Research on UAV Location based on Direction Finding Crossover and Iterative Optimization

Xiangdong Wang[1], Yuting Xie[2, #], Hao Qiao[1, #], Zikun Huang[1, #], Hao Ren[1, #]

[1]School of Engineering and Technology, Xi'an Siyuan University, Xi'an, China, 710038, China

[2]The College of Arts and Business, Xi'an Siyuan University, Xi'an, China, 710038, China

#These authors contributed equally

Abstract

This paper mainly aims to solve the problem that when UAV groups maintain electromagnetic silence and circular formation, the determination of its position in formation flight was studied, a method of passive azimuth positioning is proposed to adjust the position of UAV in formation. Firstly, based on the agreed direction information, two methods of positioning model of circular UAV formation are given and established. Secondly, the former uses the method of direction-finding crossover to express the preliminary positioning model of the orientation information of the observation UAV that needs to be solved. Then the least square estimation and Kalman filter are used to optimize the preliminary equation, and the target position is finally determined. Finally, based on the iterative optimization method, the UAV whose location can be determined is iterated continuously. By increasing the number of UAVs, the initial positioning of UAVs in the conducting formation can be achieved, the normal distribution model is used to evaluate the error results so as to meet the requirements of effective positioning of all UAVs.

Keywords

Passive Localization; Direction Finding Crossing; Kalman Filter; Iterative Optimization.

1. Introduction

Uavs need to keep electromagnetic silence when flying in formation to reduce electrical signals and avoid external interference. As drone technology continues to evolve, in drone fleets, that is, the organization mode of formation arrangement and task allocation for several UAVs to adapt to the task requirements (The concept of UAV formation is explained in the paper "Research on Problems and Key Technologies of UAV Formation Flight".). Keep the drones in circular formation with each other to maintain electromagnetic silence as much as possible, it is a basic requirement for unmanned aerial vehicles (UAVs) to send out less electromagnetic wave signals when flying in formation. The key technologies of UAV formation flight are discussed and analyzed.

In 2020, Han Xiuyou's team conducted research on "ultra-wideband RF signal monitoring technology under extreme electromagnetic silence environment of FAST", the necessity and method of extreme electromagnetic silence in the 5 km core area are analyzed[1] ;In 2021, Ye Jiesong et al. studied formation cooperative control based on azimuth-based information and grouping hierarchical model, the relevant problems of UAV cooperation are systematically expounded[2] ;In 2021, Wu Huiting, based on "Research on the Preprocessing Generalized Overrelaxation Iterative Method for Solving nonlinear Equations", the analysis model method of UAV positioning technology is studied and analyzed[3] .Based on the above relevant personnel, the UAV formation technology research methods

and related issues are interpreted. In this paper, the method of passive azimuth positioning is proposed to adjust the position of UAV in formation.

2. Uav Positioning Model based on Direction Finding Crossover Method

2.1 Establishment of Model

There are three known emitting drones, one of which is located in the center of the circle, and all of them are consistent, the remaining seven drones all had minor deviations. Assuming the UAVs stay at the same height, a positioning model of UAVs with passive reception signal is proposed. As shown in Figure 1: The signal of the receiving UAV is agreed (The interpretation of the direction information received by the UAV is agreed: namely, there are three UAVs sending signals and one of them is located in the center of the circle. There are small deviations of UAVs.). When all three transmitting drones are picked up by one of the receiving drones, their own position can be determined, and corresponding adjustment measures can be made.
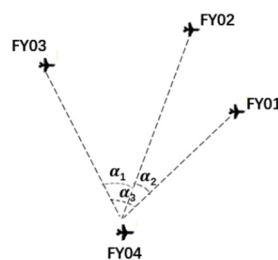

Figure 1. Define a diagram of the directional information received by the UAV

2.2 Solution of Model

One of 10 drones in a circular formation, at the center of the circle(Number as FY00),the remaining nine drones(Number as FY01-FY09)are evenly distributed around the perimeter.Polar coordinates are established as shown in Figure 2:Take the UAV located at the center of the circle as the coordinate origin $O(0,0)$,take the extension line of FY00-FY01 as the X-axis, specify that the horizontal direction to the right is positive.Make the vertical line of the X-axis as the Y-axis on the horizontal circular plane formed by the aircraft formation, specify that the horizontal upward direction is positive. Uav FY01-FY09 on the circumference is set as points A-I successively,(i,j) is their coordinates.
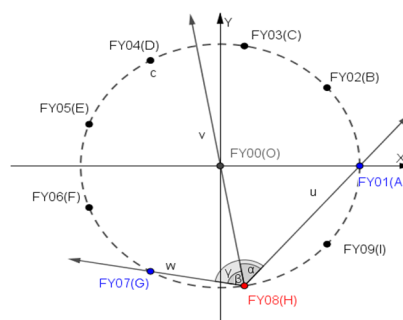

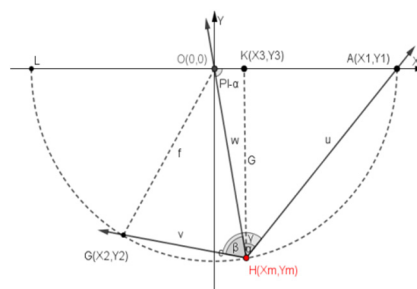**Figure 2.** Polar coordinate diagram of UAV formation

Figure 3. Principle diagram of direction finding and cross positioning method

The basic principle of direction finding and cross location method is: the use of two or more reconnaissance aircraft at the same time to the same target radiation source measured azimuth information, then the optimal state estimation method is used to calculate the location of the target radiation source [1]. First, build the schematic diagram as shown in Figure 3.Assume that the UAVs transmitting signals on the circumference are FY01 and FY02, and the circumference radius is R. The position relation of each UAV in the UAV formation was equivalent simplified by simple trigonometry(Figure 3),the position coordinates and direction information of observation UAV to be solved are presented. Let the coordinate position of the machine under test be $H(X_m, Y_m)$,the α, β

Angle between HO and AO polar coordinates calculated is $PI - \alpha$.

It can be expressed as:

$$\begin{cases} Y_m = \dfrac{R \cdot (\tan \angle OHK) y_m}{\tan \alpha - \arctan\left(\dfrac{R \cos \alpha}{y_m}\right)} \\ X_m = \tan \angle OHK \end{cases} \tag{1}$$

Using the polar coordinates and vector method,

$$\overrightarrow{GH} \cdot \overrightarrow{OA} = |GH| R \cot \alpha \cdot \cos \beta \tag{2}$$

$$\overrightarrow{OH} \cdot \overrightarrow{HA} = |R \cot \alpha| \cdot \sqrt{(Y_m \cdot \tan \angle OHK)^2} \cos \alpha \tag{3}$$

Set $\overrightarrow{OH} = (\rho, \theta)$,namely $(\overrightarrow{OH}, \alpha)$.Point H can be expressed as polar coordinates:

$H(\rho cos\alpha, \rho cos\beta)$.

It's known by the triangle,

$$X_m = Y_m \cdot \tan \angle OHK \tag{4}$$

$$Y_m = \left(\frac{R - \tan \dfrac{\alpha}{\beta}}{\tan \alpha - \arctan \dfrac{R^2 \cot \alpha}{\tan \dfrac{\alpha}{\beta}}} \right) \tag{5}$$

The coordinates of the measured points can be expressed as:

$$M \left[\tan \angle ark \tan \frac{R \cos \alpha}{\dfrac{\tan \dfrac{\beta}{\alpha}}{R \cdot \arcsin \alpha}} \cdot \left(\frac{R - \tan \dfrac{\alpha}{\beta}}{\tan \alpha - \arctan \dfrac{R^2 \cot \alpha}{\tan \dfrac{\alpha}{\beta}}} \right), \left(\frac{R - \tan \dfrac{\alpha}{\beta}}{\tan \alpha - \arctan \dfrac{R^2 \cot \alpha}{\tan \dfrac{\alpha}{\beta}}} \right) \right] \tag{6}$$

Some state variables change with time when UAV clusters are flying in formation. In the positioning process, cross positioning cannot be directly used, and continuous observation of the target is needed to estimate the motion state of the target. Data processing of a set of observations containing measurement errors with unknown parameters or states, the state and parameter estimation of the system are obtained: $PI - \alpha$ the Angle between them can be expressed as $\langle r_{x(0)}, r_{y(0)} \rangle$.

Establish the objective equation:

R' is the target initial distance, $\delta(k)$ is the target direction information at time k, formula (6) can be expressed in polar coordinates as:

$$\begin{cases} r_{x(0)} = R'\sin\alpha_{(0)} \\ r_{y(0)} = R'\cos\alpha_{(0)} \end{cases} \tag{7}$$

There is a nonlinear relationship between the azimuth state of the lateral target, and there are deviations in position and state. Thus, the azimuth measurement of the measured machine is shown:

$$\tan\delta(k) = \frac{r_{x(0)}}{r_{y(0)}} \tag{8}$$

The observation equation of passive positioning system based on azimuth measurement is nonlinear, so the passive localization algorithm is the optimal estimation problem of nonlinear system[4]. So the model is optimized, There are N parts of the UAV with known position coordinates uniformly distributed on the same UAV $M(x_m, y_m)$ carry out azimuth-only passive positioning. In Part Q $(x_j, y_j), (N > Q)$, the measurement noise of UAV directional information is negligible:

$$\beta_j = h_j(x_m, y_m) \tag{9}$$

By establishing the target azimuth state relation, the position information of part N is connected with the direction information of part Q by using nonlinear quadratic estimation method. The orientation information of each UAV about the measured aircraft α_j, β_j and the relationship between the UAV and the actual position $M(x_m, y_m)$ of the measured aircraft are established. $h_j(x_m, y_m)$ is defined as the state of the target UAV, it can be expressed as:

$$h_j(x_m, y_m) = ac\tan\frac{(y_m - y_j)}{(x_m - x_j)} \tag{10}$$

So, $h_j(x_m, y_m)$ estimates (x_m, y_m) by starting at $M(x_m, y_m)$, In combination with the expansion and linearization of the residual term (The expansion of $Taylor$ formula derived from order n with $Peano$ remainder is: $f(x) = f(x_0) + f'(x_0) + \frac{f''(x_0)}{2!}(x - x_0) + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + 0((x - x_0)^n)$). $f(x) = \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$ in $Taylor$ formula, the target location model of passive receiving UAV is established as follows:

$$\beta_j - h_j(x_m, y_m) = \left[\frac{\partial h}{\partial x_j} \cdot \frac{\partial h}{\partial y_j}\right]\left[\begin{matrix} x_j \\ y_j \end{matrix}\right] \tag{11}$$

Kalman filter algorithm was used to optimize the evaluation of the objective function, and the observation model was established:

$$y_k = x_k + \delta \tag{12}$$

Combined with the position observation of y_k at time k, establish x_k, w_k state model (initial state A is negligible) :

$$\begin{cases} x_{k+1} = d(x_k) + x_k \\ d_{k+1} = d(x_k) + w_k \end{cases} \tag{13}$$

x_k, y_k is the estimator, state and observation equations are established:

$$\begin{cases} x_{k+1} = A \cdot x_k \\ y_{k+1} = H \cdot y_k \end{cases} \tag{14}$$

Take p_{k+1} as the covariance and X_{K+1}, P_k as the covariance of X_K:

$$p_{k+1} = AP_K \big| K\, A^T \tag{15}$$

The optimal state valuation X_{K+1} at moment k is:

$$X_{K+1} = X_K + X_K \cdot \varepsilon_K, (0 < \varepsilon_K < 1, \varepsilon_K \subset R) \tag{16}$$

Finally, the positioning model of passive receiving UAV is established:

$$X_{K+1} = X_k + k \cdot \varepsilon, (0 < \varepsilon < 1) \tag{17}$$

3. Uav Location Model based on Iterative Optimization Method

3.1 Establishment of Model

To realize the effective positioning of UAV position, we need to make assumptions on the model. If the location of the drone transmitting the signal is correct, On the premise that the number and location of FY00 and FY01 are known, the iterative optimization model is used to determine the number of the minimum number of signals receiving UAVs.

3.2 Solution of Model

The iterative optimization method is used to solve the problem[5],the number of errant launch drones is increased one by one until the location of the drone can be determined. Iterate over each positionable drone, direct increase the number to effectively locate the unknown location of the remaining UAVs. The normal distribution model was used to evaluate the error results.The parameter $\mu, \sigma (\sigma > 0)$ is a constant, let X obey the normal distribution of parameter μ, σ ,so as to realize the effective positioning of UAV position [4].
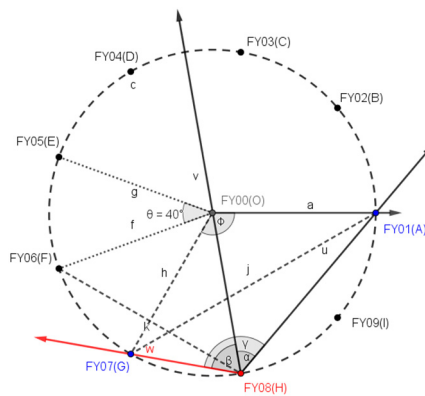

Figure 4. Neural network structure

As shown in Figure 4, given the number and position of FY00 and FY01, there are 9 UAVs on the circumference. With no deviation, the Angle between each adjacent drone and the center of the circle is 40 °.For example, the Angle between FY00-FY05 and FY00-FY06 is 40°, and the Angle between FY00-FY05 and FY00-FY07 is 80°.Suppose you take any one of the other eight dronesfirst,(Number as FY0B)with FY00 and FY01 as transmitting drones, send a signal to the remaining seven drones.

Mark the correct position of the remaining UAVs as the first iteration. at this time, the number of UAVs transmitting signals is 3.

Based on the above, the target azimuth state relationship of the UAV under test is established:

Verify that:

$$X'_{K+1} = X'_k + k \cdot \varepsilon, (0 < \varepsilon < 1) \tag{18}$$

When $2 \le B \le 5$, since a_1 is a multiple of 40°, so there is $B = a_1 / 40^o + 1$, the number of FY0B can be obtained.

When $6 \le B \le 9$, since FY0B may be in the clockwise or counterclockwise direction of FY01, can't tell the difference. Therefore, it is necessary to introduce a new launch UAV FY0C for another iteration verification. And take one of the other seven drones(Number as FY0C) and FY00 and FY01 as benchmarks, send signals to the remaining 6 UAVs (including FY0B), correct location of the remaining UAVs. Compared with the original generation, it is more consistent with:

$$X^1_{K+1} = X''_k + k \cdot \varepsilon, (0 < \varepsilon < 1) \tag{19}$$

The same applies when taking any one of the other six drones (Number as FY0D) ,with FY00 and FY01 as benchmarks, send signals to the remaining five drones. The same verification can also be satisfied:

$$X^2_{K+1} = X'''_k + k \cdot \varepsilon, (0 < \varepsilon < 1) \tag{20}$$

Because the assumptions that satisfy his criteria for measuring drones are not unique. Moreover, the effective positioning coordinates of UAVs and the number of UAVs transmitting signals can be judged as continuity, so the normal distribution statistical model must be introduced. The random variable X ob**eys a normal** distribution with a mathematical expectation of μ and a variance of σ^2, denoted $N(\mu, \sigma^2)$, so the random variable X follows the normal distribution and I'll call it $N(\mu, \sigma^2)$.

Its probability density function is the expected value μ of the normal distribution that determines its position, its standard deviation σ determines the magnitude of the distribution, As shown in Figure 5:
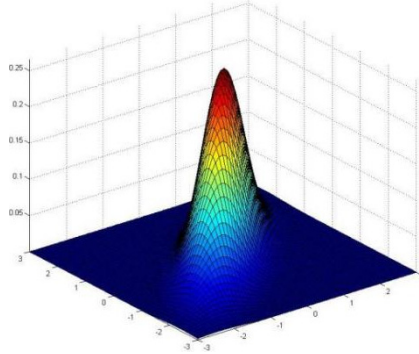

Figure 5. Neural network structure

Let $F(x)$ be the effective positioning coordinate of UAV. Where σ is the effective positioning judgment value, and μ is the number of UAVs that achieve effective positioning. Let the probability density of continuous random variable x (The number of drones transmitting signals) be :

$F(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, (2 < x < 10) \cdot$

We know from the density function property $F(x) \ge 0, \int_{-\infty}^{\infty} F(x)dx = 1$, namely:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1 \tag{21}$$

Double integrals are introduced to calculate

$$\iint_{x^2+y^2 \le R^2} F(x,y) = \int_0^R \int_0^{2\pi} F(r\sin\theta, r\cos\theta) r dr d\theta = 1 \tag{22}$$

When the normal distribution at $\mu = 0, \sigma = 1$ is the standard normal distribution. Normal distribution standardization formula:

$$I = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x)^2}{2}} \tag{23}$$

Through the definite integral of the normal distribution probability density function, it can be obtained:

$$P\{\mu - \sigma < 3 \leq \mu\} = P\{\mu < 4 \leq \mu + \sigma\} \tag{24}$$

To sum up, two more UAVs are needed to transmit signals to achieve effective positioning of UAVs. Therefore, the model can be used to determine the position of unknown UAVs in circular formation.

4. Evaluation of the Model

For the UAV positioning model of direction finding crossover method: In the process of solving the model, the nonlinear least square estimation method is used to optimize the state relation of target orientation. Then Kalman filter algorithm is used to optimize the objective function again, and a variety of models are used to analyze the problem, and the results are more accurate. However, the data used in this model is small, and the results obtained have minor errors. For the iterative optimization model: The thinking method is simple, but it cannot remove the small deviation of UAV itself, so it needs a certain number of UAV to solve.

5. Conclusion

In this paper, the passive orientation model of circular unmanned aerial vehicle formation flight is studied, and an approach to solve the problem is proposed. A large number of mathematical methods are used to solve the positioning problem of unmanned aerial vehicles (UAVs).As drone formation becomes the new direction of UAV development in the future, it can be used in more fields. For example, it has a good application prospect in logistics management and warehousing as well as UAV tracking technology. At the same time, whether the model method can meet other shapes of formation flying UAVs needs to be further verified.

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