ISSN: 2414-1895 DOI: 10.6919/ICJE.202304_9(4).0023

Radon Transform based Reflection Wave Extraction for Advanced Detection

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Abstract

Seismic advance detection is one of the applications of seismic wave detection technology. This technology mainly analyzes and predicts the reflected waves of different interfaces in front of the tunnel. In practical applications, the acquired seismic data are often full of a large number of interference waves, such as direct wave, surface wave, etc. Therefore, the processing of advanced seismic data needs to take multiple steps to extract effective waves and suppress interference waves. This paper focuses on the reflection wave extraction methods in these steps, tests and analyzes the Radon transform algorithm used in this paper through theoretical data, and carries out the reflection wave extraction operation on the research data to complete the research on the reflection wave extraction method.

Keywords

Seismic Advance Detection; Reflected Wave Extraction; Radon Transform.

1. Introduction

The main problems in seismic exploration include surface wave, multiple, random noise and multiple attenuation. These problems need to be well dealt with in seismic data processing, otherwise they will interfere with effective information or even completely cover. Therefore, the effectiveness of processing seismic data is very important. Whether it is suppressing noise and multiples, or extracting effective waves, it is of great value to the practical application and economic benefits of seismic exploration.

In order to solve these problems, Radon transform has been introduced into many fields such as geophysics, and has been greatly developed among domestic scholars. Niu Binhua, Liu Xiwu, Xiong Deng, etc. proposed high-resolution parabola Radon transform and hybrid domain high-resolution Radon transform. At the same time, the conjugate gradient algorithm and sparse constraints are also given to improve the efficiency of Radon transform. In recent years, Li Zhina and Shi Ying have introduced λ- High-resolution Radon transform in f domain is used to suppress multiples and reconstruct seismic data. Although the research direction of Radon transform is mainly divided into theoretical research and application research, Radon transform still has a very broad prospect in the field of geophysics. This paper will start from theory and combine the application of Radon transform in seismic data processing, mainly in multiple suppression and reflection wave extraction, and realize the transformation processing of seismic data through MATLAB programming language to verify and evaluate the algorithm.

2. Basic Principle of Radon Transform

2.1 Mathematical Theory of Radon Transformation

The classical Radon transformation proposed by J. Radon in 1917 is essentially a linear integral transformation of a multivariate function along a specific line in a certain direction. In the two-

ISSN: 2414-1895 DOI: 10.6919/ICJE.202304_9(4).0023

dimensional plane, there are numerous straight lines in a certain direction, so you can get countless integrals and countless one-dimensional projections. Therefore, in the two-dimensional plane, the two-dimensional linear Radon transformation of the function is the integral obtained by the one-dimensional projection of the entire plane and all directions of the function.

The basic idea of two-dimensional Radon transform is that the experimenter can integrate the function f (x, y) according to the given path [1-3]. If the integration path is set as curve L:y = φ(x), according to the nature of the first type of curve integration, the Radon transformation form of the function f (x, y) along the curve L is:

$$\mathrm{R}\big(\mathrm{f(x,y)}\big) = \int_L f(x,y)dL = \int_{-\infty}^{\infty} f(x,\varphi(x))\sqrt{1+[\varphi'(x)]^2}dx$$

When φ(x) =τ + px is, the integration path is a straight line, which also becomes the straight Radon transformation mentioned above.

If φ(x is a nonlinear function and the integral path is a nonlinear curve, then the generalized Radon transform of the function f (x, y) in the above formula becomes a nonlinear Radon transform. Here are two mathematical formulas for nonlinear Radon transform:

1. When $\varphi(x) = \tau + px^2$,

$$\mathrm{R}\big(\mathrm{f(x,y)}\big) = \int_L f(x,y)dL = \int_{-\infty}^{\infty} f\big(x,\varphi(x)\big)\sqrt{1+[\varphi'(x)]^2}dx$$
$$= \int_{-\infty}^{\infty} f(x,\tau+px^2)\sqrt{1+(2qx)^2}dx$$

This form is called parabolic Radon transformation.

2.When $\varphi(x) = \sqrt{\tau^2+qx^2}$,

$$\mathrm{R}\big(\mathrm{f(x,y)}\big) = \int_L f(x,y)dL = \int_{-\infty}^{\infty} f\big(x,\varphi(x)\big)\sqrt{1+[\varphi'(x)]^2}dx$$
$$= \int_{-\infty}^{\infty} f\big(x,\sqrt{\tau^2+qx^2}\big)\sqrt{1+\frac{q^2x^2}{\tau^2+qx^2}}dx$$

This form is called hyperbolic Radon transformation.

2.2 Basic Principle of Radon Transform in Time Domain

The basic principle of Radon transformation is to integrate a function on a given path, which is essentially an integral transformation [4-6]. In the processing of seismic data, the specific application idea of Radon transform in noise reduction is as follows: Now transform the original data into a new domain (that is, the domain where noise and effective signal can be easily distinguished) through Radon transform. After processing the seismic wave data in the new domain, transform the seismic data into the original domain through anti-Radon transform, and then complete the processing of noise reduction and reflection wave extraction of seismic wave data. The general form of Radon transform is:

Positive transformation:

$$\mathrm{u(\tau,q)} = \int_{-\infty}^{\infty} d(t=\tau+q\varphi(x),x)dx$$

Inverse transformation:

$$\mathrm{d'(t,x)} = \int_{-\infty}^{\infty} u(\tau=t-q\varphi(x),q)dq$$

In the above formula, d(t,x) and u(τ,q)They are the original seismic data and the corresponding data in the new domain after Radon transform, d'(t,x) is the data after anti-Radon transform, x is the spatial variable (offset), t is the two-way travel time, τ is the intercept time, and t =τ+qφ(x) it means the integration path given by Radon transform, and the key is qφ(x The expression of determines whether the Radon transformation is a linear transformation, a parabolic transformation or a hyperbolic transformation, where q represents curvature, and when q=1/v, it is the slope. The discrete form of the above formula is as follows:

Positive transformation:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202304_9(4).0023

$$\mathrm{u}(\tau, q_j) = \sum_{n=1}^{Nx} d(t = \tau + q_j\varphi(x_n), x_n)$$

Inverse transformation:

$\mathrm{d}'(\mathrm{t}, x_n) = \sum_{j=1}^{Nq} d(\tau = t - q_j\varphi(x_n), q_j)$ Where Nx represents the number of seismic channels in the t-x domain, and Nq represents the Radon domain (i.e τ-p field). The matrix form of Radon transform in discrete case is as follows:

Positive transformation:

$$\mathrm{u} = L^H \mathrm{d}$$

Inverse transformation:

$$\mathrm{d}' = \mathrm{Lu}$$

In the above formula, d, u and d' respectively correspond to the data d (t, x) in the original data (i.e., t-x domain) and the seismic data u in the Radon domain (τ,q) And inverse transform back to the seismic data d '(t, x) in the t-x domain, the dimension of vector d is nt*nx, and the dimension of vector u is nτ*nq and L are transformation operators and L^H operators respectively.

2.3 Radon Transform in Frequency Domain

(1) The form and implementation of linear Radon transform in frequency domain

Although the time domain Radon transform has little change compared with the classical mathematical form Radon transform and its principle is relatively easy to understand, in practical applications, the time domain Radon transform takes a lot of time and is inefficient, so in order to improve its calculation efficiency and reduce the calculation time, it can be converted to the frequency domain for Radon transform[7].

The frequency domain Radon transform is to transform the time-domain Radon transform with time-invariant characteristics into the frequency domain through Fourier transform for calculation, and only the time-invariant Radon transform (that is, linear Radon transform, parabolic Radon transform, time-invariant hyperbolic Radon transform, polynomial Radon transform mentioned in this article) can be transformed into the frequency domain Radon transform.

Perform FFT on the general formula of discrete form of Radon transform in time domain:

$$\mathrm{U}(\omega, q_j) = \sum_{n=1}^{Nx} D(\omega, x_n) e^{i\omega q_j \varphi(x_n)}$$

$$\mathrm{D}'(\omega, x_n) = \sum_{j=1}^{Nq} U(\omega, q_j) e^{-i\omega q_j \varphi(x_n)}$$

In the above formula, D, D' and U represent the frequency domain data of d, d' and u respectively, ω Indicates frequency. The above discrete form is divided into separate frequency components, which can be written as the following matrix form:

$$\mathrm{M}(\omega, q_j) = \sum_{n=1}^{N} D(\omega, x_n) e^{i\omega q_j x_n}$$

$$\mathrm{D}'(\omega, x_n) = \sum_{j=1}^{J} M(\omega, q_j) e^{-i\omega q_j x_n}$$

The matrix form of discrete Radon transform in frequency domain is given:

$$\mathrm{m}(p_j) = L^H d(x_n)$$

$$\mathrm{d}(x_n) = L m(p_j)$$

In the above formula:

$$\mathrm{L} = e^{-i\omega q_j x_n}, i = 1,2,\dots,N; j = 1,2,\dots,J$$
$$L^H = e^{i\omega q_j x_n}, i = 1,2,\dots,N; j = 1,2,\dots,J$$

Use the least square method to make $\mathrm{J} = ||\mathrm{d} - \mathrm{Lm}||^2$minimum to obtain m. At the same time, take the derivative of $\mathrm{J} = ||\mathrm{d} - \mathrm{Lm}||^2$ and get the following two cases:

When $\mathrm{N} \geq \mathrm{J}$, the result of positive transformation of linear Radon transformation is:

$$\mathrm{m} = (L^H \mathrm{L})^{-1} L^H d$$

When A=B, the result is:

$$\mathrm{m} = L^H (LL^H)^{-1} d$$

In the actual calculation, and may be singular, so a damping factor needs to be added in the calculation, so the above formula is rewritten as:

$$\mathrm{m} = (L^H \mathrm{L} + \mathrm{\mu I})^{-1} L^H d$$
$$\mathrm{m} = L^H (LL^H + \mu I)^{-1} d$$

Where,μ is the damping factor, which $L^H\mathrm{L}$ is generally 0.01 of the main diagonal value

The high-resolution implementation of linear Radon transform is discussed below.

The damping factor of the least square regularization method is fixed, so in the actual transformation, if the size of the damping factor is not set properly, there will be problems: when the damping factor is large enough, the result of the frequency domain transformation is very close to the result of the time domain transformation; When the damping factor is very small, the energy will become divergent in the Radon domain. To solve this problem, Mauricio D proposed an improved method, that is, by improving the regularization matrix, making the damping factor change with p, and giving an appropriate value of the damping factor to achieve the purpose of improving the resolution.

Replace the above damping factorμI with $\mathrm{W}W^H$, so the formula becomes:

$$(\mathrm{R} + W^H W)m = L^H d$$

Where A=B, W is a diagonal matrix, and

$$\mathrm{W(l,m)} = w_l \delta_{l,m}$$

The diagonal element of matrix W is related to m, and W is determined by the result of each m iteration:

$$\mathrm{Q} = \mathrm{R} + W^H W$$

$$Q_{j,m} = \sum_{n=1}^{N} e^{-2i\pi f (p_j - p_m) x_n^2} + w_j^2 \delta_{j,m} \quad j, m = 1, \dots, M$$

The weight matrix W can be determined by the speed distribution range. The solution vector matrix within the speed range is given a smaller weight, and the solution vector matrix outside the range is given a larger weight.

(2) The form and implementation of parabolic Radon transform in frequency domain

In terms of solution method, the linear Radon transform in frequency domain is very similar to the parabolic Radon transform in frequency domain, but their integration paths are slightly different, and even their solution formulas are "the same". However, in order to make a difference, the implementation methods of parabolic Radon transform in frequency domain will be introduced in detail below [8].

Due to the low accuracy of linear Radon transform in practical use, the hyperbolic Radon transform does not have the characteristic of sometimes invariance, and the time invariance hyperbolic Radon transform is too complex in comparison, so the parabolic Radon transform in frequency domain has always been the focus of Radon transform research. It has the characteristics of high transformation accuracy, low complexity and short calculation time.

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The form and basic principle of parabolic Radon transform in frequency domain are introduced below [9-11]. The discrete form of parabolic Radon forward and inverse transformation in frequency domain is:

$$\mathrm{M}(\omega, \mathrm{q}_j) = \sum_{k=1}^{N} D(\omega, x_k) e^{-j\omega q x^2}$$

$$\mathrm{D}(\omega, \mathrm{x}_k) = \sum_{j=1}^{M} M(\omega, q_j) e^{-j\omega q x^2}$$

The matrix form corresponding to the above formula is:

$$\mathrm{M} = L^H D$$

$$\mathrm{D} = LM$$

Among them, L and L^H are inverse transformation operators and inverse transformation operators respectively, which together constitute a pair of conjugate operators. The matrix form of L and L^H is:

$$L_{m*n} = \begin{pmatrix} e^{-j\omega q_1 x_1^2} & \cdots & e^{-j\omega q_1 x_n^2} \\ \vdots & \ddots & \vdots \\ e^{-j\omega q_m x_1^2} & \cdots & e^{-j\omega q_m x_n^2} \end{pmatrix}$$

$$L_{n*m}^H = \begin{pmatrix} e^{j\omega q_1 x_1^2} & \cdots & e^{-j\omega q_m x_1^2} \\ \vdots & \ddots & \vdots \\ e^{j\omega q_1 x_n^2} & \cdots & e^{-j\omega q_m x_n^2} \end{pmatrix}$$

Where m is the number of seismic data channels, and n is the number of data requiring Radon transform. It can be seen that L and are not square, so there is no inverse matrix of L and, so it is not directly calculated.

Before introducing the high-resolution parabolic Radon transform, first introduce the realization method of the frequency domain Radon transform, namely the least square method. First, the formula of the inverse parabolic Radon transformation in matrix form mentioned in the previous section is given:

$$\mathrm{d} = Lm$$

L is a matrix, and the general formula of its elements is:$L_{k*j} = e^{-j\omega q_j x_k^2}$. Use the least square method for the above formula to minimize $\mathrm{J} = ||\mathrm{d} - \mathrm{Lm}||^2$and obtain m.

Take the derivative of the objective function J to m and make its derivative 0. You can also get two cases:

When $\mathrm{N} \geq \mathrm{M}$, the positive transformation result of parabolic Radon transformation is:

$$\mathrm{m} = (L^H \mathrm{L})^{-1} L^H d$$

When $\mathrm{N} \leq \mathrm{M}$, the result is:

$$\mathrm{m} = L^H (LL^H)^{-1} d$$

At the same time, a damping factor should be added to the solution, so that the equations in the above two cases become the following form:

$$\mathrm{m} = (L^H \mathrm{L} + \mu \mathrm{I})^{-1} L^H d$$

$$\mathrm{m} = L^H (LL^H + \mu I)^{-1} d$$

In the above formula, μ Is the damping factor, and the value is generally 0.1~1.

The high-resolution implementation of parabolic Radon transform in frequency domain will be discussed below. The least square method is often used in solving parabolic Radon transformation,

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DOI: 10.6919/ICJE.202304_9(4).0023

which can improve the resolution of transformation results and eliminate spatial convolution, but there are still many problems that cannot meet our requirements, which is the birth of the high-resolution Radon transformation method.

The realization method of high-resolution linear Radon transform is introduced in the front. The idea of the least square parabolic Radon transform is consistent with that mentioned in it, and it is precisely because of this, because the regularization matrix is constant, the resolution of the transform is poor. The method to be introduced in this paper is the method to improve the regularization matrix proposed by Maurucio D, so that the damping factor changes with the change of q, that is, the sparse solution of the least squares parabolic Radon transformation is given and based on this, the resolution after transformation is improved.

Unlike the previous section, in the parabolic Radon transformation at high resolution, m is obtained from the following equation:

$$(\mathrm{R} + W^H W)m = L^H d$$

In the above formula, W is a diagonal matrix, and the general formula of elements is:

$$\mathrm{W}(\mathrm{l},\mathrm{m}) = w_l \delta_{l,m}$$

It can be seen that its elements are related to m. This shows that there is an iterative process, and W is determined by the results of the previous iteration. This method improves the resolution by restricting the weighted ordinal number through the W matrix: when q_j is located in the reflection area, select a smaller weight value, otherwise select a larger weight value, which can achieve the purpose of improving the resolution.

3. Radon Transform Algorithm and Reflected Wave Extraction Principle

3.1 Reflected Wave Extraction Process

According to the results of Radon changes, the steps of using Radon changes to extract reflected waves can be divided into:

(1) Firstly, the original seismic data is transformed by Radon transform, and its image is displayed in Radon domain.

(2) Based on the principle of Radon transform, according to different types of Radon transform, the seismic data in the Radon domain is first identified by resolution, and then the reflected wave is extracted or removed from the data.

Carry out anti-Radon transform on seismic data, transform the data in Radon domain into t-x domain, observe the extraction and separation effect, and confirm whether the reflection wave extraction of seismic data is completed.

3.2 Implementation Algorithm of Radon Transform in MATLAB

Next, the Radon transform MATLAB algorithm used in this paper is given. This algorithm is a linear Radon transform in the frequency domain. First, the forward transform algorithm is given [1]:

```
function m = Forradon(b,dt,x,q,flag,noise)
% B is seismic data (b [nt, nx], nt is time sampling point, nx is channel number)
    dt is the time sampling interval
    x is offset
    q is the ray parameter
    flag is the transform type
    Noise is the white noise coefficient[nt,nx] = size(b);    %Find the dimension of input
seismic data matrix
nq = length(q);    %Find the number of traces of q
uf = fft(b,[],1);    %Fast Fourier transform of data
```

ISSN: 2414-1895 DOI: 10.6919/ICJE.202304_9(4).0023

```
R = zeros(nq,nx);
teop = zeros(nq,nx);
g = zeros(nq,1);
ufgg = zeros(nt,nq);

for k = 1:nt
    if k <=nt/2+1
        omega = 6.28318530717959*(k-1)/(nt*dt);
        R = exp (i * omega * q'* (x.^flag));    %Calculate L matrix
        MATRIX = R*R';
        toep = inv (MATRIX + noise*eye(nq))*R;
        g = toep * reshape(uf(k,1:nx),nx,1);
        ufpp(k,:) = g.';
    else
        ufpp(k,:) = conj(ufpp(nt+2-k,:));
    end
end
m = real(ifft(ufpp,[],1));    %Inverse Fourier transform, complete Radon transform
```

The above is the algorithm of linear Radon forward transformation, and the following is the algorithm of linear Radon inverse transformation:function b = invradon(m,dt,x,q,flag)

```
% M is the seismic data (m [nt, nq], nt is the time sampling point, nq is the channel number of q)
    dt is the time sampling interval
    x is offset
    q is the ray parameter
    flag is the transform type
[nt,nq] = size(m);    %Find the number of seismic traces
nx = length(x);    %Find the number of seismic traces
fp = fft(m,[],1);    %fast Fourier transform
R = zeros(nq,nx);
temp = zeros(nx,1);
ufxx = zeros(nt,nx);
for k = 1:nt
    if k<=nt/2+1
        omega = 6.28318530717959*(k-1)/(nt*dt);
        R = exp(i*omega*q'*(x.^flag));
        temp = R'* reshape(fp(k,1:nq),nq,1);
        ufxx(k,:) = temp.';
    else
        ufxx(k,:) = conj(ufxx(nt + 2-k,:));
    end
end
b = real(ifft(ufxx,[],1)); %Inverse Fourier transform, complete Radon transform
```

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DOI: 10.6919/ICJE.202304_9(4).0023

4. Reflected Wave Extraction

4.1 Seismic Wave in Advance Detection

According to the seismic data, the seismic profile (Fig. 1) obtained by MATLAB is as follows:

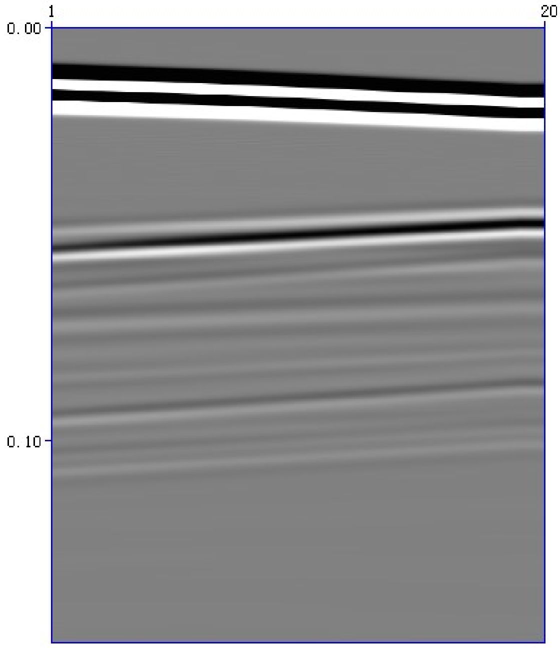
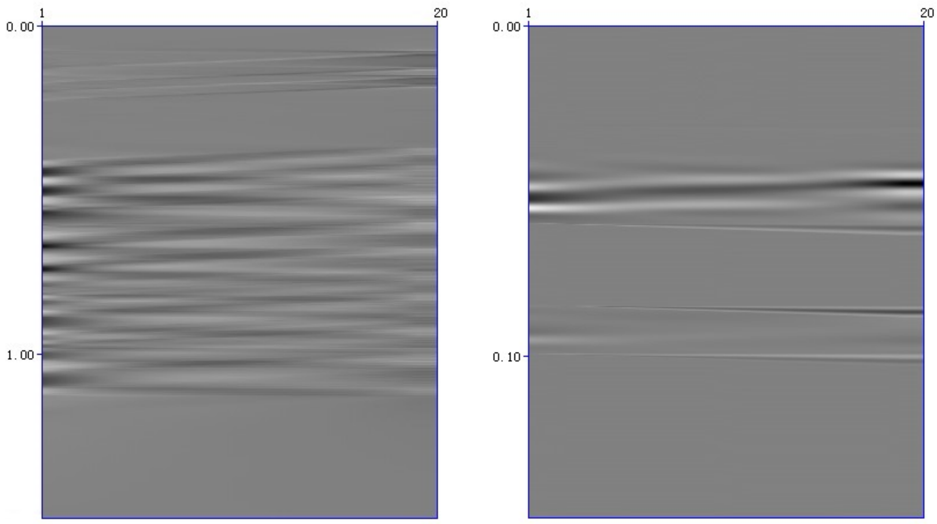


Figure 1. Seismic Profile of Model Data

It can be seen from the figure that the direct wave is obvious, but the reflected wave is accompanied by many invalid waves, but the primary reflected wave in the figure can be distinguished by the naked eye.

4.2 Reflected Wave Extraction

Transform seismic data using Radon transform algorithm and extract reflected wave.



(a) (b)

Figure 2. (a is the data in Radon domain, and b is the reflected wave extraction result)

Figure 2 (a) is the Radon domain transformation form of seismic forward modeling data of advanced detection, which can be seen the energy point distribution of the original data. Figure 2 (b) is the seismic profile data after the reflection wave extraction, which is the data obtained after the reverse transformation in the Radon domain, which can be seen that the extraction of the first reflection wave of the first interface with strong energy is obvious, but the reflection wave of the second interface has appeared energy divergence, It is difficult to distinguish them in Radon domain.

4.3 Conclusion

1. The shape and position of the time-distance curve of the leading seismic wave are different at Radon, so the reflected wave can be separated. Therefore, the reflected wave extraction method based on Radon transform proposed in this paper can complete the reflected wave extraction operation of advanced seismic exploration data.

2. When the front reflection interface is getting closer and closer to the vertical in the seismic advance detection, the reflection wave shape of its time-distance curve is also getting closer and closer to the straight line, and the time-distance relationship is also getting closer to the linear relationship from the nonlinear hyperbolic shape. Under the linear Radon transform adopted in this paper, its energy point separation and Radon domain separation are also simpler.

References

[1] Shen Cao, Niu Binhua, Yu Qinfan. MATLAB implementation of Radon transformation [J]. Geophysical and geochemical exploration calculation technology,2000,(04):58-62.

[2] Cao Lun. High-resolution Radon transform and its application in seismic data processing [D]. Chengdu University of Technology.2017.

[3] Zhou Wenjing. Radon transform and its application in seismic data analysis [D]. Southwest Petroleum University2017.

[4] Zhang Weiwei. Radon transform and its application in seismic data processing [D]. Chang'an University 2007:6-47.

[5] Hu Hao. Radon transform and its application in seismic data denoising [D]. Chengdu University of Technology. 2014: 5-45.

[6] Wang Chaoling. Research on numerical simulation of wave field separation and inversion methods in tunnel earthquake prediction [D]. Southwest Jiaotong University. 2012: 72-79.

[7] Wang Chaoling, Liu Zhengping τ- Numerical simulation study on the application of p-transform in wave field separation of tunnel reflection seismic prediction [J]. Progress in Geophysics. 2012 (05): 2216-2225.

[8] Lin Yi. Research on numerical simulation of tunnel earthquake advance prediction technology [D]. Southwest Jiaotong University. 2015: 43-59.

[9] Li Lingyun. Applying Radon transform to eliminate multiple waves [J]. Journal of Petroleum and Natural Gas (Journal of Jianghan Petroleum Institute), 2006 (04): 264-266.

[10] Liu Xiwu, Liu Hong, Li Youming High-resolution Radon transform method and its application in seismic signal processing [J] Progress in Geophysics, 2004,19 (1): 8-15.

[11] Lin Wen, Wei Dali, Wang Jianmin, Liu Jianying, Wang Weihong. Research on multiple suppression method and application of Radon transform [J]. Geophysical and geochemical exploration calculation technology, 2009,31 (04): 344-348+289.