characteristics of gear systems, scholars almost always study the gear rotor bearing transmission system. For example, Wang Lihua et al. [3] established a dynamic model for the coupled vibration of helical gear bending torsion axial torsion pendulum, derived the vibration differential equation of the system, calculated the vibration response of the transmission system, and conducted good numerical simulation of the helical gear transmission system, providing an effective method for the dynamic design of the helical gear transmission system. Ren Chaohui et al. [4] applied the lumped mass parameter method to establish a dynamic model of multi degree of freedom helical gear rotor bearing bending torsion shaft coupling, taking into account factors such as time-varying input/output torque, gear eccentricity, comprehensive transmission error, gravity excitation, and nonlinearity of supporting bearings. On this basis, the dynamic differential equation of the helical gear transmission system was derived, and the effects of parameters such as speed, gear eccentricity, and bearing clearance on the vibration response characteristics of the transmission system were analyzed. Kahraman [5] established a three-dimensional dynamic model that includes lateral, torsional, axial, and rotational (rocking) movements of flexible mounting gears.

Due to lubrication, temperature compensation, manufacturing and installation errors, gear systems inevitably have backlash on the tooth side. Clearance is a factor that cannot be ignored in the gear system, and it also affects the dynamic characteristics of the gear system. Wang Xin et al. [6] established a three degree of freedom single stage helical cylindrical gear axial torsional coupling nonlinear dynamic model considering nonlinear factors such as backlash, time-varying meshing stiffness, and comprehensive meshing error. Based on the "segmented linear" tooth side clearance function used in straight cylindrical gear transmission, a tooth side clearance function suitable for the meshing characteristics of helical cylindrical gears is obtained through high-order fitting. The variable step Runge kutta method is used to solve the dynamic equation of the helical cylindrical gear system, and the nonlinear dynamic response results of the system under two different tooth side clearance functions are obtained. Gao Haodong et al. [7] established a three gear Torsional vibration model considering such factors as backlash, tooth surface friction and time-varying meshing stiffness. The influence of layout parameters on tooth surface friction and time-varying meshing stiffness was analyzed, and the influence of different friction factors on the dynamic response of the system and the influence of the presence or absence of friction factors on the chaotic motion of the system were studied.

This article establishes an 8-degree of freedom nonlinear torsional vibration model for helical gear pairs; The fixed step Runge kutta method was used to numerically solve the differential equations of the model, and the effects of velocity excitation, initial backlash, and time-varying center distance factors on the dynamic characteristics of the gear system were explored. The steady-state vibration response and transmission error response of the system were obtained.

2. Dynamic Model of Helical Gears

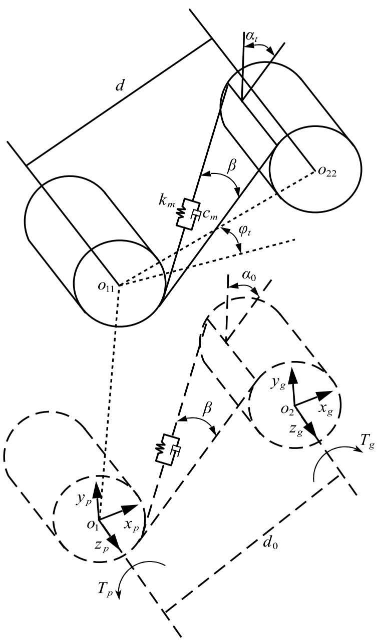

Fig 1. Dynamic model of helical gear wheel pair

In the paper, the multi degree of freedom helical gear dynamic model studied is shown in Fig. 1.; The system under study consists of two gears installed on a well aligned input and output shaft. The gear is a standard error free involute helical gear; The gear pair is modeled as a generalized lumped parameter Torsional vibration system; o_1 and o_2 is the gear center at the initial moment; d_0 is the distance between the centers of the initial gear pair, β Is the initial helix angle of the gear pair. In most previous studies, these values were considered constant and independent of time; But usually, due to the vibration of the rotating shaft, the center of the gear will change; As shown in Fig. 1, the center of the gear starts from o_1 and o_2 moved to o_{11} and o_{22}; The change of the gear center and the lateral vibration of the gear system in all directions make the distance between the gear pair centers and the end face pressure angle change with time;

In this paper, the influence of time-varying center distance on the nonlinear dynamics of gears is mainly discussed d_t and α_t represents; To simplify the calculation, tooth surface friction is not considered.

$m_{p,g}$ represents the mass of the small wheel and the driven gear; k_{ix} and $c_{ix}(i=p,g)$ represent the bearing stiffness and bearing damping of gear i in the x direction respectively; k_{iy} and c_{iy} represent the bearing stiffness and bearing damping of gear i in the y direction respectively; k_{iz} and c_{iz} represent the bearing stiffness and bearing damping of gear i in the z direction respectively; And consider them as linear springs and viscous dampers. k_m and c_m representing the comprehensive stiffness and damping of gear meshing, respectively; R_{bi} represents the base circle radius of gear i; $\dot{\theta}_p$ represents the angular velocity of the pinion; $\dot{\theta}_g$ indicates the angular velocity of the gear.

For this model, its dynamic transmission error z_t is

$$z_t=\left[\left(x_p-x_g\right)\sin\left(\alpha_t\right)+\left(y_p-y_g\right)\cos\left(\alpha_t\right)+R_{bp}\theta_p-R_{bg}\theta_g\right]\cos\left(\beta\right)+\left(z_g-z_p\right)\sin\left(\beta\right) \tag{1}$$

here, β represents the helix angle, which is a constant value; α_t represents the time-varying pressure angle of helical gear at any time. According to the geometric relationship, then

$$\alpha_t=\arccos\left(\frac{R_{bp}+R_{bg}}{\sqrt{\left(x_g-x_p+d_0\right)^2+\left(y_g-y_p\right)^2}}\right) \tag{2}$$

Here, $d_0=R_p+R_g$; R_p and R_g Represent the pitch radius of the pinion and gear respectively.

In the process of gear transmission, the backlash is almost composed of two parts: constant backlash and time-varying backlash; This article considers two parts of backlash; Constant backlash is the initial backlash; It represents the total backlash assuming that the surfaces of two gears are smooth, usually caused by gear installation errors and tooth thickness deviations; In this study, based on the meshing principle of involute gears,the half of the total backlash b_t can be expressed as

$$b_t=b_0/2+\Delta b_t \tag{3}$$

$$\Delta b_t=\left(R_{bp}+R_{bg}\right)\left(inv\left(\alpha_t\right)-inv\left(\alpha_0\right)\right) \tag{4}$$

Here, inv(x) is an involute function, inv(x)=tan(x)-x, α_0 is the initial end face pressure angle.

The Torsional vibration and axial vibration of the gear system are considered; Therefore, the differential equation of motion for a gear pair can be derived from eight generalized coordinates, whose vector form is

$$q=\begin{bmatrix}x_p & y_p & z_p & \theta_p & x_g & y_g & z_g & \theta_g\end{bmatrix}^T \tag{5}$$

x_i, y_i and z_i represents the lateral vibration of gear i relative to the gear center in the x, y, and z directions, respectively

Through the generalized coordinates mentioned above, the kinetic energy T, potential energy U, and dissipation function R of the system can be derived accordingly.

$$T=\frac{m_p(\dot{x}_p^2+\dot{y}_p^2+\dot{z}_p^2)+I_p\dot{\theta}_p^2}{2}+\frac{m_g(\dot{x}_g^2+\dot{y}_g^2+\dot{z}_g^2)+I_g\dot{\theta}_g^2}{2} \tag{6}$$

$$U=\frac{k_{px}x_p^2+k_{py}y_p^2+k_{pz}z_p^2}{2}+\frac{k_{gx}x_g^2+k_{gy}y_g^2+k_{gz}z_g^2}{2}+\frac{\left(k_m+k_{mz}\left(\sin(\beta)/\tan(\alpha_t)\right)^2\right)f^2(\delta,b_t)}{2} \tag{7}$$

$$R=\frac{c_{px}\dot{x}_p^2+c_{py}\dot{y}_p^2+c_{pz}\dot{z}_p^2}{2}+\frac{c_{gx}\dot{x}_g^2+c_{gy}\dot{y}_g^2+c_{gz}\dot{z}_g^2}{2}+\frac{\left(c_m+c_{mz}\left(\sin(\beta)/\tan(\alpha_t)\right)^2\right)f_1^2(\delta,b_t)}{2} \tag{8}$$

Using Lagrange's theorem, the differential equation of a gear system satisfies

$$m_p\ddot{x}_p+k_{px}x_p+c_{px}\dot{x}_p+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,x_p}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{x}_p}\left(z_t,b_t\right)=0 \tag{9}$$

$$m_p\ddot{y}_p+k_{py}y_p+c_{py}\dot{y}_p+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,y_p}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{y}_p}\left(z_t,b_t\right)=-m_pg \tag{10}$$

$$m_p\ddot{z}_p+k_{pz}z_p+c_{pz}\dot{z}_p+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,z_p}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{z}_p}\left(z_t,b_t\right)=0 \tag{11}$$

$$I_p\ddot{\theta}_p+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,\theta_{pz}}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{\theta}_p}\left(z_t,b_t\right)=0 \tag{12}$$

$$m_g\ddot{x}_g+k_{gx}x_g+c_{gx}\dot{x}_g+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,x_g}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{x}_g}\left(z_t,b_t\right)=0 \tag{13}$$

$$m_g\ddot{y}_g+k_{gy}y_g+c_{gy}\dot{y}_g+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,y_g}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{y}_g}\left(z_t,b_t\right)=-m_gg \tag{14}$$

$$m_g\ddot{z}_g+k_{gz}z_g+c_{gz}\dot{z}_g+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,z_g}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{z}_g}\left(z_t,b_t\right)=0 \tag{15}$$

$$I_g\ddot{\theta}_g+\sum_{\tau=1}^{n}\left(F_m^\tau+F_{mz}^\tau\right)\dot{f}_{,\theta_g}\left(z_t,b_t\right)+\sum_{\tau=1}^{n}\left(F_c^\tau+F_{cz}^\tau\right)\dot{f}_{1,\dot{\theta}_g}\left(z_t,b_t\right)=-T_g \tag{16}$$

Here, $F_{m,c}^{\tau}$ represents the meshing force and meshing damping of gear engagement τ; $F_{mz,cz}^{\tau}$ represents the meshing force and meshing damping of the axial direction of gear engagement τ;

$$F_m^\tau=k_m^\tau f\left(z_t,b_t\right),F_c^\tau=c_m^\tau f_1\left(z_t,b_t\right) \tag{17}$$

$$F_{mz}^\tau=\left(\frac{\sin(\beta)}{\tan(\alpha_t)}\right)^2k_{mz}^\tau f\left(z_t,b_t\right),F_{cz}^\tau=\left(\frac{\sin(\beta)}{\tan(\alpha_t)}\right)^2c_{mz}^\tau f_1\left(z_t,b_t\right) \tag{18}$$

Here, $c_m^\tau=2\xi_m\sqrt{k_m^\tau I_pI_g/\left(I_pR_{bg}^2+I_gR_{bp}^2\right)}$, $f(z_t,b_t)$ is a nonlinear function of gear backlash, it can be expressed as

$$f(z_t,b_t)=\begin{cases}z_t-b_t & (z_t>b_t)\\ 0 & (|z_t|\le b_t)\\ z_t+b_t & (z_t<-b_t)\end{cases} \tag{19}$$

$$f_1(z_t,b_t)=\begin{cases}\dot{z}_t-\dot{b}_t & (z_t>b_t)\\ 0 & (|z_t|\le b_t)\\ \dot{z}_t+\dot{b}_t & (z_t<-b_t)\end{cases} \tag{20}$$

Here,

$$\dot{z}_t = \dot{x}_p \frac{\partial z_t}{\partial x_p} + \dot{y}_p \frac{\partial z_t}{\partial y_p} + \dot{z}_p \frac{\partial z_t}{\partial z_p} + \dot{\theta}_p \frac{\partial z_t}{\partial \theta_p} + \dot{x}_g \frac{\partial z_t}{\partial x_g} + \dot{y}_g \frac{\partial z_t}{\partial y_g} + \dot{z}_g \frac{\partial z_t}{\partial z_g} + \dot{\theta}_g \frac{\partial z_t}{\partial \theta_g} \tag{21}$$

$$\dot{b}_t = \left(R_{bp} + R_{bg}\right)\tan^2\left(\alpha_t\right)\left(\dot{x}_p \frac{\partial \alpha_t}{\partial x_p} + \dot{y}_p \frac{\partial \alpha_t}{\partial y_p} + \dot{x}_g \frac{\partial \alpha_t}{\partial x_g} + \dot{y}_g \frac{\partial \alpha_t}{\partial y_g}\right) \tag{22}$$

Here,

$$\frac{\partial z_t}{\partial x_p} = \left(\sin\left(\alpha_t\right) + C\frac{\partial \alpha_t}{\partial x_p}\right)\cos\left(\beta\right) \tag{23}$$

$$\frac{\partial z_t}{\partial y_p} = \left(\cos\left(\alpha_t\right) + C\frac{\partial \alpha_t}{\partial y_p}\right)\cos\left(\beta\right) \tag{24}$$

$$\frac{\partial z_t}{\partial x_g} = \left(-\sin\left(\alpha_t\right) + C\frac{\partial \alpha_t}{\partial x_g}\right)\cos\left(\beta\right) \tag{25}$$

$$\frac{\partial z_t}{\partial y_g} = \left(-\cos\left(\alpha_t\right) + C\frac{\partial \alpha_t}{\partial y_g}\right)\cos\left(\beta\right) \tag{26}$$

$$\frac{\partial z_t}{\partial z_p} = -\sin\left(\beta\right), \frac{\partial z_t}{\partial z_g} = \sin\left(\beta\right), \frac{\partial z_t}{\partial \theta_p} = R_{bp}\cos\left(\beta\right), \frac{\partial z_t}{\partial \theta_g} = -R_{bg}\cos\left(\beta\right) \tag{27}$$

$$\frac{\partial b_t}{\partial x_{p,g}} = \left(R_{bp} + R_{bg}\right)\tan^2\left(\alpha_t\right)\cdot\frac{\partial \alpha_t}{\partial x_{p,g}} \tag{28}$$

$$\frac{\partial b_t}{\partial y_{p,g}} = \left(R_{bp} + R_{bg}\right)\tan^2\left(\alpha_t\right)\cdot\frac{\partial \alpha_t}{\partial y_{p,g}} \tag{29}$$

$$\frac{\partial b_t}{\partial \theta_{p,g}} = 0 \tag{30}$$

$$C = \left(x_p - x_g\right)\cos\left(\alpha_t\right) - \left(y_p - y_g\right)\sin\left(\alpha_t\right) \tag{31}$$

$$\frac{\partial \alpha_t}{\partial x_p} = -\frac{\partial \alpha_t}{\partial x_g} = -\frac{\left(R_{bp} + R_{bg}\right)\left(x_g - x_p + d_0\right)}{\lambda\eta} \tag{32}$$

$$\frac{\partial \alpha_t}{\partial y_p} = -\frac{\partial \alpha_t}{\partial y_g} = -\frac{\left(R_{bp} + R_{bg}\right)\left(y_g - y_p\right)}{\lambda\eta} \tag{33}$$

Here,

$$\begin{cases} \lambda = \left(x_g - x_p + d_0\right)^2 + \left(y_g - y_p\right)^2 \\ \eta = \sqrt{\left(x_g - x_p + d_0\right)^2 + \left(y_g - y_p\right)^2 - \left(R_{bp} + R_{bg}\right)^2} \end{cases} \tag{34}$$

Through the partial derivative relationship, the following can be derived

$$\dot{f}_{,k_j}\left(z_t, b_t\right) = \dot{f}_{1,\dot{k}_j}\left(z_t, b_t\right), \dot{f}_{,\theta_j}\left(z_t, b_t\right) = \dot{f}_{1,\dot{\theta}_j}\left(z_t, b_t\right), \mathrm{k} = x, y, z, \mathrm{j} = p, g \tag{35}$$

Table 1. Main parameters of helical gear pairs

parameter	pinion	gear
number of teeth	23	48

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mass(kg)	0.67	2.94
rotational inertia(kg· m^2)	3.07e4	5.83e-3
normal modol(mm)	2.5	
initial face pressure angle α_0(°)	20	
Meshing damping coefficient ξ_m	0.025	
tooth width(mm)	30	
helix angle β (°)	17.75	
bearing support stiffness (N/m)	6.56e7	
bearing damping (N/m)	1000	
Initial backlash b_0(mm)	100	

3. Dynamic Response Results and Analysis of Helical Gears

The analysis of this article focuses on the dynamic characteristics of gear pairs with time-varying center distance, as shown in Fig. 1; The relevant gear parameters are shown in Table 1. Unlike previous scholars, the Lagrangian method is used to obtain the nonlinear dynamic model of gear pairs; On this basis, the dynamic effects of velocity excitation on helical gear systems with time-varying center distance were explored, and a constant center distance gear model was used for comparison; For the dynamic model of helical gear pairs, namely formulas (9) - (16) the Runge Kutta method is used for numerical solution; Explore the dynamic effects of different parameters on helical gear systems.

3.1 The Effect of Velocity Excitation on Vibration Response

In gear systems, velocity excitation is one of the key factors affecting the dynamic behavior of mechanical systems, and studying the nonlinear dynamic characteristics of the system is of great significance; Here, the small gear speed is used as the excitation source of the system; Similarly, the driving torque and load torque excitation sources of the system cannot be ignored; In this section, the main focus is on exploring the impact of velocity excitation sources on the dynamic characteristics of helical gear systems. The following data graphs are all steady-state data of the gear system.

Next, in order to understand the vibration response of the entire gear transmission system under speed excitation, the transverse and axial vibration response sweep frequency of the gear system is given under speed excitation from 500r/min to 9000r/min, and the maximum vibration response in each direction is taken as the ordinate of the coordinate system. During this process, the driving torque of the small gear is 10N · m, and the initial backlash is 100 μm; The response results of the small gear are shown in Fig. 2.

Fig. 2 shows the vibration response diagram of the small gear in the x, y, and z directions, taking into account the time-varying center distance factor. By comparing with and without time-varying center distance, the vibration response trend in each corresponding direction is roughly the same, indicating that time-varying center distance has almost no effect on the vibration characteristics of the system. The displacement vibration amplitude in the y direction is significantly greater than that in the x and z directions. It can also be observed that the vibration response of the system in various directions undergoes resonance (3200rpm) and jumping (7000rpm) under some speed excitation. The resonance phenomenon is due to the consistency between the speed excitation frequency and the natural frequency of the gear system, and the jumping is due to the presence of gear backlash. The existence of time-varying center distance also delays the occurrence of these two phenomena, resulting in higher velocity excitation. From the figure, it can be seen that the time-varying center

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distance has a certain impact on the vibration amplitude in the x, y, and z directions; In this model, the time-varying center distance has a significant impact on the vibration response of the small gear in the x direction, while the influence in the y, and z directions is very small. It can be seen from the figure that the time-varying center distance increases the vibration response amplitude in the x direction and decreases the vibration response amplitude in the y and z directions, which is caused by the time-varying backlash and time-varying pressure angle. The vibration displacement of the pinion in the x and y directions is negative, and the displacement in the z direction is positive, indicating that the vibration response direction in the x and y directions is the same as the positive direction of the coordinate system established by the system, and the z direction is in the opposite direction.

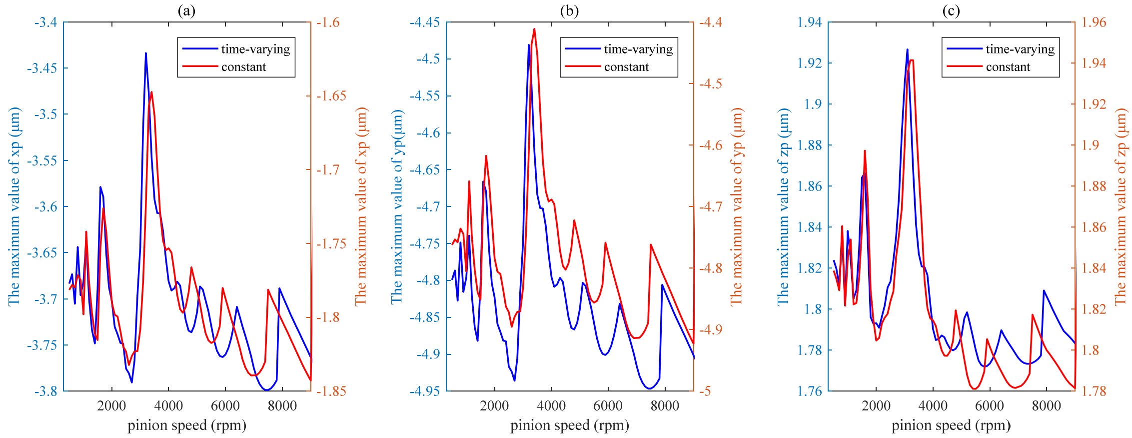

Fig 2. Scanning frequency map of lateral and axial vibration response of pinion

In order to gain a deeper understanding of the difference between the time-varying center distance model and the constant center distance model, the nonlinear dynamic response of the gear system under 2000 r/min and 6000 r/min speed excitation was analyzed.

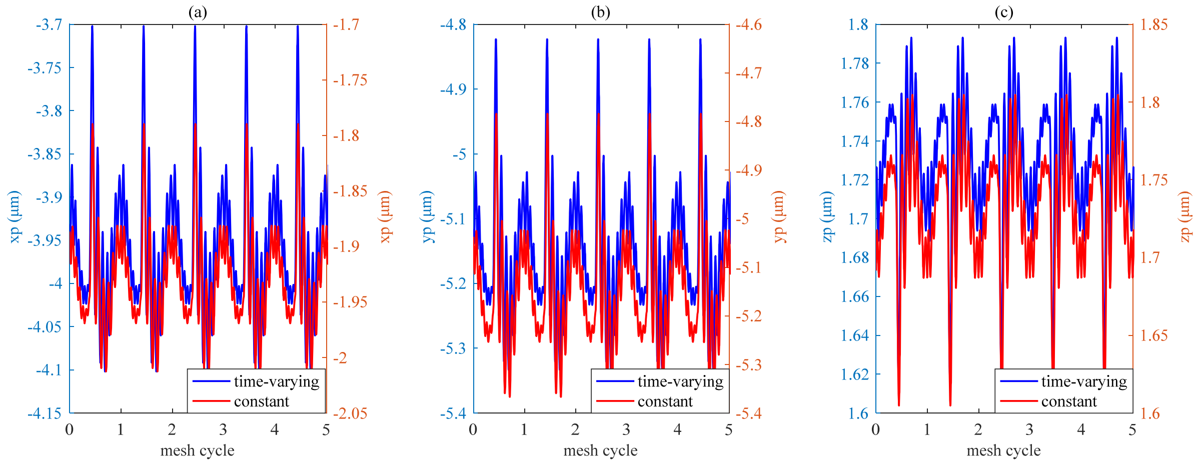

Fig 3. Time domain diagram of lateral and axial vibration response of pinion at 2000rpm

Fig. 3 and Fig. 4 show the time-domain comparison of lateral and axial vibration responses of the small gear under speed excitation of 2000r/min and 6000r/min, respectively. It can be seen that time-varying center distance does not change the vibration characteristics in the x, y and z directions, but it will affect the vibration amplitude; The influence on the vibration amplitude in the x, direction is significantly greater than that in the y and z directions, which increases the vibration displacement in the x, direction by almost twice. This is due to the existence of time-varying center distance leading to the generation of time-varying backlash, which increases the backlash during the meshing process of the gear system; However, for the $y$and z directions, the impact is exactly the opposite, reducing the vibration response, but the reduction is relatively small. For speed excitation, it can be observed that speed excitation changes the vibration characteristics of the system in all directions, and as the speed of the pinion increases, the vibration amplitudes in these three directions will increase. The corresponding frequency domain diagram also exhibits the same pattern. During the vibration response process of the entire gear system, the vibration response in the y-direction is greater than that in the other two directions, which is caused by the meshing form of the helical gear and the established coordinate system.

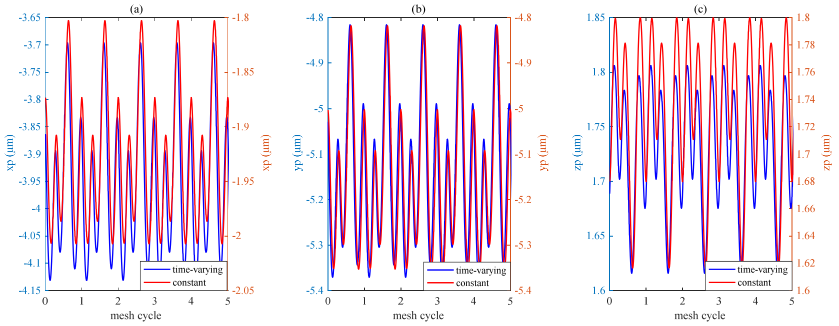

Fig 4. Time domain diagram of lateral and axial vibration response of pinion at 6000rpm

4. Conclusion

Taking into account factors such as constant driving torque excitation, constant load torque excitation, gear backlash, support bearing, support damping, and gravity, an 8-degree-of-freedom nonlinear dynamic model of helical gear bearing bending torsion axis coupling was established using Lagrange's theorem. Analyzed the impact of velocity excitation on the vibration response characteristics and dynamic transmission error of a helical gear system with or without considering time-varying center distance. The main conclusion are as follows:

The time-varying center distance has a significant impact on the dynamic characteristics of helical gear systems. It has a significant impact on the vibration response of the system. The vibration response amplitude in the x, y, and z directions is different, which increases the vibration response amplitude in the x direction and has a significant impact, while the vibration response amplitude in the y and z directions decrease; And as the speed excitation increases, the vibration response of the system will increase.

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