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Linear Vibration Analysis of a Elastic Conductive Plate in Magnetic Field with Three-direction Displacements

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Abstract

This article employs Maxwell equations in quasi-static electromagnetic field and D'Alembert's principle to deduce the differential equations of motion governing the behavior of an elastic plate experiencing Lorentz force, and study on vibrational characteristics of three-direction displacements. The article provides vibration mode functions for the boundary conditions of simply supported thin plates with immovable and movable edges. To obtain the vibration equation of three-direction displacements, space coordinates and time coordinates are separated using the Galerkin method. Subsequently, the corresponding ordinary differential equation is solved to determine the form of all displacement solutions.

Keywords

Elastic Plate; Three-Direction Displacements; Lorentz Force; Galerkin Method.

1. Introduction

Plates are one of the most popular continuous structural systems used in many engineering applications including aircraft structures, nuclear vessels, bridges, power hydraulics etc.The study of the dynamic characteristics of a plate in a magnetic field environment can help to understand and reveal the effects of electromagnetic fields on the complex dynamic behavior of the plate.Librescu [1] investigates the three-dimensional coupling of magneto-thermo-elasticity in a magnetic field for a conductive plate.Li[2] discusses the internal resonance of rectangular thin plates with different size ratios of 1:1 or 1:3 in a transverse magnetic field.Gao[3]gets an analytical solution for eddy currents and electromagnetic forces of a circular plate based on the T-method.M. Higuchi[4] obtained dynamic and quasi-static analytical solutions for the controlled stress field and eddy currents of a conductor plate in the presence of arbitrary variable magnetic fields.Ipakarand M Bayat[5] used the HPM to study the vibration frequency of a truss shell under simply supported boundary conditions. Qasem M[6] applied the extended homotopy perturbation method to obtain the analytical solution for the boundary layer flow of condensing vapor.Hu[7] used the double parameter perturbation method to solve the large deflection bending problem of functionally graded thin plates with different tensile. Zhong[8] studied the large deflection problem of a circular thin plate under uniform external pressure using HAM.

Although many studies have been conducted by scholars, research on the linear vibration analysis of the elastic plates with three-direction displacements is still scarce. Analyzing the convergence of vibration of elastic plates under different magnetic induction intensities is significant for controlling the vibration of elastic plate in practical engineering problems.

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2. Differential Equations of Vibration for Plates

2.1 Basic Equations of Electromagnetic Fields

In a quasi-static electromagnetic field, the impact of displacements current can be disregarded, and since the current flowing through the thin plate is isotropic, there is no charge accumulation, and the charge density equals 0. Thus, Maxwell equations can be expressed in the following form:

$$\begin{gathered}\nabla\cdot\boldsymbol{B}=0,\nabla\times\boldsymbol{H}=\boldsymbol{J},\\ \nabla\cdot\boldsymbol{D}=0,\nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t}=0\end{gathered}\tag{1}$$

Lorentz force density expression:

$$\boldsymbol{f}_v=\boldsymbol{J}\times\boldsymbol{B}\tag{2}$$

In cases where the velocity of the material is much lower than the speed of light, the constitutive equation for electromagnetic materials in isotropic homogeneous mediums is linear. However, the relationship between current density and electric field strength is nonlinear, and it can be expressed in the following form:

$$\begin{gathered}\boldsymbol{D}=\varepsilon\boldsymbol{E}\\ \boldsymbol{B}=\mu\boldsymbol{H}\\ \boldsymbol{J}=\sigma(\boldsymbol{E}+\dot{\boldsymbol{U}}\times\boldsymbol{B})\end{gathered}\tag{3}$$

In the formula, ε, μ and σ represent permittivity, permeability and conductivity, respectively. The geometric relationship of the displacements field of the thin plate under Kirchhoff's hypothesis:

$$\begin{aligned}u(x,y,z,t)&=u(x,y,t)-z\frac{\partial w}{\partial x}\\ v(x,y,z,t)&=v(x,y,t)-z\frac{\partial w}{\partial y}\\ w(x,y,z,t)&=w(x,y,t)\end{aligned}\tag{4}$$

The three-direction displacements u, v and w are the displacement fields in the x, y, and z directions, respectively. When the Lorentz force acts on a thin plate with a thickness of h, ignoring the electromagnetic disturbance term, substituting the equation (4) into (2) and integrating in the z direction yields the electromagnetic force and torque per unit area:

$$\begin{cases}F_x=\int_{-h/2}^{+h/2}f_{vx}dz=\sigma h\left(\frac{\partial w}{\partial t}B_xB_z-\frac{\partial u}{\partial t}B_z^2\right)\\ F_y=\int_{-h/2}^{+h/2}f_{vy}dz=\sigma h\left(\frac{\partial w}{\partial t}B_yB_z-\frac{\partial v}{\partial t}B_z^2\right)\\ F_z=\int_{-h/2}^{+h/2}f_{vz}dz=\sigma h\left(\frac{\partial v}{\partial t}B_zB_y-\frac{\partial w}{\partial t}B_y^2-\frac{\partial w}{\partial t}B_x^2+\frac{\partial u}{\partial t}B_zB_x\right)\end{cases}\tag{5}$$

And electromagnetic torque

$$\begin{cases}M_x=\int_{-h/2}^{+h/2}f_{vx}zdz=\sigma\frac{h^3}{12}\frac{\partial w}{\partial x\partial t}B_z^2\\ \mathrm{M}_y=\int_{-h/2}^{+h/2}f_{vy}zdz=\sigma\frac{h^3}{12}\frac{\partial w}{\partial y\partial t}B_z^2\end{cases}\tag{6}$$

2.2 Differential Equations of Vibration for Rectangular Plates

Assuming there is a conductive thin plate with no internal current or electric field and is not subject to external loads, it is placed in a uniform magnetic field with a magnetic induction intensity of

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(B_x,B_y,B_z) in the x, y, and z directions. The four sides of simply supported plate are immovable edges, as shown in the figure..

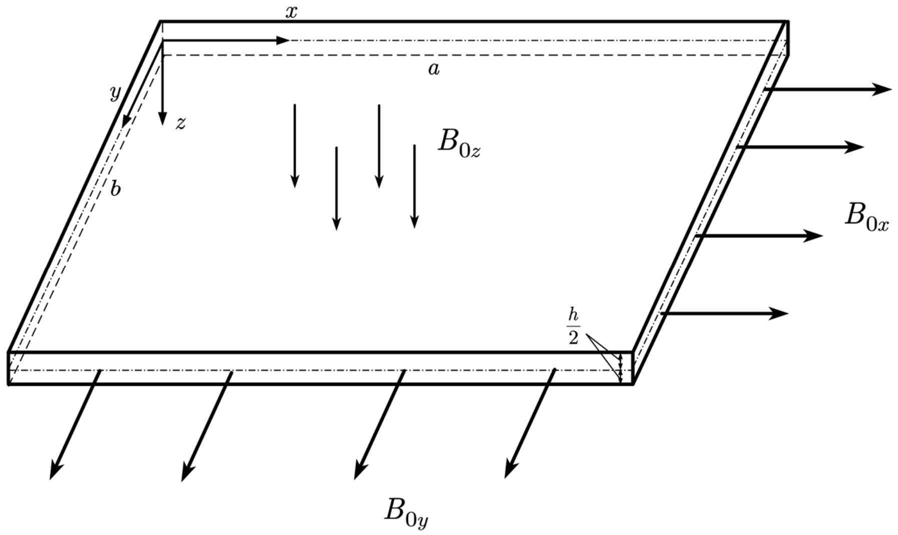

Figure 1. A fixed simply supported plate in a uniform magnetic field.

The static equilibrium differential equation of the elastic rectangular plate is:

$$\begin{aligned}&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0,\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,\\&\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial y\partial x}+\frac{\partial^2 M_y}{\partial y^2}+\left(N_{\text{x}}\frac{\partial^2 w}{\partial x^2}+2N_{\text{xy}}\frac{\partial^2 w}{\partial x\partial y}+N_{\text{y}}\frac{\partial^2 w}{\partial y^2}\right)\\&+\left(\frac{\partial N_{\text{x}}}{\partial x}+\frac{\partial N_{\text{xy}}}{\partial y}\right)\frac{\partial w}{\partial x}+\left(\frac{\partial N_{\text{y}}}{\partial y}+\frac{\partial N_{\text{xy}}}{\partial x}\right)\frac{\partial w}{\partial y}=0\end{aligned}\tag{7}$$

Here, the relationships between force and displacement are

$$\begin{aligned}N_x&=Qh\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\upsilon\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)\right]\\N_y&=Qh\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\upsilon\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)\right]\\N_{xy}&=Lh\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\end{aligned}\tag{8}$$

the relationships between torque and displacement are

$$M_x=-\frac{Qh^3}{12}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right),\ M_y=-\frac{Qh^3}{12}\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right),\ M_{xy}=-\frac{Lh^3}{6}\frac{\partial^2 w}{\partial x\partial y}\tag{9}$$

Among them tensile modulus $Q=E/(1-\upsilon^2)$, shear modulus $L=E/2(1+\upsilon)$, where υ is the Poisson ratio and E is Elastic modulus. When establishing the differential equation of motion of the object, considering the inertial force exerted by the elastic body due to the acceleration and the Lorentz force (5) and torque (6) generated by the vibration in the electromagnetic field, and omitting nonlinear terms, the linear differential equations of three-direction motion of the elastic plate can be obtained according to D'Alembert's principle:

$$\frac{\partial^2 u}{\partial x^2}+\frac{1-\upsilon}{2}\frac{\partial^2 u}{\partial y^2}+\frac{1+\upsilon}{2}\frac{\partial^2 v}{\partial x\partial y}+\frac{\sigma(1-\upsilon^2)}{E}\left(\frac{\partial w}{\partial t}B_xB_z-\frac{\partial u}{\partial t}B_z^2\right)=\frac{\rho(1-\upsilon^2)}{E}\frac{\partial^2 u}{\partial t^2}\tag{10}$$

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$$\frac{\partial^2 v}{\partial y^2}+\frac{1-\upsilon}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+\upsilon}{2}\frac{\partial^2 u}{\partial x\partial y}+\frac{\sigma(1-\upsilon^2)}{E}\left(\frac{\partial w}{\partial t}B_yB_z-\frac{\partial v}{\partial t}B_z^2\right)=\frac{\rho(1-\upsilon^2)}{E}\frac{\partial^2 v}{\partial t^2} \tag{11}$$

$$\begin{gathered}-\frac{h^3E}{12\left(1-\upsilon^2\right)}\nabla^2\nabla^2 w+\sigma h\left(\frac{\partial v}{\partial t}B_zB_y+\frac{\partial u}{\partial t}B_zB_x-\frac{\partial w}{\partial t}B_y^2-\frac{\partial w}{\partial t}B_x^2\right)\\ +\sigma\frac{h^3}{12}\frac{\partial^3 w}{\partial x^2\partial t}B_z^2+\sigma\frac{h^3}{12}\frac{\partial^3 w}{\partial y^2\partial t}B_z^2=h\rho\frac{\partial^2 w}{\partial t^2}-\frac{h^3}{12}\nabla^2\frac{\partial^2 w}{\partial t^2}\end{gathered} \tag{12}$$

The combined motion in multiple directions generates Lorentz force, which acts in various directions and influences each other. Furthermore, for a thin plate with isotropic characteristics, its own equation system has high coupling. When incorporating Lorentz force into the motion differential equation of a conductive thin plate, which includes coupling in the first-order time derivative term, it can increase the difficulty of finding a solution.

3. Vibration of the Simply Supported Plate under Electromagnetic Field with Different Boundaries

3.1 Galerkin Method

The complexity of problem-solving has increased due to the advancement of dynamics theory, and effective mathematical models are essential for research. The Galerkin method is commonly employed in the analysis of continuous system dynamics. In this method, a partial differential equation is assumed, and it can be expressed as::

$$\frac{\partial^2\tilde{w}}{\partial t^2}+D(\tilde{w})=0 \tag{13}$$

Take a set of shape functions $\varphi_r(x)\ r=1,2...,n$ that satisfy the boundary conditions, where, construct the following function

$$\tilde{w}(x,t)=\sum_{r=1}^{n}\varphi_r(x)q_r(t) \tag{14}$$

Where $q_r(t)$ is the generalized coordinate. For any given function $\tilde{w}(x,t)$, substituting it into the partial differential equation (13) usually results in a non-zero difference between the two sides, which becomes a functional related to the function $\tilde{w}(x,t)$ and is called the residue of the vibration equation:

$$R[\tilde{w}(x,t),x,t] \tag{15}$$

For elasticity, the residue also reflects the residual force. In order to minimize the residual force as much as possible, the unknown function qr(t) can be selected to make the residual force do zero work on the displacements corresponding to each shape function r(x), that is:

$$\int_0^l R\left[\sum_{r=1}^{n}\varphi_r(x)q_j(t),x,t\right]\varphi_i(x)\mathrm{d}x=0(i=1,2,\cdots,n) \tag{16}$$

By utilizing the Galerkin method, the objective of discretizing the continuous body is achieved, and the partial differential equation can be transformed into a system of ordinary differential equations. Consequently, the subsequent analysis of the system becomes feasible.

3.2 Vibration Analysis of a Fixed Simply Supported Boundary Plate under a Magnetic Field

Assuming a conductive plate is placed in a uniform magnetic field of (B_{0x},B_{0y},B_{0z}), where its length, width, and thickness are denoted by a, b, and h, respectively. The boundary conditions for the simply supported plate with immovable edges, as shown in the figure:

In order to simplify the system to a finite dimension, an approximate function is used to expand the displacements u, v and w. The system is discretized using two coordinates of panel displacement, namely the spatial and temporal coordinates, to account for different boundary conditions. For a simply-supported plate with immovable edges, the following boundary conditions hold:

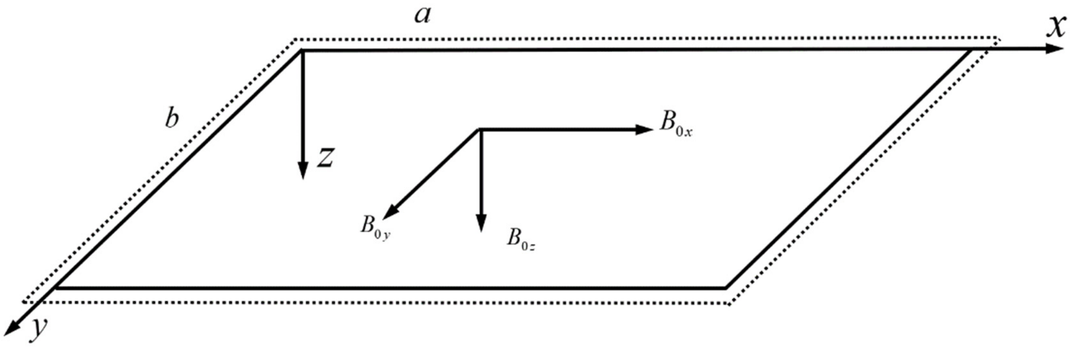

Figure 2. Simply supported plate with immovable edges in a magnetic field.

$$\begin{aligned} &at\ \ x=0,a:\ \ u=v=w=M_x=0 \\ &at\ \ y=0,b:\ \ u=v=w=M_y=0 \end{aligned} \tag{17}$$

To ensure that the approximate function satisfies the geometric boundary conditions for the above situation, the three-direction displacements u, v and w can be expanded using temporal and shape function expressions of the following form:

$$\begin{aligned} u(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} u_{m,n}(t)\sin(2m\pi x/a)\sin(2n\pi y/b) \\ v(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} v_{m,n}(t)\sin(2m\pi x/a)\sin(2n\pi y/b) \\ w(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{m,n}(t)\sin[(2m-1)\pi x/a]\sin[(2n-1)\pi y/b] \end{aligned} \tag{18}$$

Here, m and n are the half-waves in the x and y directions, respectively, both of which are positive integers. The functions $u_{m,n}(t)$, $v_{m,n}(t)$ and $w_{m,n}(t)$ are unknown functions with respect to the generalized time coordinate t. Alternatively, they can be expressed as the (m,n)-th degree of freedom of the respective expansion functions. M and N represent the required terms in the plate displacement expansion. In the linear problem discussed in this article, we can make $M, N \to \infty$ .Write out the residual functions up to order M and N:

$$\begin{aligned} R_{uA} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{ k_{11}\sin\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b} + k_{12}\cos\frac{2m\pi x}{a}\cos\frac{2n\pi y}{b} \right. \\ &\left. + k_{13}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b} \right\} \\ R_{vA} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{ k_{14}\sin\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b} + k_{15}\cos\frac{2m\pi x}{a}\cos\frac{2n\pi y}{b} \right. \\ &\left. + k_{16}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b} \right\} \\ R_{wA} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{ k_{17}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b} + k_{18}\sin(\frac{2m\pi x}{a})\sin(\frac{2n\pi y}{b}) \right. \\ &\left. + k_{19}\sin(\frac{2m\pi x}{a})\sin(\frac{2n\pi y}{b}) \right\} \end{aligned} \tag{19}$$

For convenience in subsequent calculations, the shape functions in trigonometric form with respect to spatial units are separated, where the coefficients are only functions of time and half-waves m and n, and independent of spatial coordinates:

$$k_{11} = u_{m,n}\left[\left(\frac{2m\pi}{a}\right)^2 + \frac{1-\upsilon}{2}\left(\frac{2n\pi}{b}\right)^2\right] + \frac{\partial u_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_z^2 + \frac{\partial^2 u_{m,n}}{\partial t^2}\frac{\rho(1-\upsilon^2)}{E}$$

$$k_{12} = -v_{m,n}\frac{2(1+\upsilon)mn\pi^2}{ab},\quad k_{13} = -\frac{\partial w_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_xB_z,$$

$$k_{14} = v_{m,n}\left[\left(\frac{2n\pi}{b}\right)^2 + \frac{1-\upsilon}{2}\left(\frac{2m\pi}{a}\right)^2\right] + \frac{\partial v_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_z^2 + \frac{\partial^2 v_{m,n}}{\partial t^2}\frac{\rho(1-\upsilon^2)}{E},$$

$$k_{15} = -u_{m,n}\frac{2(1+\upsilon)mn\pi^2}{ab},\quad k_{16} = -\frac{\partial w_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_yB_z,$$

$$k_{17} = \frac{h^3 E w_{m,n}}{12(1-\upsilon^2)}\left[\frac{(2m-1)^2\pi^2}{a^2} + \frac{(2n-1)^2\pi^2}{b^2}\right]^2 + \frac{\partial w_{m,n}}{\partial t}\sigma h B_x^2 + \frac{\partial w_{m,n}}{\partial t}\sigma h B_y^2$$
$$+\frac{\partial^2 w_{m,n}}{\partial t^2}h\rho + \frac{\partial w_{m,n}}{\partial t}\sigma\frac{h^3}{12}B_z^2\frac{\pi^2}{a^2} + \frac{\partial w_{m,n}}{\partial t}\sigma\frac{h^3}{12}B_z^2\frac{\pi^2}{b^2}$$
$$+\frac{\partial^2 w_{m,n}}{\partial t^2}\frac{h^3\rho}{12}\left[\frac{(2m-1)^2\pi^2}{a^2} + \frac{(2n-1)^2\pi^2}{b^2}\right], \tag{20}$$

$$k_{18} = -\frac{\partial v_{m,n}}{\partial t}\sigma h B_z B_y,\quad k_{19} = -\frac{\partial u_{m,n}}{\partial t}\sigma h B_z B_x\ .$$

According to the Galerkin method, multiplying Eq.(19) by the shape functions and integrating yields, with the requirement that the resulting average work done by the integrated displacements is zero:

$$\int_0^b\int_0^a R_{uA}\sin\left(\frac{2i\pi x}{a}\right)\sin\left(\frac{2j\pi y}{b}\right)dxdy = 0$$
$$\int_0^b\int_0^a R_{vA}\sin\left(\frac{2i\pi x}{a}\right)\sin\left(\frac{2j\pi y}{b}\right)dxdy = 0 \tag{21}$$
$$\int_0^b\int_0^a R_{wA}\sin\left(\frac{(2i-1)\pi x}{a}\right)\sin\left(\frac{(2j-1)\pi y}{b}\right)dxdy = 0$$

Here,i=1,2,3...M and j=1,2,3...N.Computing the above expression yields the differential equation governing the vibration of a conductor plate in an electromagnetic field with immovable-edge simply-supported edges:

$$\frac{d^2 u_{m,n}}{\partial t^2} + f_{uA}\frac{du_{m,n}}{dt} + g_{uA}u_{m,n} = 0$$
$$\frac{d^2 v_{m,n}}{dt^2} + f_{vA}\frac{dv_{m,n}}{dt} + g_{vA}v_{m,n} = 0 \tag{22}$$
$$\frac{d^2 w_{m,n}}{dt^2} + f_{wA}\frac{dw_{m,n}}{dt} + g_{wA}w_{m,n} = 0$$

The coefficient is a function solely dependent on time and half-waves m and n, and independent of spatial coordinates:

$$f_{uA}=\frac{\sigma}{\rho}B_z^2, g_{uA}=\left[\left(\frac{2m\pi}{a}\right)^2+\frac{1-\upsilon}{2}\left(\frac{2n\pi}{b}\right)^2\right]\frac{E}{\rho(1-\upsilon^2)},$$

$$f_{vA}=\frac{\sigma}{\rho}B_z^2,\ g_{vA}=\left[\left(\frac{2n\pi}{b}\right)^2+\frac{1-\upsilon}{2}\left(\frac{2m\pi}{a}\right)^2\right]\frac{E}{\rho(1-\upsilon^2)},$$

$$f_{wA}=\frac{1}{\varpi_{wA}}\left(\sigma hB_x^2+\sigma hB_y^2+\sigma\frac{h^3}{12}B_z^2\frac{(2m-1)^2\pi^2}{a^2}+\sigma\frac{h^3}{12}B_z^2\frac{(2n-1)^2\pi^2}{b^2}\right), \tag{23}$$

$$g_{wA}=\frac{h^3E}{12\varpi_{wA}\left(1-\upsilon^2\right)}\left[\frac{(2m-1)^2\pi^2}{a^2}+\frac{(2n-1)^2\pi^2}{b^2}\right]^2,$$

$$\varpi_{wA}=h\rho+\frac{h^3\rho}{12}\left[\frac{(2m-1)^2\pi^2}{a^2}+\frac{(2n-1)^2\pi^2}{b^2}\right].$$

Eq.(22) represents a second-order constant coefficient homogeneous linear differential equation system consisting of three independent equations. The system can be easily solved for an infinite number of solutions as functions of the half-waves *m* and *n*. In other words, the complete form of the vibration function Eq.(18) can be obtained by solving the equation system Eq.(22). Since the coefficients of each term vary with the values of the half-waves *m* and *n*, the characteristic equation of the second-order differential equation is obtained by solving for them:

$$s^2+f_rs+g_r=0\ (r=uA、vA、wA) \tag{24}$$

For the quadratic equation in the above formula, its solutions can be classified into the following three cases based on the discriminant of the equation $\Delta=f_r^2-4g_r$:

The first case is when $\Delta=0$ In this case, Eq.(22) has two identical real roots, and the solutions can be expressed as:

$$\begin{aligned}&\left\{u_{m,n}\ v_{m,n}\ w_{m,n}\right\}\\&=\left(C_{1r}+C_{2r}t\right)\exp\left(-\frac{f_r}{2}t\right)\end{aligned} \tag{25}$$

The second case is when $\Delta>0$. In this case, Eq.(22) has two distinct real roots, and the solutions can be expressed as:

$$\begin{aligned}&\left\{u_{m,n}\ v_{m,n}\ w_{m,n}\right\}\\&=C_{1r}\exp\left(\frac{-f_r+\sqrt{f_r^2-4g_r}}{2}t\right)+C_{2r}\exp\left(\frac{-f_r-\sqrt{f_r^2-4g_r}}{2}t\right)\end{aligned} \tag{26}$$

The third case is when $\Delta<0$. In this case, Eq.(22) has a pair of complex conjugate roots, and the solutions can be expressed as:.

$$\begin{aligned}&\left\{u_{m,n}\ v_{m,n}\ w_{m,n}\right\}\\&=\exp\left(-\frac{f_r}{2}t\right)\left(C_{1r}\cos\frac{\sqrt{4g_r-f_r^2}}{2}t+C_{2r}\sin\frac{\sqrt{4g_r-f_r^2}}{2}t\right)\end{aligned} \tag{27}$$

The constants C_{1r} and C_{2r} are real numbers, and their values depend on the initial conditions.

The complete expressions for the three-dimensional displacements *u*, *v* and *w* of the motion of a simply supported plate with immovable edges in an electromagnetic field can be obtained by substituting all the solutions obtained from the above judgments into the vibration mode function Eq.(22), where the solutions refer to all the possible values of $u_{m,n}$, $v_{m,n}$ and $w_{m,n}$.

3.3 Vibration Analysis of an Active Simply Supported Boundary Plate under a Magnetic Field.

Assuming a conductive thin plate is placed in a uniform magnetic field with magnetic induction intensity of (B_{0x},B_{0y},B_{0z}), and the boundary around the simply supported plate is movable edges, as shown in the figure:

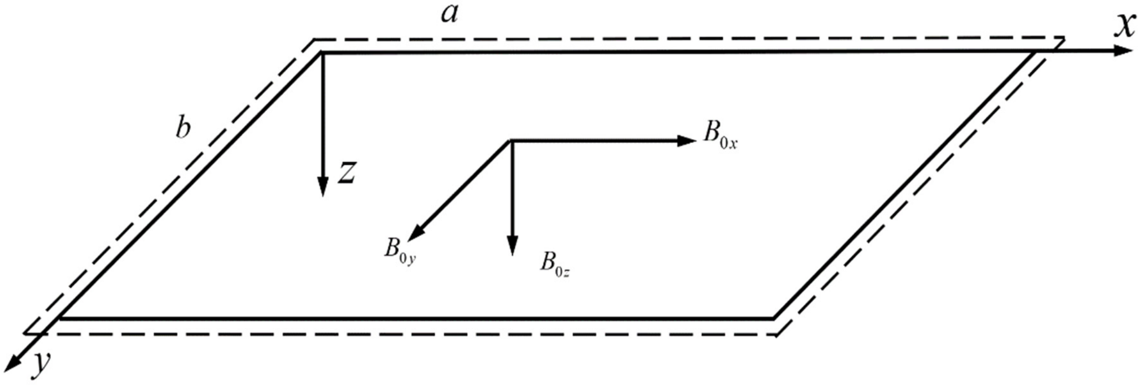

Figure 3. Simply supported plate with movable edges in a magnetic field.

For a simply supported plate with immovable edges, the following boundary conditions apply:

$$\begin{aligned} &at\ \ x=0,a:\ \ v=w=N_x=M_x=0 \\ &at\ \ y=0,b:\ \ u=w=N_y=M_y=0 \end{aligned} \tag{28}$$

To ensure that the approximate function satisfies the geometric boundary conditions for the above situation, the three-direction displacements u, v, and w can be expanded using temporal and shape function expressions of the following form:

$$\begin{aligned} u(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} u_{m,n}(t)\cos(2m\pi x/a)\sin(2n\pi y/b) \\ v(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} v_{m,n}(t)\sin(2m\pi x/a)\cos(2n\pi y/b) \\ w(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{m,n}(t)\sin[(2m-1)\pi x/a]\sin[(2n-1)\pi y/b] \end{aligned} \tag{29}$$

Substituting the above expressions into the original equation yields the residual function:

$$\begin{aligned} R_{uB} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{k_{21}\sin\frac{2m\pi x}{a}\cos\frac{2n\pi y}{b}+k_{22}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b}\right\} \\ R_{vB} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{k_{23}\cos\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b}+k_{24}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b}\right\} \\ R_{wB} &= \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{k_{25}\sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b}+k_{26}\sin\frac{2m\pi x}{a}\cos\frac{2n\pi y}{b}\right. \\ &\quad \left.+k_{27}\cos\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b}\right\} \end{aligned} \tag{30}$$

Where the coefficient is a function solely dependent on time and half-waves m and n, and independent of spatial coordinates.

$$\begin{aligned} k_{21} &= u_{m,n}\left[\left(\frac{2m\pi}{a}\right)^2+\frac{1-\upsilon}{2}\left(\frac{2n\pi}{b}\right)^2\right]+\frac{\partial u_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_z^2 \\ &\quad +\frac{\partial^2 u_{m,n}}{\partial t^2}\frac{\rho(1-\upsilon^2)}{E}+v_{m,n}\frac{2(1+\upsilon)mn\pi^2}{ab} \end{aligned}$$

$$k_{23}=v_{m,n}\left[\left(\frac{2n\pi}{b}\right)^2+\frac{1-\upsilon}{2}\left(\frac{m\pi}{a}\right)^2\right]+\frac{\partial v_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_z^2$$
$$+\frac{\partial^2 v_{m,n}}{\partial t^2}\frac{\rho(1-\upsilon^2)}{E}+u_{m,n}\frac{2(1+\upsilon)mn\pi^2}{ab}$$
$$k_{22}=-\frac{\partial w_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_xB_z,\quad k_{24}=-\frac{\partial w_{m,n}}{\partial t}\frac{\sigma(1-\upsilon^2)}{E}B_yB_z \tag{31}$$
$$+\frac{\partial w_{m,n}}{\partial t}\sigma hB_y^2+\frac{\partial w_{m,n}}{\partial t}\sigma\frac{h^3}{12}B_z^2\left[\frac{(2m-1)^2\pi^2}{a^2}+\frac{(2n-1)^2\pi^2}{b^2}\right]$$
$$+\frac{\partial^2 w_{m,n}}{\partial t^2}h\rho+\frac{\partial^2 w_{m,n}}{\partial t^2}\frac{h^3\rho}{12}\left[\frac{(2m-1)^2\pi^2}{a^2}+\frac{(2n-1)^2\pi^2}{b^2}\right]$$
$$k_{26}=-\frac{\partial u_{m,n}}{\partial t}\sigma hB_zB_x,\quad k_{27}=-\frac{\partial v_{m,n}}{\partial t}\sigma hB_zB_y.$$

According to the Galerkin method, multiplying Eq.(29) by the shape functions and integrating yields, with the requirement that the resulting average work done by the integrated displacements is zero:

$$\int_0^b\int_0^a R_{uB}\sin\left(\frac{2i\pi x}{a}\right)\cos\left(\frac{2j\pi y}{b}\right)dxdy=0$$
$$\int_0^b\int_0^a R_{vB}\cos\left(\frac{2i\pi x}{a}\right)\sin\left(\frac{2j\pi y}{b}\right)dxdy=0 \tag{32}$$
$$\int_0^b\int_0^a R_{wB}\sin\left[\frac{(2i-1)\pi x}{a}\right]\sin\left[\frac{(2j-1)\pi y}{b}\right]dxdy=0$$

Here,i=1,2,3...M and j=1,2,3...N.By computing the above equation, the vibration differential equation of the conductive thin plate in an electromagnetic field with a movable-edge simply-supported boundary can be obtained:

$$\frac{d^2u_{m,n}}{dt^2}+f_{uB}\frac{du_{m,n}}{\partial t}+g_{uB}u_{m,n}+h_{uB}v_{m,n}=0$$
$$\frac{d^2v_{m,n}}{dt^2}+f_{vB}\frac{dv_{m,n}}{dt}+g_{vB}v_{m,n}+h_{vB}u_{m,n}=0 \tag{33}$$
$$\frac{d^2w_{m,n}}{dt^2}+f_{wB}\frac{dw_{m,n}}{dt}+g_{wB}w_{m,n}=0$$

Where the coefficient is a function of the half-waves m and n and is expressed as follows:

$$f_{uB}=f_{vB}=\frac{\sigma}{\rho}B_z^2,\ g_{uB}=\left[\left(\frac{2m\pi}{a}\right)^2+\frac{1-\upsilon}{2}\left(\frac{2n\pi}{b}\right)^2\right]\frac{E}{\rho(1-\upsilon^2)},$$
$$h_{uB}=h_{vB}=\frac{2mn\pi^2E}{ab\rho(1-\upsilon)},\ g_{vB}=\left[\left(\frac{2n\pi}{b}\right)^2+\frac{1-\upsilon}{2}\left(\frac{2m\pi}{a}\right)^2\right]\frac{E}{\rho(1-\upsilon^2)},$$
$$f_{wB}=\frac{1}{\varpi_{wB}}\left(\sigma hB_x^2+\sigma hB_y^2+\sigma\frac{h^3}{12}B_z^2\left[\frac{(2m-1)\pi}{a}\right]^2+\sigma\frac{h^3}{12}B_z^2\left[\frac{(2n-1)\pi}{b}\right]^2\right), \tag{34}$$

$$g_{wB}=\frac{h^3E}{12\varpi_{wB}\left(1-\upsilon^2\right)}\left(\left[\frac{(2m-1)\pi}{a}\right]^2+\left[\frac{(2n-1)\pi}{b}\right]^2\right)^2,$$

$$\varpi_{wB}=h\rho+\frac{h^3\rho}{12}\left(\left[\frac{(2m-1)\pi}{a}\right]^2+\left[\frac{(2n-1)\pi}{b}\right]^2\right)$$

The above Eq. (33) is a second-order constant-coefficient homogeneous linear differential equation system, with the first and second equations coupled with respect to $u_{m,n}$(t) and $v_{m,n}$(t), which can be transformed into a fourth-order constant-coefficient homogeneous linear differential equation for solution. The third equation, which is about $w_{m,n}$, is an independent equation whose solution has already been discussed in Eq.(25)-(27). For the motion differential equation Eq.(33) it can be easily solved for an infinite number of terms with respect to half-waves *m* and *n*, so the vibration function Eq.(33) can be fully expressed by solving the equation system. Now we will discuss the fourth-order constant differential homogeneous equation, which is obtained by transforming the first and second equations into them each other:

$$\begin{aligned}&\frac{d^4u_{m,n}}{dt^4}+a_0\frac{d^3u_{m,n}}{dt^3}+b_0\frac{d^2u_{m,n}}{dt^2}+c_0\frac{du_{m,n}}{dt}+d_0u_{m,n}=0\\&\frac{d^4v_{m,n}}{dt^4}+a_0\frac{d^3v_{m,n}}{dt^3}+b_0\frac{d^2v_{m,n}}{dt^2}+c_0\frac{dv_{m,n}}{dt}+d_0v_{m,n}=0\end{aligned}\tag{35}$$

As the above two equations have the same form, their characteristic equations are:

$$s^4+a_0s^3+b_0s^2+c_0s+d_0=0\tag{36}$$

Here

$$\begin{aligned}a_0&=f_{uB}+f_{vB}\\b_0&=g_{uB}+g_{vB}+f_{uB}f_{vB}\\c_0&=g_{uB}f_{vB}+g_{vB}f_{uB}\\d_0&=g_{uB}g_{vB}-h_{vB}h_{uB}\end{aligned}\tag{37}$$

It is obvious that here a_0、b_0、c_0 and d_0 are related to the half-waves *m* and *n*.The above mentioned quartic characteristic equation can be solved using Ferrari's method, denoted as:

$$\begin{cases}\Re_1=b_0^2-3a_0c_0+12d_0\\\Re_2=2b_0^3-9a_0b_0c_0+27c_0^2+27a_0^2d_0-72b_0d_0\end{cases}\tag{38}$$

And

$$\Re=\frac{\sqrt[3]{2}\Re_1}{3\sqrt[3]{\Re_2+\sqrt{-4\Re_1^3+\Re_2^2}}}+\frac{\sqrt[3]{\Re_2+\sqrt{-4\Re_1^3+\Re_2^2}}}{3\sqrt[3]{2}}\tag{39}$$

Therefore, the solution of the Eq.(36) can be expressed as:

$$s_1=-\frac{a_0}{4}-\frac{1}{2}\sqrt{\frac{a_0^2}{4}-\frac{2b_0}{3}+\Re}-\frac{1}{2}\sqrt{\frac{a_0^2}{2}-\frac{4b_0}{3}-\Re-\frac{-a_0^3+4a_0b_0-8c_0}{4\sqrt{a_0^2-2b_0+\Re}}}\tag{40}$$

$$s_2=-\frac{a_0}{4}-\frac{1}{2}\sqrt{\frac{a_0^2}{4}-\frac{2b_0}{3}+\Re}+\frac{1}{2}\sqrt{\frac{a_0^2}{2}-\frac{4b_0}{3}-\Re-\frac{-a_0^3+4a_0b_0-8c_0}{4\sqrt{a_0^2-2b_0+\Re}}}\tag{41}$$

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$$s_3 = -\frac{a_0}{4} + \frac{1}{2}\sqrt{\frac{a_0^2}{4} - \frac{2b_0}{3} + \Re} - \frac{1}{2}\sqrt{\frac{a_0^2}{2} - \frac{4b_0}{3} - \Re - \frac{-a_0^3 + 4a_0b_0 - 8c_0}{4\sqrt{a_0^2 - 2b_0 + \Re}}} \tag{42}$$

$$s_4 = -\frac{a_0}{4} + \frac{1}{2}\sqrt{\frac{a_0^2}{4} - \frac{2b_0}{3} + \Re} + \frac{1}{2}\sqrt{\frac{a_0^2}{2} - \frac{4b_0}{3} - \Re - \frac{-a_0^3 + 4a_0b_0 - 8c_0}{4\sqrt{a_0^2 - 2b_0 + \Re}}} \tag{43}$$

Considering that the values of f_{uB}, f_{vB}, g_{uB}, $g_{vB}$are all positive, and $d_0=g_{uB}g_{vB}-h_{uB}h_{vB}>0$ on a large scale, it follows that the coefficient terms a_0, b_0, c_0 and d_0 of the characteristic equation Eq.(36) are all positive. Sun[9] has said that the Eq(36) does not have any real positive roots as solutions, and there are only three possible scenarios: four negative real roots, two negative real roots and one pair of complex conjugate roots, or two pairs of complex conjugate roots.

For the case of four negative real roots, the solution is an exponentially decaying solution. The system is over-damped and will not oscillate. The general solution can be written as:

$$\begin{aligned} u_{m,n} &= A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3 e^{s_3 t} + A_4 e^{s_4 t} \\ v_{m,n} &= B_1 e^{s_1 t} + B_2 e^{s_2 t} + B_3 e^{s_3 t} + B_4 e^{s_4 t} \end{aligned} \tag{44}$$

Here, s_1,s_2,s_3 and s_4 are the roots of the equation, their values are related to the half-waves m and n. The constants A_r and B_r (r=1,2,3,4) are real numbers, and their values depend on the initial conditions and the roots of Eq.(36).

For the case of two negative real roots and one pair of complex conjugate roots, the general solution can be written as follows:

$$\begin{aligned} u_{m,n} &= A_1 e^{s_1 t} + A_2 e^{s_2 t} + e^{o_1 t}(A_3 \cos\omega_1 t + A_4 \sin\omega_1 t) \\ v_{m,n} &= B_1 e^{s_1 t} + B_2 e^{s_2 t} + e^{o_1 t}(B_3 \cos\omega_1 t + B_4 \sin\omega_1 t) \end{aligned} \tag{45}$$

Here, s_1,s_2, and $o_1 \pm \omega_1$ are the roots of the equation, their values are related to the half-waves m and n. Since the first two terms of each equation in Eq.(45) are exponential decay and disappear as time increases, the system will only oscillate at the damped natural frequency. The constants A_r and B_r (r=1,2,3,4) are real numbers, and their values depend on the initial conditions and the roots of Eq.(36).

The third case has two pairs of complex conjugate roots, and the solution can be expressed in the following form:

$$\begin{aligned} u_{m,n} &= e^{o_1 t}(A_1 \cos\omega_1 t + A_2 \sin\omega_1 t) + e^{o_2 t}(A_3 \cos\omega_2 t + A_4 \sin\omega_2 t) \\ v_{m,n} &= e^{o_1 t}(B_1 \cos\omega_1 t + B_2 \sin\omega_1 t) + e^{o_2 t}(B_3 \cos\omega_2 t + B_4 \sin\omega_2 t) \end{aligned} \tag{46}$$

Here the solutions are given by $o_1 \pm \omega_1$ and $o_2 \pm \omega_2$, their values are related to the half-waves m and n. Then o_1 and o_2 are two damping exponential functions of the oscillation and ω_1 and ω_2 are the two natural frequencies of the damped oscillation. The constants A_r and B_r (r=1,2,3,4) are real numbers, and their values depend on the initial conditions and the roots of Eq.(36).

The three possible damping vibration cases discussed above depend on the values of the coefficients a_0, b_0, c_0 and d_0. For a single-degree-of-freedom system, the system's motion whether it vibrates or not depends on whether the damping coefficient is less than or greater than the defined critical damping coefficient. For a two-degree-of-freedom system, the use of an equivalent critical damping concept exists, but its expression is very complicated. According to the theory of equations to determine the properties of the roots, let us denote the roots as follows.

$$\hat{\Delta} = K^3 - 27P^2 \tag{47}$$

where

$$K = d_0 - \frac{a_0 c_0}{4} + \frac{b_0^2}{12}$$
$$P = \frac{b_0 d_0}{6} + \frac{a_0 b_0 c_0}{48} - \left(\frac{c_0}{4}\right)^2 - d_0\left(\frac{a_0}{4}\right)^2 - \left(\frac{b_0}{6}\right)^3 \tag{48}$$

The characteristics of system motion are represented by the properties of the roots. If $\hat{\Delta}<0$, the equation will have two negative real roots and a pair of complex conjugate roots, and the system will oscillate at a single natural damping frequency. This is commonly referred to as a degenerate system. If $\hat{\Delta}>0$ and satisfies the following relationship:

$$G = \frac{b_0}{6} - \left(\frac{a_0}{4}\right)^2 < 0$$
$$V = K - 12G^2 < 0 \tag{49}$$

Eq.(35) will have four negative real roots, indicating that the system has excessive damping and no vibration will occur. If the $\hat{\Delta}>0$ and Eq.(49) is not satisfied, then Eq.(35) will have two pairs of complex conjugate roots, indicating an underdamped system oscillating at two natural frequencies. When $\hat{\Delta}=0$, there will be at least two equal roots.

The complete expressions for the three-direction displacements u, v and w of the motion of a simply supported plate with movable edges in an electromagnetic field can be obtained by substituting all the solutions obtained from the above judgments into the vibration mode function(33), where the solutions refer to all the possible values of $u_{m,n}$, $v_{m,n}$ and $w_{m,n}$.

4. Conclusion

This article derives the motion differential equation of an elastic plate with three-direction displacements (u, v and w) in a magnetic field using D'Alembert's principle. Two different boundary conditions are set: the boundary for simply supported plate with immovable edges and the boundary for simply supported plate with movable edges. The corresponding vibration mode functions are given for each boundary condition, and the Galerkin method is used to separate the spatial and temporal coordinates. The original initial-boundary value problem is simplified into an initial value problem, obtaining the vibration equation with respect to time. It is found that the solution of the vibration equation is closely related to the half-waves m and n, and the solution forms of the vibration equation for the immovable-edges simply supported boundary and the movable-edges simply supported boundary are discussed.

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