Study on the Formation Mechanism of In-seam Seismic Waves in Coal Seam

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Abstract

This study deeply analyzes and explores the generation principle and formula derivation of half-space surface wave and underground trough wave. Most of the previous literature use the method of surface wave to study the ISS wave in coal seam, so in order to verify its rationality, this paper makes a comparative analysis of the function derivation process of the two, and finds out their similarities and differences. At the same time, according to the understanding of the formula, combined with the literature, the mechanism of ISS wave generation is analyzed. By comparing the formula derivation process of half-space surface wave and coal seam trough wave, it is found that their corresponding displacement equations are the same, which is the result of the determination of the physical properties of the medium in the process of wave propagation, so it can be considered reasonable. Although their dispersion equations are also different because of their different models, their dispersion characteristics are the same.

Keywords

In-seam Wave; Surface Wave; Dispersion.

1. Introduction

ISS is a new detection technology in recent years [1-2], but there are still great deficiencies in some principal research. At present, ISS is still studied and analyzed by some theoretical methods of surface wave. The theory of surface wave has been basically formed, but the study of surface wave is based on the surface model. The generation of 'surface wave' in coal seam with a depth of several hundred meters is very different from that of normal surface wave. Therefore, it is very necessary to explore whether the surface wave theory can be copied to the underground coal seam.

2. The Formation of ISS Wave

The physical parameters of coal seam are very different from those of surrounding rock. The density of coal seam is much smaller than that of surrounding rock, and the velocity of seismic wave is closely related to the density. Accordingly, the velocity of seismic wave in the interior of coal seam is much smaller than that of surrounding rock. The lower density coal seam and the higher density surrounding rock constitute the "guide wave" model[3]. When the source is excited in the coal seam, part of the seismic wave will be constrained in the interface of the roof and floor and propagate in the way of total reflection, and the energy will be trapped in the coal seam, and only a small amount of attenuation will occur with the propagation of the seismic wave. Interference and superposition will occur in the process of propagation of these waves. We can use the disturbance formed by this process to detect the coal seam, which is called in-seam seismic (ISS) detection.
3. Characteristics of Surface Wave

The surface wave refers to the wave propagating along the interface of the medium, and its amplitude decreases gradually in the direction away from the interface. There are two basic types of surface waves: Love wave and Rayleigh wave. The difference between them is that Love wave is a particle motion composed of SH wave, and Rayleigh wave is a motion composed of P-SV wave. The following is a schematic diagram of Love wave and Rayleigh wave.

\[
\begin{align*}
\mathbf{u}_p &= A(l\mathbf{a}_x + n\mathbf{a}_z) \exp \left[ i\omega \left( t - \frac{tx + nz}{\alpha} \right) \right], \\
\mathbf{u}_{SV} &= B(-n\mathbf{a}_x + l\mathbf{a}_z)\exp \left[ i\omega \left( t - \frac{tx + nz}{\beta} \right) \right], \\
\mathbf{u}_{SH} &= C\mathbf{a}_y\exp \left[ i\omega \left( t - \frac{tx + nz}{\beta} \right) \right]
\end{align*}
\]
In the above formula, \( n^2 + l^2 = 1 \). This shows that they can represent both surface wave displacement and body wave displacement. The difference between the surface wave and the body wave is that \(|u|\)’s dependent on the depth \( z \), and the displacement of the surface wave tends to zero with the deepening of \( z \). In equation (1) - (3), if \( n \) is a pure imaginary number, then there is the characteristic of surface wave for \(|u|\), so the displacement of surface wave can be derived by modifying the index in formula (1) - (3). Before that, it is rewritten as \((lx + nz)/\delta\) when \( n \) and \( l \) are real numbers, \( \delta \) can be \( \alpha \) or \( \beta \).

\[
\frac{lx + nz}{\delta} = \frac{x + nz}{\delta} = \frac{x + z\sqrt{1/l^2 - 1}}{\delta/l} = \frac{x + z\sqrt{c^2/\delta^2 - 1}}{c}
\]

\[\delta = \alpha, \beta \]  

\[n = \pm\sqrt{1 - l^2} \]  

\[c = \frac{\delta}{l} \]  

In this case \( c \) refers to phase velocity. Introduce \( \eta_\delta \), set up

\[\eta_\delta = \sqrt{\frac{c^2}{\delta^2} - 1} \]  

Derived from equations (5) and (6)

\[\frac{n}{l} = \pm\sqrt{1/l^2 - 1} = \pm\eta_\delta \]  

Using \( \eta_\alpha, \eta_\beta \) and \( k = \frac{\omega}{c} \), can rewrite the formula (1) - (3) into

\[u_p = A(a_x \pm \eta_\alpha a_z)exp [ik(ct - x \mp \eta_\alpha z)] \]  

\[u_{SV} = B(\mp \eta_\beta a_x + a_z)exp [ik(ct - x \mp \eta_\beta z)] \]  

\[u_{SH} = Ca_yexp [ik(ct - x \mp \eta_\beta z)] \]  

The coefficients \( A \) and \( B \) in equations (8) and (9) contain factors \( 1/l \), so that they do not lose generality because they satisfy the boundary conditions. The positive and negative numbers in the index represent the positive and negative movements of the \( z \)-axis, respectively.

When \( n \) is an imaginary number, \( n \) and \( l \) must satisfy the following conditions

\[n^2 + l^2 = 1 \]  

Because \( n \) is an imaginary number, \( l^2 \) must be greater than 1, and

\[n = \pm\sqrt{1 - l^2} = \pm i\sqrt{l^2 - 1} \]  

\[\frac{n}{l} = i\sqrt{1 - \frac{1}{l^2}} = \pm i\sqrt{1 - \frac{c^2}{\delta^2}}, \delta = \alpha, \beta \]  

\[\gamma_\delta = \sqrt{1 - \frac{c^2}{\delta^2}} \]  

\[\frac{n}{l} = \mp i\gamma_\delta \]  

Using \( \gamma_\alpha, \gamma_\beta \) and \( k \) to rewrite (1)-(3)
\[ u_p = A(\alpha_x + i\gamma_\alpha \alpha_z)\exp[\mp\gamma_\alpha kz + ik(ct - x)] \quad \text{(16)} \]

\[ u_{SV} = B(\pm i\gamma_\beta \alpha_x + \alpha_z)\exp[\mp\gamma_\beta kz + ik(ct - x)] \quad \text{(17)} \]

\[ u_{SH} = C\alpha_y \exp[\mp\gamma_\beta kz + ik(ct - x)] \quad \text{(18)} \]

(16)-(18) indicate that the amplitude of the surface wave in the positive axis decreases exponentially with the depth.

4. The Similarities and Differences between ISS Wave and Half-Space Surface Wave

4.1 Model

In the study of ISS waves, it is generally the study of coal seams, which are located in the deep underground. The model established is what scholars call the full-space model, that is, the three-layer symmetry model is shown in figure 4 below. The coal seam density is small, the wave propagation speed is slow, it is a low velocity layer, but its surrounding rock density is higher, the wave propagation speed is faster, it is a high-speed layer. Its simple schematic diagram is shown in figure 4. When the mathematical model is established for calculation, a certain position in the middle of the thickness of the coal seam is generally selected as the coordinate origin.

Fig 4. Schematic diagram of three-layer symmetric model

The model of surface wave on the ground is a model in half-space, with hard rock below the interface, air above the interface, or a loose cover as shown in figure 5 below. The model for the study of surface waves established by scholars is the half-space model.

Fig 5. Half Space Model

The above discussion is a difference between the ISS wave in the coal seam and the half-space surface wave, and this difference is the most fundamental difference between them. It can be said that at
present, most of the studies on coal seam trough wave and half-space surface wave are carried out on the basis of these models [5-7].

### 4.2 Boundary Condition

In the half-space model, the upper part of the rock interface is a vacuum, and the stress vector on the free surface is zero, but the free surface can have displacement. The earth surface is a good example of the free surface, because the parameters of rock and water are much larger than the elastic parameters of the atmosphere. Natural earthquakes produce waves that propagate in the atmosphere, and air explosions produce surface waves in the earth. Another case on the ground surface wave is that there is a cover layer on the half-space, in which case the stress vector on the interface is not zero. In the real case, there will be a soil cover layer above the stratum. To sum up, the boundary conditions are: (1) the stress on the free surface is zero; (2) the stress vector and displacement are continuous at the interface. If it is deep underground, the stress vector and displacement vector are continuous on the two kinds of solid interface. Continuous displacement does not allow interfacial materials to infiltrate (superimpose) and have voids (separation) on the interface. The surface wave corresponds to the case that the upper part of the interface is vacuum, and the boundary conditions are obtained by combining the stress vector and displacement of the upper and lower layers of the ISS wave.

### 4.3 Attenuation Characteristic

Whether it is half-space or full-space, they produce the same type of waves, which will produce Rayleigh waves and Love waves. The main causes of energy attenuation are dispersion, wavefront diffusion and absorption of incomplete elastic media. The following will be discussed from these three aspects.

The first is the energy attenuation caused by dispersion, which will lead to the attenuation of energy due to the dispersion of waves with the increase of propagation distance. Both the elastic half space surface wave and the ISS wave are composed of the superposition of several simple harmonics with different frequencies and have a continuous spectrum, which is expressed as

\[ f(x, t) = \int_{-\infty}^{\infty} g(k) e^{ik(x-ct)} \, dk \]  

\[ g(k) \] is the amplitude spectrum of wave, and the phase of wave is \( \theta \) and \( \theta = k(ct - x) \). The principle of wave synthesis and superposition is shown in figure 6. The idea of Fourier transform is used for reference here. The red waves represent the synthesized waves and the blue ones are simple harmonics. Among the many simple harmonics superimposed on each other, each has its own phase velocity \( c \), which is a function of \( \omega \) and \( k \). All the simple harmonics constrained in the coal seam interfere with each other, and the amplitude will be enhanced at some point due to superposition.

![Fig 6. Composite superposition of waves](image)

After a series of transformations, Taylor expansion and processing, the final result is
\[ f(x,t) = \sqrt{\frac{2\pi}{x}} g(k_0) e^{i\left[\text{Re}(c(k_0)t - x)\right]} \]  

(20)

It is known from the equation (20) that with the propagation of the wave, the attenuation of the amplitude is inversely proportional to the 1/2 power of the distance, and the Airy phase is inversely proportional to the 1/3 of the distance. No matter ISS wave in the coal seam or the surface wave in the half space have this characteristic.

The second is the attenuation caused by wavefront diffusion. When the ISS wave and surface wave propagate outward, their wavefronts are cylindrical, and when they propagate outward, the cylindrical surface is constantly expanding, which will lead to the redistribution of energy and the continuous decrease of energy density, which can only reduce the energy in all directions when the total energy is constant. The wavefront diffusion of spherical waves has long been studied, and the wavefront diffusion can be summarized as

\[ \frac{l_2}{l_1} = \frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_1}{r_2}\right)^m \]  

(21)

In the formula, if the plane wave \( m \) is 0, the cylindrical wave \( m \) is 1, and the spherical wave \( m \) is 2.

Both ISS waves and surface waves are cylindrical waves, so the attenuation caused by the front of the wave is inversely proportional to the 1/2 power of the distance, which is much smaller than that of spherical waves.

The third is the energy attenuation caused by the absorption of the medium, which is caused by the physical parameters of the rock.

According to previous theories, body waves spread according to the reciprocal of the epicenter distance. The third-order expansion of the equation (5-3) is carried out in Fu Chengyi's <Foundation of Geophysics>, and it is considered that the amplitude attenuation near the Airy phase is inversely proportional to the 1/3 power of the distance. Imagine that in a certain stratum, the attenuation of the amplitude of a wave is proportional to the distance. If the attenuation degree of this beam is known to 200m, it is such a stratum with the same energy and amplitude. The attenuation caused by dispersion is 100m relative to the spherical wave, plus 100m due to wavefront diffusion, a total of 200m. And the attenuation of the Airy phase is even smaller. This means that the attenuation of surface waves is much smaller than that of body waves. Therefore, in long-distance detection, the signal received by surface wave is much stronger than that of body wave, which is a common characteristic of surface wave and ISS wave.

4.4 Displacement Equation

Whether it is the solution of surface wave or ISS wave, it is assumed that under an ideal condition, the arbitrary motion expression of the medium is written, then the matching model is selected according to the real situation of the simulation, and then the boundary condition is selected. this is a general step in the analysis of surface waves.

In the study, because the SH wave can be separated and considered separately, the surface wave formed by it is divided into one kind, namely Love wave, whose particle vibration direction is parallel to the formation and perpendicular to the direction of motion. P and SV waves are coupled with each other, and they should be considered at the same time in the study. This kind of surface wave is called Rayleigh wave and its particle vibration is elliptical. The displacement expressions of these three waves are shown below [8-9].

\[ u_P = A(la_x + na_x) \exp\left[i\omega\left(t - \frac{lx+ny}{a}\right)\right] \]  

(22)

\[ u_{SV} = B(-na_x + la_x) \exp\left[i\omega(t - \frac{lx+ny}{\beta})\right] \]  

(23)
\[ \mathbf{u}_{SH} = C \mathbf{a}_y \exp \left[ i \omega \left( t - \frac{ix + nz}{\beta} \right) \right] \] (24)

(1) The displacement equation of Love wave of surface wave with a cover layer
\[ \mathbf{u}_1 = a_y \left( A e^{-i \eta_1 k z} + B e^{i \eta_1 k z} \right) e^{ik(c t - x)} , 0 < z < H \] (25)
\[ \mathbf{u}_2 = C \mathbf{a}_y e^{-\gamma_2 k z + i k (c t - x)} , z > H \] (26)
\[ \eta_1 = \frac{c^2}{\beta^2_1} - 1 \] (27)
\[ \gamma_2 = \sqrt{1 - \frac{c^2}{\beta^2_2}} > 0 \]

0 is the surface coordinate, H is the elastic rock layer.

(2) the displacement equation of Rayleigh wave of surface wave with a cover layer is as follows, which is the combination of equation (22) and equation (23). For A and B, they can be obtained by boundary condition.
\[ \mathbf{u} = [A a_y - i \gamma a_x] e^{-\gamma z} + B (i \gamma_1 a_y + a_z) e^{-\gamma_1 z} e^{ik(c t - x)} \] (28)

\[ \gamma_\delta = \sqrt{1 - \frac{c^2}{\delta^2}}, \delta = \alpha, \beta \] (29)

(3) Love wave displacement equation of ISS wave.
\[ \begin{cases} v_a = C e^{[\beta_1(z + k)]} & -\infty < z < -\hat{k} \\ v_b = A e^{\beta_2 \hat{k} \hat{k}} + \sin \beta_2 \hat{k} & -\hat{k} < z < \hat{k} \\ v_c = D e^{[-\beta_1(z - k)]} & \hat{k} < z < \infty \end{cases} \] (30)

\[ \begin{cases} \beta_1 = \omega/c_L (1 - c^2_L/v^2_{a1})^{1/2} \\ \beta_2 = \omega/c_L (c^2_L/v^2_{b1} - 1)^{1/2} \end{cases} \] (31)

(4) Rayleigh wave displacement equation of ISS wave.
Its displacement equation is the same as the Rayleigh wave displacement equation of surface wave, which is the combination of P wave and SV wave formula, but its coefficient is different because of its different environment.

It can be seen from the above that their displacement equation is the same, that is, Rayleigh waves and Love waves in half-space are the same as those of ISS waves, but their environments are different and their boundary equations are different. resulting in their different dispersion equations.

5. Conclusion
When the mathematical method is used to analyze the ISS wave, the basic method used is the same as the surface wave, and its displacement equation is also similar, except that the ISS wave is underground and its boundary conditions are different from that of the ground. In this way, the characteristics of underground ISS waves obtained by the same method are consistent with those of surface waves, whether the half-space surface wave theory can be directly applied to underground ISS waves, the answer is yes. Although there are great differences between above-ground and underground models, this method is logically feasible, and the simulation results obtained by using the theoretical formula are often consistent with those measured in the real scene. However, the derivation of the formula is based on the premise of ideal conditions, which is very different from the
actual situation. Before there is no better method to simulate the occurrence of underground trough waves, I think that this is a more appropriate method at present.

References