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Study on Lamb Wave Complex Dispersion Relations with out-of-Plane Amplitude Information in Plate Structure

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Abstract

In order to solve the problem that the dispersion curve of the existing plate structure is lack of out-of-plane amplitude information, a semi-analytical method for calculating the complex dispersion curve of the plate structure is proposed in this paper. This method is based on Bloch-Floquent theory and finite element intrinsic frequency method. By normalizing the maximum displacement of each vibration mode, the dispersion curve in complex wave number domain that can reflect the relative magnitude of out-of-plane displacement is obtained. In addition, the dispersion relationship of complex wave number is verified by using finite element software to excite evanescent wave in single mode in frequency domain, and the significance of relative amplitude for the excitation and reception of Lamb wave in practical applications is pointed out.

Keywords

Non-destructive Testing; Finite Element; Plate Structure; Bloch-Floquent Theory; Finite Element Intrinsic Frequency Method; Lamb Wave; Complex Wave Number; Dispersion Characteristics; Evanescent Wave.

1. Introduction

finite element intrinsic frequency method The plate-like or quasi-plate-like structures find widespread applications in critical national infrastructures and high-end equipment, such as aviation/aerospace, high-speed railways, ships, and wind power systems. These structures are susceptible to various environmental loads during their long-term operation, leading to damages such as corrosion, fatigue, and cracks, which accumulate and eventually cause equipment failure and even catastrophic accidents. To prevent safety incidents, non-destructive testing or structural health monitoring is necessary for plate structures. Extensive research has shown that Lamb waves, with their long propagation distance and high sensitivity to damages, have great potential in NDT&SHM applications for plate-like structures [1-5].Effective acquisition of Lamb wave dispersion curves, which exhibit multi-mode and dispersion characteristics, is the basis for mode selection, excitation, and separation of Lamb wave modes in plate structures. Researchers have conducted extensive studies on Lamb wave dispersion relations [6-9]. Currently, the Lamb wave dispersion curves obtained by existing methods mainly show the relationship between the wavenumber/phase velocity/group velocity and frequency of modes, based on which fundamental information such as excitation conditions of specific modes of Lamb waves can be derived. However, the relative off-plane displacement information of different Lamb wave modes in plate structures cannot be effectively reflected. To address this issue, this study further developed the method for solving the complex dispersion relation of Lamb wave modes with relative off-plane amplitude in plate structures based on the previous research on Lamb wave complex frequency dispersion relations using finite element intrinsic frequency method and Bloch-Floquet theory [10], and validated the effectiveness of the dispersion curves.

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2. Theory and Method

2.1 Bloch-Floquent Theory and Finite Element Intrinsic Frequency Method

Bloch-Floquent theory points out that when the wave propagates from one unit to another unit, it does not depend on the position of periodic lattice unit, and the displacement of two adjacent units in the lattice can be expressed as [11]:

$$\boldsymbol{u}(\boldsymbol{r}+\boldsymbol{d},\boldsymbol{k})=\boldsymbol{u}(\boldsymbol{r},\boldsymbol{k})\mathrm{e}^{\mathrm{i}\left(\boldsymbol{k}^{T}d\right)} \quad (1)$$

Where: $\boldsymbol{r}=(x,y,z)$ is the position vector; $\boldsymbol{u}=(u_x,u_y,u_z)$ is displacement vector; $\boldsymbol{d}=(d_x,d_y,d_z)$ is a lattice vector; $\boldsymbol{k}=(k_x,k_y,k_z)$ is wave vector. The real part of wave vector $\boldsymbol{k}$ reflects the phase velocity, while the imaginary part reflects the attenuation of wave propagation. Lamb waves with real wave vectors are called propagation Lamb waves, while Lamb waves with imaginary parts in wave vectors are called evanescent Lamb waves. According to Bloch-Floquent theory, the wave propagation in the whole periodic structure can be described by considering a basic element, which greatly reduces the computational load. Assuming that the medium has no loss, the motion equation of periodic structure can be expressed in the form of matrix

$$\left(\boldsymbol{K}-\omega^{2}\boldsymbol{M}\right)\boldsymbol{u}=\boldsymbol{f}_{i}+\boldsymbol{f}_{e} \quad (2)$$

Where: $\boldsymbol{K}$ is the dynamic stiffness matrix of the periodic element, $\boldsymbol{M}$ is the mass matrix of the periodic element, and ω is the angular frequency. The combination of internal force $\boldsymbol{f_i}$ and external force $\boldsymbol{f_e}$ is the force vector acting on the element. Use equation (1) to constrain the displacement of the two boundaries of the periodic element and make them interrelated. The constraint relation is described by lattice vector $\boldsymbol{d}$, which will help to reduce the quantity of motion equation. It can be proved that the internal force can be eliminated by applying Bloch-Floquent theory [12]. The simplified equation of motion is

$$\left(\boldsymbol{K}^{'}-\omega^{2}\boldsymbol{M}^{'}\right)\boldsymbol{u}^{'}=\boldsymbol{f}_{e}^{'} \quad (3)$$

Where: $\boldsymbol{K'},\boldsymbol{M'},\boldsymbol{u}^{'},\boldsymbol{f}'_{\boldsymbol{e}}$ they are simplified stiffness matrix, mass matrix, displacement vector and external force vector. The eigenvalue problem of periodic structure can be solved by setting the external force as zero

$$\left(\boldsymbol{K}^{'}-\omega^{2}\boldsymbol{M}^{'}\right)\boldsymbol{u}^{'}=0 \quad (4)$$

Use equation (4) to solve a series of wave numbers $\mathrm{k_i}$, and the result is a set of intrinsic frequency and intrinsic modes. The real part of phase velocity of each intrinsic mode is $c_{i,j}=2\pi f_{i,j}/Re(k_i)$

2.2 IBZ Dispersion Curve

If the lattice constant of the structure in the direction is a, the reciprocal lattice vector of the lattice is

$$K=\frac{2\pi}{a} \quad (5)$$

Equation (5) is the period of the wave number domain, then any set of solutions of equation (4) $[\mathrm{f},\mathrm{k_x}]$ can be expressed as the wave number in the period of other wave number domains

$$[f,k_x+mK] \quad (6)$$

In formula (6), Bloch wave vector k_x is set in Bloch-Floquent periodic boundary condition, $m$is an arbitrary integer. When the wavenumber domain is limited in the irreducible Brillouin zone (IBZ)$[f,K/2]$, the dispersion relationship of the periodic structure can be calculated.

2.3 Normalization of Maximum Displacement of Intrinsic Mode

In order to evaluate the relative size of each component of each vibration mode, it is necessary to normalize the maximum displacement in the vibration mode. The vibration component of the particle is:

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$$u_i\left(t\right)=A_i\cdot e^{\mathrm{i}\theta_i}\cdot e^{\mathrm{i}\omega t} \tag{7}$$

Where,A_i is the amplitude in each direction, $\theta_i$is the initial phase of vibration, and $e^{i\omega t}$ is the time factor. The relationship between particle displacement$disp(t)$ and each component is:

$$\begin{aligned} disp\left(t\right)^2 &= \sum_{i=1}^{3}\mathrm{Re}\left(u_i\left(t\right)\right)^2 \\ &= \frac{1}{2}\sum_{i=1}^{3}A_i^2\cos\left(2\omega t+2\theta_i\right)+\frac{1}{2}\sum_{i=1}^{3}A_i^2 \end{aligned} \tag{8}$$

Obviously, when $disp(t)$ takes the maximum value, the first item of equation (8) must take the maximum value, making it U_{max}:

$$U_{\max}=\left|\sum_{i=1}^{3}A_i^2\cdot \mathrm{e}^{\mathrm{i}\left(2\omega t+2\theta_i\right)}\right| \tag{9}$$

Therefore, the maximum displacement of any particle $disp_{max}$ can be expressed as:

$$disp_{\max}=\sqrt{\frac{1}{2}\sum_{i=1}^{3}U_i+\frac{1}{2}U_{\max}} \tag{10}$$

The maximum displacement D_{max} in the vibration mode is:

$$D_{\max}=\mathrm{Max}_V\left(disp_{\max}\right) \tag{11}$$

Divide all displacements in each mode by the maximum displacement D_{max} of the intrinsic mode, and then normalize the maximum displacement of the mode.

2.4 Extended IBZ Dispersion Curve

After extending the displacement of a single lattice periodic mode to N periods, the expanded displacement field $u(x+na)$ meets the following requirements:

$$\mathbf{u}(x+na)=\mathbf{u}_{IBZ}(x)\mathrm{e}^{\mathrm{i}\left(nk_x a\right)} \quad n=1\ldots\mathrm{N} \tag{12}$$

Where,$\mathrm{u_{IBZ}}$is the displacement in the initially calculated space where x is. Furthermore, the high precision wavenumber spectrum can be obtained by fast Fourier transform on $\boldsymbol{u}(x+na)$, and the position of the wavenumber peak corresponds to the actual wavenumber k in the structure, so as to obtain the dispersion curve of the structure.

It should be noted that when equation (12) is used to expand the displacement of plate structure, especially the displacement of evanescent wave mode, it is necessary to first eliminate the attenuation of wave propagation. The exponential attenuation term in the equation is eliminated, and the displacement after elimination of attenuation can be expressed as:

$$\boldsymbol{u}_i^{'}(x)=\frac{\boldsymbol{u_i}(x)}{\mathrm{e}^{-\mathrm{i}\,\mathrm{Im}\left(k_x\right)\cdot x}} \quad i=1,2,3 \tag{13}$$

Where $\boldsymbol{u}_i'(x)$ is the corrected displacement component of each particle, then the correction of evanescent wave expansion formula (12) is:

$$\boldsymbol{u}(x+na)=\boldsymbol{u}^{'}(x)\mathrm{e}^{\mathrm{i}\left[n\mathrm{Re}\left(k_x\right)a\right]} \quad n=1\ldots N \tag{14}$$

Fourier transform the expanded displacement field:

$$Amp_l\left(k_i,\omega\right)=2\pi\sum_{j=1}^{N}\boldsymbol{u_l}\left(x_j,\omega\right)e^{-\boldsymbol{i}\frac{1}{N}k_{i-1}t_{i-1}} \tag{15}$$

Where $Amp_l(k_i,\omega)$ is the amplitude obtained at the coordinate k_i in the wavenumber domain, and $\boldsymbol{u_l}(x_j,\omega)$ is the displacement at the position x_j in the discrete space. k_i and ω correspond to the dispersion relationship of the plate structure. For evanescent waves, the real part of the wave number is obtained by formula (14). The imaginary part of the wave number will not be aliased, so it is still $Im(k)$. The period is extended to improve the accuracy of wavenumber spectrum. Finally, the wave

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number peak (the imaginary part of evanescent wave needs to be added) and its amplitude $Amp_l(k_i,\omega)$ obtained in the wave number spectrum are recorded to obtain the true dispersion relationship of the plate structure. Since the calculated intrinsic modes are normalized by the maximum displacement, the amplitudes of different modes also have certain comparative significance. This paper mainly considers the displacement in the out-of-plane direction, which enables this amplitude to describe the relative magnitude of the out-of-plane displacement of the intrinsic mode.

3. Results and Discussion

The finite element software is used for modeling. The infinite aluminum plate can be regarded as a periodic structure composed of infinitely small periodic elements. Its SH mode is decoupled from the Lamb wave mode [13]. The SH wave does not produce out-of-plane displacement. Therefore, to solve the Lamb wave dispersion relationship of the infinite aluminum plate containing the relative amplitude of out-of-plane displacement only needs to model the periodic element in two-dimensional, as shown in Figure 1. The density of the model plate is 2700 kg/m3, Young's modulus is 6.85×10^{10} Pa, Poisson's ratio is 0.34, the thickness of the model plate is 1 mm, and the lattice constant a is 7.5 mm.

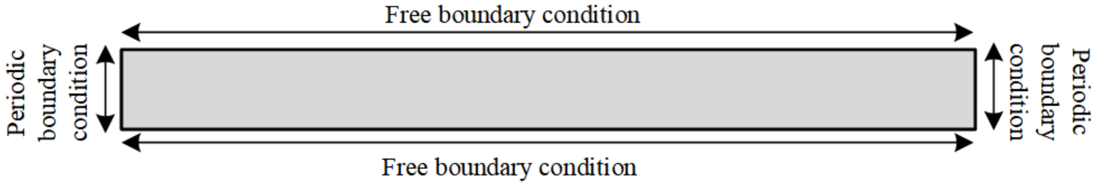

Fig 1. Aluminum plate finite element two-dimensional simulation model.

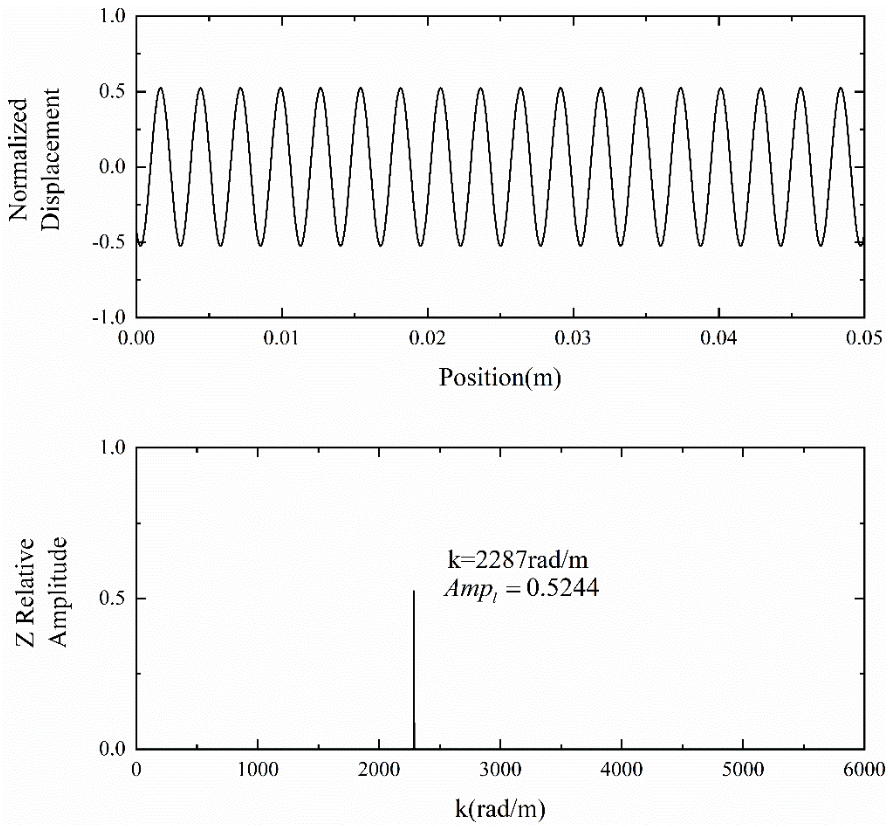

Fig 2. (a) Expanded N = 1000 cycles of off-plane displacement; (b) wave number spectrum after displacement space extension

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In Figure 1, the upper and lower boundaries of the model are set as free boundaries; The left and right boundaries are periodically connected boundaries, which are set as Fluent periodic boundary conditions. Calculate the eigenfrequencies of different Bloch wave vectors, and obtain the aliased dispersion curve in IBZ. Take the vibration mode corresponding to the eigenfrequency f=1.7841×10^6 Hz as an example, whose Bloch wave vector is 226.19rad/s. According to equation (11), normalize the maximum displacement displacement value of the vibration mode, and sample the out-of-surface displacement on the upper surface, and use equation (16) to expand the out-of-surface displacement, with the number of expansion cycles N of 1000. Some results are shown in Figure 2 (a). It is a continuous sine wave shape, which can expand the total sampling space range, far greater than the simulation space shown in Figure 1, and can obtain high-resolution beam spectrum. Figure 2 (b) shows the wave number spectrum containing normalized out-of-plane amplitude information obtained by the spatial Fourier transform of the waveform in Figure 2 (a), which shows that the actual wave number corresponding to the above vibration mode is 2287rad/m, and the relative amplitude of out-of-plane displacement is 0.5244

Based on the above method, the real wave number in the extended space is obtained for all intrinsic modes in the IBZ, and the real frequency-wave number dispersion curve is obtained, as shown in Figure 3. In addition, the relative amplitude of the out-of-plane displacement of each mode is recorded as $Amp_l(k_i, \omega)$. The magnitude of the amplitude is reflected on the dispersion curve by the magnitude of the symbol on the curve. That is, in Figure 3, the marker point indicates that Lamb wave mode exists under the condition of this frequency wave number. The larger the mark point of the mode, the greater the relative amplitude of out-of-plane displacement generated by the mode.

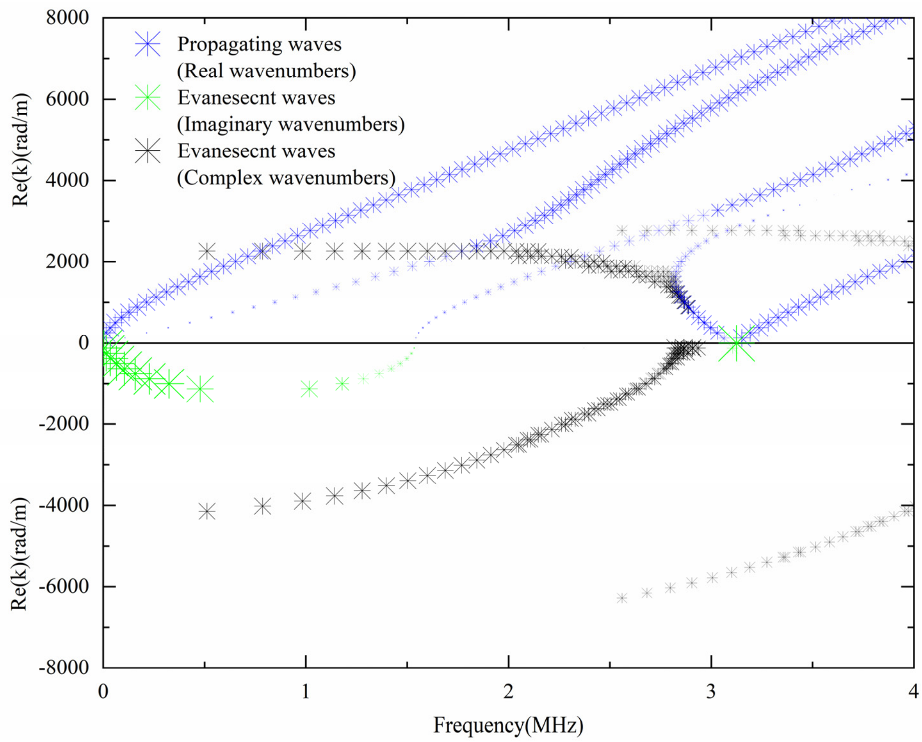

Fig 3. Dispersion curve of aluminum plate with amplitude information

4. Result Verification

The propagation wave mode in the real number domain is widely studied in the conventional research and application, and its propagation properties are well known, while the evanescent wave mode with imaginary part of the wave number is rarely studied. Therefore, we will conduct a more complete verification of the obtained complex dispersion relationship, the evanescent wave component with complex number.

By using the method of Yan et al [14]. After the single-mode evanescent Lamb wave is excited in the frequency domain, the wavenumber of excited single-mode is extracted and compared with the wavenumber obtained by our method to verify the correctness and accuracy of the complex dispersion relationship studied in this paper.

Figure 4 (a) shows the excitation of single-mode evanescent wave mode in 1mm thick aluminum plate structure at 3.6486e6 Hz (as shown in Figure 4). It can be seen that the vibration only exists near the excitation end and cannot propagate to the far field. It is a typical evanescent wave. According to the calculation results in Figure 3, the wave number of this mode is 2638.9-i4630 rad/m. To determine whether the excited evanescent wave number is consistent with the single-mode wave number excited in Figure 4. The wave number needs to be measured. However, the wave number of evanescent wave has both real and imaginary parts, which need to be measured. The out-of-plane displacement of the upper surface of the plate is sampled. Since the real part of the wave number represents the phase change of the wave propagation, the attenuation of the wave has no effect on it. The real part of the wave number is easy to obtain. Just measure the distance between two vibration zeros, which is half a wavelength, to obtain the actual real part of the wave number. The imaginary part of the wave number represents the attenuation of the wave propagation. The attenuation coefficient of the evanescent wave amplitude is shown in Figure 4 (b), which is related to, and the slope is, so the actual imaginary part can be obtained. To sum up, the actual wave number of single-mode evanescent wave excited at 3.6486e6 Hz is 2640.59-i4632.12 rad/m. Comparing the wave number, it is found that the error of the real part and the imaginary part is not more than 0.7%, and the error of the imaginary part is less than 0.01%. We have also verified other evanescent wave models, and the error of the results is not more than 1%. Therefore, we can think that the calculation results in Figure 3 are basically consistent with the simulation results, with a certain degree of reliability.At the same time, we also compared the complex dispersion curve calculated by Takiuti et al. [15] to verify the correctness of the method in this paper. The reliability of this method is also verified

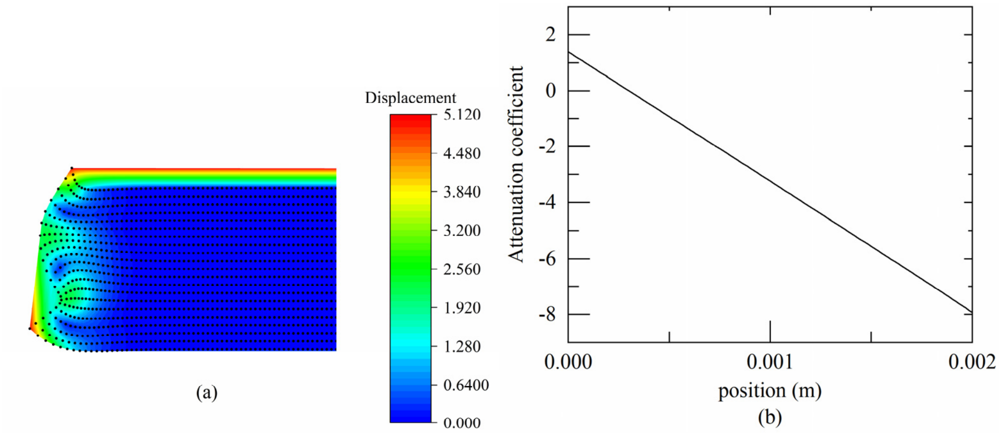

Fig 4. (a) Frequency-domain single-mode excitation evanescent wave; (b) wave amplitude decay coefficient at the near field.

5. Conclusion

Based on Bloch-Floquent theory and finite element intrinsic frequency method, a method of complex dispersion relation with out-of-plane amplitude information is studied in this paper. The wavenumber domain is expanded from pure real number and pure imaginary number domain to complete complex wavenumber domain. By analyzing the displacement distribution of vibration modes, the relative strength of out-of-plane displacement of each vibration mode at different frequencies is determined. The accuracy of the calculated results of the complex dispersion relation is successfully verified by using the excited single-mode evanescent wave method. The relative out-of-plane amplitude of each vibration mode can also be used to analyze the out-of-plane displacement of each vibration mode when the structure is subjected to normal excitation, which has guiding significance for how to select the normal excitation frequency of the structure in practical application. This method for calculating the complex dispersion relationship of plate structure with amplitude can quickly and accurately reflect the physical characteristics of wave propagation in the structure, and provide theoretical basis for frequency selection and vibration analysis in ultrasonic Lamb wave nondestructive testing. This method can be applied to traditional plate structure and some more complex periodic structures, and has certain scientific significance and research value and practical application value.

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