

# Optimization Research on Online Scheduling of Four-Way Shuttle Type Dense Storage System

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## Abstract

In this paper, a travel time model of a four-way shuttle dense storage system in the single operation and compound operation scenarios is established, and the inbound and outbound operation scheduling model is constructed with the goal of shortening the total operation time, considering the same layer and different layers of compound operations. The advantages of the gray wolf optimization algorithm compared with the genetic algorithm in solving the offline scheduling problem are compared, and the gray wolf online optimization algorithm based on insertion is proposed to solve the scheduling problem in the online case for the actual warehouse scheduling problem. Experimental results show that the proposed algorithm has better results in a large-scale environment, is more in line with the actual warehousing application scenarios, and has stronger generalizability.

## Keywords

Four-Way Shuttle; Dense Storage System; Online Scheduling; Online Optimization Algorithm.

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## 1. Introduction

As a novel storage system, the operational procedures of a four-way shuttle dense storage system are relatively intricate, and effectively realizing task scheduling poses a significant challenge for numerous scholars. The scheduling problem associated with its operation is classified as NP-hard (non-deterministic polynomial, NP)[1]. Consequently, there is a lack of an efficient polynomial-time algorithm capable of solving the problem model. Presently employed solving methods predominantly optimize the offline scenario of the problem, wherein all relevant information is known prior to the solution. The devised algorithms exhibit robust problem-solving capabilities in this research context. However, in real-world scenarios, decision-makers are unable to possess complete information about the problem before making decisions. Furthermore, various details of the problem are subject to change at any given time, and online order arrivals inevitably introduce disruptions to the optimization and resolution of the offline scheduling problem. Challenges necessitating decision-makers to make choices without full information are termed online problems, and real-world problems exhibit online characteristics as opposed to offline problems[2].

Hence, to align with practical storage scenarios, this paper delves into the scheduling optimization quandary of the four-way shuttle dense storage system. It formulates a travel time model for diverse operations, contemplates the scheduling dynamics of online orders, and puts forth a gray-wolf online optimization algorithm grounded in insertion to tackle the predicament. The effectiveness of the proposed algorithm is scrutinized through a series of carefully designed experiments.

## 2. Scheduling Model for Four-Way Shuttle Type Dense Storage System

### 2.1 Model Assumptions

To formulate a mathematical model abstracting the scheduling problem of the four-way shuttle dense storage system and considering the real-world context, this paper makes the following assumptions about the system:

- 1) The storage location of each commodity is known and organized according to a predefined arrangement. Each row of shelves consists of  $(x, y, z)$  with fixed values for length, width, and height.
- 2) Both the hoist and the four-way shuttle undergo uniform acceleration movements, and the analysis does not account for collision issues.
- 3) The four-way shuttle and hoist are capable of carrying only a single pallet of goods at a time, and once a task is initiated, it cannot be interrupted.
- 4) The initial position of the four-way shuttle is the I/O point of each layer, and the initial position of the hoist is the I/O point of the first floor.
- 5) The time required for loading and unloading commodities on the storage position of the four-way shuttle, the time for interaction with the hoist, and the steering time are all considered fixed values.
- 6) The dimensions (length, width, and height) of the cargo compartment define the moving distance of both the shuttle and the hoist.

### 2.2 Model Parameters.

Table 1 enumerates the fundamental parameters of the model.

### 2.3 Model Construction

#### 1) Single job

The singular operation of the four-way shuttle dense storage system can be bifurcated into an individual inbound operation and an individual outbound operation, with both being considered a mutually reversible operational process[3]. In the context of a single operation, the inbound operation task encompasses the process from task initiation to transporting goods from the I/O station to the designated storage location. Conversely, the outbound task involves the process from task initiation to conveying goods from the designated storage location to the I/O station. The models for a single job task are depicted in equation (1):

$$T^D = \sum_{i=1}^n (\max(t_{si}, t_{cxi} + t_{cyi} + t_{ri}) + nt_{sc}) \quad (1)$$

The calculation method for  $t_{ri}$  is as in equation (2):

$$t_{ri} = t_{pi} + t_{ii} \quad (2)$$

Based on the distance to the destination, there are two categories of travel times for shuttles and hoists, computed in a manner analogous to the above-described approach.

#### 2) Compound jobs

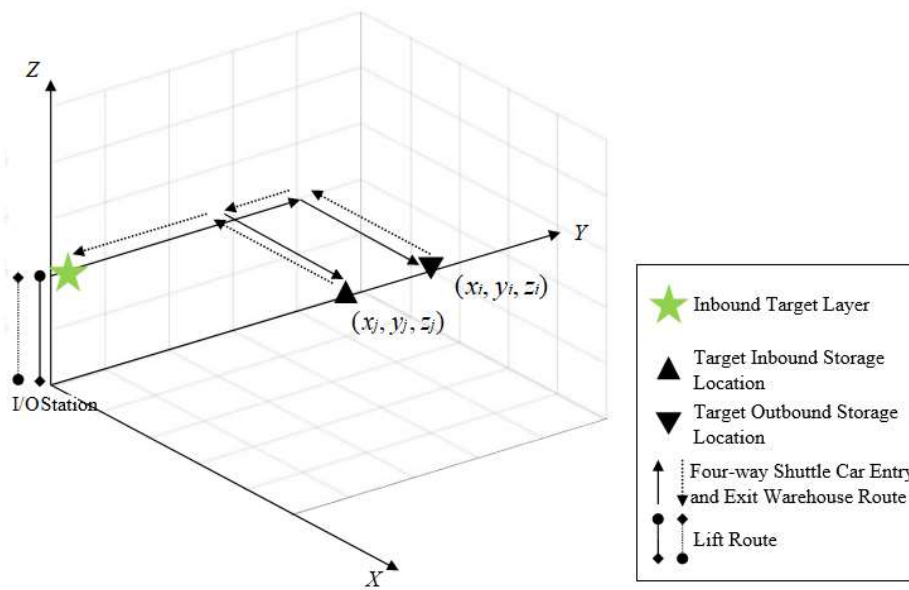
For composite jobs, two scenarios exist: compound job tasks on the same floor and compound job tasks on different floors.

Scenario 1, same floor compound job tasks:

After the commodity reaches the I/O platform, the hoist loads the goods to be stored at the I/O platform. The hoist, carrying the loaded commodity, moves to the I/O point of the target layer. At the I/O point, the four-way shuttle on the target layer interacts with the hoist, completing the connection of the commodity. Subsequently, the four-way shuttle transports the commodity to the target storage location, placing it in the inbound location to conclude the inbound operation. Following this, the four-way shuttle exits from the storage location, retrieves the target commodity via reverse movement to the outbound storage location, and transports the commodity to the layer I/O point. The hoist interacts with the four-way shuttle at the I/O point on the floor, completing the connection of the goods. The hoist then moves the goods to the I/O platform, completing the outbound operation. Figure 1 illustrates the operational flow.

**Table 1.** Basic parameter list

| Symbols                      | Meanings   |
|------------------------------|--|
| $(x, y, z)$                  | Storage location   |
| $v_c$                        | Four-way shuttle maximum speed   |
| $v_s$                        | Elevator maximum speed   |
| $a_c$                        | Four-way shuttle plus (minus) speed  |
| $a_s$                        | Boost the speed of adding (decreasing).  |
| $h$                          | The height of the cargo compartment  |
| $l$                          | The length of the cargo compartment  |
| $w$                          | The width of the cargo compartment   |
| $t_{si}$                     | The running time of the hoist to perform the task i  |
| $t_{cxi}$                    | The running time of the four-way shuttle to perform the task i on the mother road  |
| $t_{cyi}$                    | The running time of the four-way shuttle to perform the task i in the sub-lane   |
| $t_{sc}$                     | The interaction time between the hoist and the shuttle   |
| $t_{ri}, t_{rj}$             | The shuttle performs tasks i and j at other times except for moving  |
| $t_{pi}, t_{pj}$             | The loading and unloading time of the four-way shuttle to perform tasks i and j  |
| $t_{ti}, t_{tj}$             | The steering time of the four-way shuttle to perform tasks i and j   |
| (compound jobs)<br>$t_{cxi}$ | The time when the four-way shuttle performs the task i in the mother lane to the storage $(x_i, y_i, z_i)$ sub-lane      |
| (compound jobs)<br>$t_{cyi}$ | The time when the $(x_i, y_i, z_i)$ four-way shuttle performs the task i in the sub-road to the storage position         |
| $t_{cxj}$                    | The four-way shuttle performs the task J on the mother road to the time it takes to store the sub-road $(x_i, y_i, z_i)$ |
| $t_{cyj}$                    | The time when the $(x_i, y_i, z_i)$ four-way shuttle performs the task J in the sub-road to the storage position         |
| $T^D$                        | The amount of time the system executes a single job  |
| $T^T$                        | The time when the system executes the same layer composite job   |
| $T^B$                        | The amount of time the system executes the different layers of composite jobs  |



**Figure 1.** Inbound and outbound on the same floor

The comprehensive travel time model is presented in Equation (3):

$$T^T = \sum_{i=1}^n \sum_{j=1}^n (\max(t_{si}, t_{cxi} + 2t_{cyi} + t_{ri} + t_{cxj} + t_{cyj} + t_{rj}) + 2nt_{sc}) \quad (3)$$

Depending on the distance length, two types of travel times for the shuttle and the hoist are computed as per Equations (4)-(6), subject to constraints  $z_i = z_j$ .

$$t_{si} = \begin{cases} 2\sqrt{\frac{z_i \times h}{a_s}} & (z_i \leq \frac{v_s^2}{a_s \times h}) \\ \frac{z_i \times h}{v_s} + \frac{v_s}{a_s} & (z_i > \frac{v_s^2}{a_s \times h}) \end{cases} \quad (4)$$

$$t_{cxi} = \begin{cases} 2\sqrt{\frac{x_i \times l}{a_c}} & (x_i \leq \frac{v_c^2}{a_c \times l}) \\ \frac{x_i \times l}{v_c} + \frac{v_c}{a_c} & (x_i > \frac{v_c^2}{a_c \times l}) \end{cases} \quad (5)$$

$$t_{cyi} = \begin{cases} 2\sqrt{\frac{y_i \times w}{a_c}} & (y_i \leq \frac{v_c^2}{a_c \times w}) \\ \frac{y_i \times w}{v_c} + \frac{v_c}{a_c} & (y_i > \frac{v_c^2}{a_c \times w}) \end{cases} \quad (6)$$

Scenario 2, different layer compound job tasks:

Following the arrival of the commodity at the I/O platform, the hoist loads the goods to be stored at the I/O platform. The hoist, along with the loaded commodity, moves to the I/O point of the target warehousing layer. At the I/O point, the four-way shuttle on the target layer interacts with the hoist, completing the connection of the commodity. Subsequently, the four-way shuttle transports the commodity to the target warehousing storage location, placing it in the inbound location to conclude the inbound operation. Simultaneously, while the warehousing operation is ongoing, the outbound operation task commences. The four-way shuttle of the outbound task layer moves to the target outbound storage location, retrieves the target commodity from the storage location, and transports the loaded commodity to the layer I/O point. After the hoist completes interaction with the four-way shuttle of the inbound task layer, it moves to the I/O point of the outbound task layer without a load. The hoist then interacts with the four-way shuttle of the outbound task layer, completing the connection of the goods. Lifting the airborne goods, the hoist moves to the I/O platform to conclude the outbound operation. Figure 2 depicts the operational flow of the job.

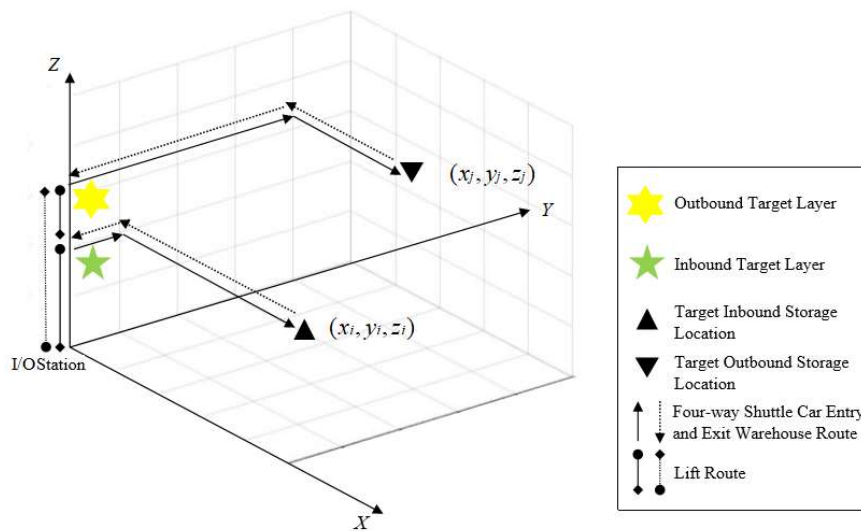


Figure 2. Inbound and outbound on the different floor

Hence, when there are different layers involved in warehousing and outbound operations, the comprehensive travel time model for the compound operation can be derived, as illustrated in Equation (7):

$$T^T = \sum_{i=1}^n \sum_{j=1}^n (\max(t_{si}, t_{xi}) + 2t_{cyi} + t_{ri} + t_{cxj} + t_{cyj} + t_{rj}) + 2nt_{sc} \quad (7)$$

Based on the distance length, two categories of travel time for the shuttle and the hoist are computed as outlined in Equations (8)-(11), subject to specified constraints  $z_i \neq z_j$ .

$$t_{si} = \begin{cases} 2\sqrt{\frac{z_i \times h}{a_s}} & (z_i \leq \frac{v_s^2}{a_s \times h}) \\ \frac{z_i \times h}{v_s} + \frac{v_s}{a_s} & (z_i > \frac{v_s^2}{a_s \times h}) \end{cases} \quad (8)$$

$$t_{sj} = \begin{cases} 2\sqrt{\frac{z_j \times h}{a_s}} & (z_j \leq \frac{v_s^2}{a_s \times h}) \\ \frac{z_j \times h}{v_s} + \frac{v_s}{a_s} & (z_j > \frac{v_s^2}{a_s \times h}) \end{cases} \quad (9)$$

$$t_{cxi} = \begin{cases} 2\sqrt{\frac{x_i \times l}{a_s}} & (x_i \leq \frac{v_c^2}{a_c \times l}) \\ \frac{x_i \times l}{v_c} + \frac{v_c}{a_c} & (x_i > \frac{v_c^2}{a_c \times l}) \end{cases} \quad (10)$$

$$t_{cyi} = \begin{cases} 2\sqrt{\frac{y_i \times l}{a_s}} & (y_i \leq \frac{v_c^2}{a_c \times l}) \\ \frac{y_i \times l}{v_c} + \frac{v_c}{a_c} & (y_i > \frac{v_c^2}{a_c \times l}) \end{cases} \quad (11)$$

Considering the two operation scenarios described above, the overall travel time for the operation of the four-way shuttle-type dense storage system in a batch order is determined by the following Equation (12):

$$T = T^D + T^T + T^B \quad (12)$$

### 3. Solving the Scheduling Model.

#### 3.1 Grey Wolf Optimization Algorithm

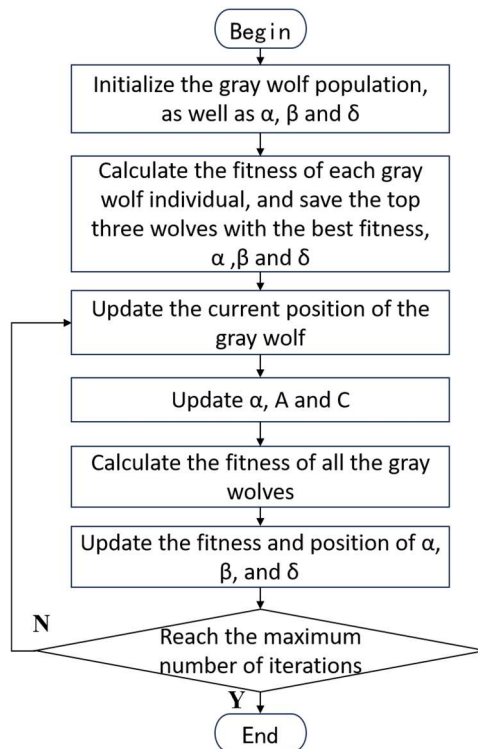


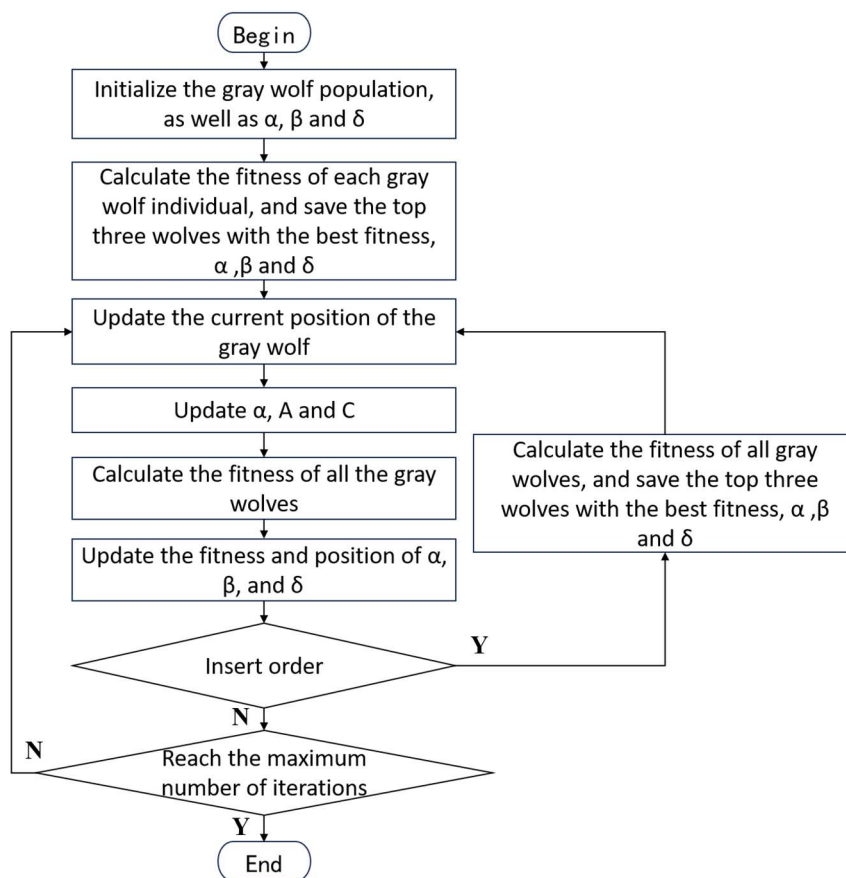
Figure 3. Step diagram of gray wolf optimization algorithm

In 2014, Mirjalili et al.[4] introduced the Grey Wolf Optimizer (GWO), demonstrating its superior convergence compared to other algorithms through testing with various functions. The Grey Wolf Optimizer (GWO) has garnered significant interest from scholars due to its attributes of having few parameters, a straightforward structure, easy implementation, and having found applications in diverse areas such as PID controller parameter optimization, economic scheduling, and workshop scheduling.

The procedural steps of the GWO algorithm are illustrated in Figure 3.

### 3.2 Insertion-based Grey Wolf Online Optimization Algorithm

In this paper, an enhanced gray wolf online optimization algorithm, incorporating the insertion algorithm based on the GWO algorithm, is proposed to address the scheduling optimization challenge of the four-way shuttle dense storage system. The algorithmic flowchart is illustrated in Figure 4.



**Figure 4.** Step diagram of online optimization algorithm of gray wolf based on insertion

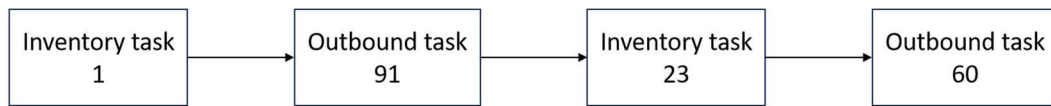
The crucial technologies for addressing the scheduling of a four-way shuttle dense storage system utilizing the inserted gray wolf online optimization algorithm are as follows:

1) Initialize the gray wolf population code:

The  $x$  inbound tasks and  $y$  outbound tasks of the compound job are encoded as integers. For the inbound tasks, a random number within the range  $[1, x]$  is assigned, and for the outbound tasks, a random number within the range  $[1, y]$  is assigned. If the total number of inbound tasks does not match the total number of outbound tasks, a virtual task is employed to align the discrepancy. The coordinates of the virtual task are specified, and the execution of the virtual task is equivalent to the execution of a single job.

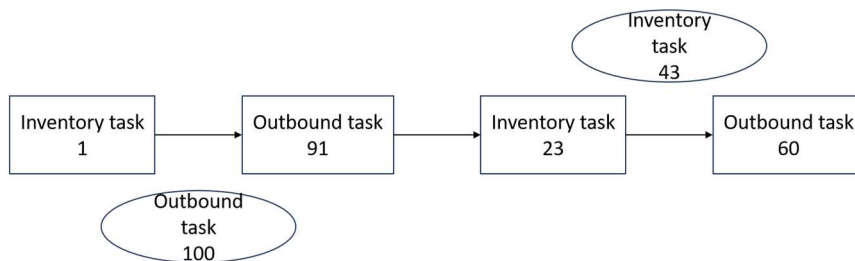
2) Insert strategy:

Assuming a known order task has completed the compound job pairing before insertion, illustrated in Figure 5.



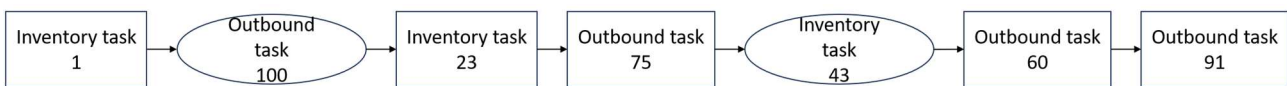
**Figure 5.** Paired composite job diagram before insertion

Upon the arrival of an online task, the sequence of inserted tasks necessitates re-optimization, as illustrated in Figure 6.



**Figure 6.** Composite job diagram of inserted tasks and completed pairings

The fitness of the entire gray wolf population is recalculated until the termination condition is met, ultimately yielding the overall composite operation, as depicted in Figure 7.



**Figure 7.** Completion of the optimized insert task compound job graph

For inserted orders, the following conditions are considered:

Condition 1, inserted outbound order and optimized inbound order:

If they are on the same layer, they are paired.

If not on the same layer, the algorithm searches for the inbound operation order on the adjacent layer.

If no suitable inbound operation is found for pairing, the inserted outbound order is placed at the end of the order sequence as a standalone outbound job.

Condition 2, inserted inbound order and optimized outbound order:

If they are on the same layer, they are paired.

If not on the same layer, the algorithm searches for the outbound operation order on the adjacent layer.

If no suitable outbound operation is found for compounding, the inserted inbound order is placed at the end of the order sequence as a standalone inbound job[5].

#### 4. Experimental Case Validation and Result Analysis

To assess the effectiveness of the gray wolf optimization algorithm in addressing the scheduling optimization problem of the four-way shuttle dense storage system, experiments are categorized into small batches of 25 orders, medium batches of 50 orders, and large batches of 100 orders. Both the gray wolf optimization algorithm and the genetic algorithm are employed for solving the problem in offline scenarios. The experimental setup details are presented in Table 2.



### 4.1 Offline Scheduling Experiment

In offline scheduling scenarios, complete information about each order is available. Tables 3-8 present the order details for Experiment 1, Experiment 2, and Experiment 3. Each order consists of 2-10 different products, with each product corresponding to one task.

**Table 2.** Experimental setup

| Serial number | Order quantity | Scheduling type    | Algorithm   |
|---------------|----------------|--------------------|---|
| Experiment 1  | 25             | Offline scheduling | Grey Wolf Optimization Algorithm                        |
|               |                |                    | Genetic Algorithm                                       |
|               |                | Online scheduling  | Insertion-based Grey Wolf Online Optimization Algorithm |
| Experiment 2  | 50             | Offline scheduling | Grey Wolf Optimization Algorithm                        |
|               |                |                    | Genetic Algorithm                                       |
|               |                | Online scheduling  | Insertion-based Grey Wolf Online Optimization Algorithm |
| Experiment 3  | 100            | Offline scheduling | Grey Wolf Optimization Algorithm                        |
|               |                |                    | Genetic Algorithm                                       |
|               |                | Online scheduling  | Insertion-based Grey Wolf Online Optimization Algorithm |

**Table 3.** Experiment 1: Offline Inbound Orders

| Order number | Product number                   |
|--------------|----------------------------------|
| 1            | 28,92,53,105,156,37,93,66,19     |
| 2            | 158,145                          |
| 3            | 120,69,111,99,49,123             |
| 4            | 74,135                           |
| 5            | 120,56,30,44,98                  |
| 6            | 39,31                            |
| 7            | 61,113,111,85,96,75,21,110,13,35 |

**Table 4.** Experiment 1: Offline Outbound Orders

| Order number | Product number                   |
|--------------|----------------------------------|
| 1            | 124,68,13,114,58,152,106,10      |
| 2            | 29,134,79,61,26,100,112          |
| 3            | 117,118,143                      |
| ...          | ...                              |
| 16           | 56,143,36,37,139,16,66,42,27,150 |
| 17           | 116,72,115,85,33                 |
| 18           | 77,60                            |

**Table 5.** Experiment 2: Offline Inbound Orders

| Order number | Product number            |
|--------------|---------------------------|
| 1            | 15,27,98                  |
| 2            | 26,87,32,49,10,16,98,62   |
| 3            | 55,95,19,98,3,64,50,21,53 |
| ...          | ...                       |
| 12           | 62,136,46,20              |
| 13           | 92,16                     |

**Table 6.** Experiment 2: Offline Outbound Orders

| Order number | Product number                 |
|--------------|--------------------------------|
| 1            | 33,57,120,153,17,96,36         |
| 2            | 46,88,33                       |
| 3            | 83,40,39,77,19,56,35,2,107,125 |
| ...          | ...                            |
| 36           | 46,88,33                       |
| 37           | 83,40,39,77,19,56,35,2,107,125 |

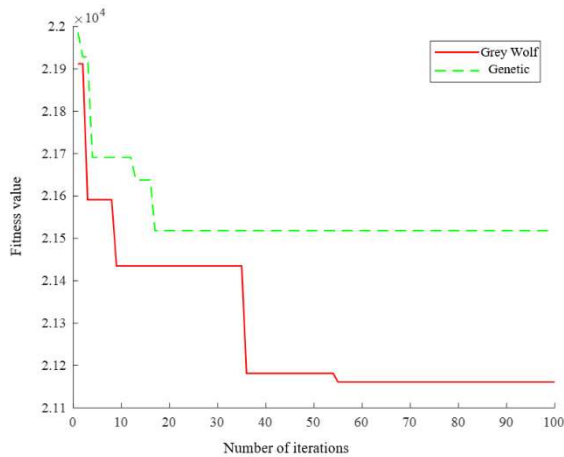
**Table 7.** Experiment 3: Offline Inbound Orders

| Order number | Product number                   |
|--------------|----------------------------------|
| 1            | 11,145,102,132,31,125,104,50     |
| 2            | 127,133,59,83,111,151,28,30,94   |
| 3            | 152,9,26,92,54,20,148,12         |
| 4            | 88,51,62,144,25,115,112          |
| ...          | ...                              |
| 38           | 13,16,152,134,32,102,76          |
| 39           | 103,120,92,66,48,74              |
| 40           | 95,132,78,38,45,112,42,23,13,119 |

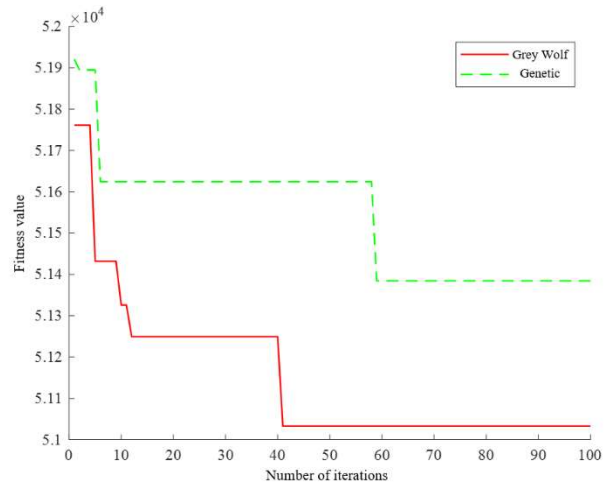
**Table 8.** Experiment 3: Offline Outbound Orders

| Order number | Product number                    |
|--------------|-----------------------------------|
| 1            | 70,120,67,53,3                    |
| 2            | 92,97,108,40,116,151              |
| 3            | 88,28,139,127,33,108,153          |
| 4            | 71,109,158,24                     |
| 5            | 127,107,30,33,145,129,91,12,94,36 |
| ...          | ...                               |
| 58           | 136,5,12,156,26,7,122             |
| 59           | 98,35,119,81,87,129,34,80         |
| 60           | 115,82,130,52,131                 |

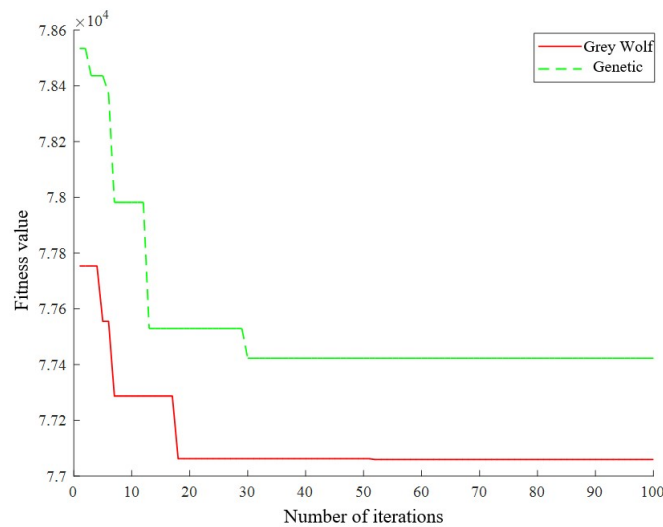
In MATLAB, both the genetic algorithm and the gray wolf optimization algorithm were employed to solve the offline scheduling problem. The parameters for the genetic algorithm were configured with a population size of 100, a maximum number of iterations set at 100, a crossover probability of 0.8, and a mutation probability of 0.1. For the gray wolf optimization algorithm, the parameters included a population size of 100 and a maximum number of iterations set at 100. Figures 8-10 illustrate the iterative convergence diagrams for each algorithm.



**Figure 8.** Convergence plot for Algorithm in Offline Scheduling Problem - Experiment 1



**Figure 9.** Convergence plot for Algorithm in Offline Scheduling Problem - Experiment 2



**Figure 10.** Convergence plot for Algorithm in Offline Scheduling Problem - Experiment 3

The experimental results reveal that the gray wolf optimization algorithm outperforms the traditional genetic algorithm in solving offline scheduling problems. Moreover, as the number of orders increases, the gray wolf optimization algorithm demonstrates a reduced time consumption for problem resolution, indicating higher efficiency. This suggests that the gray wolf optimization algorithm is particularly well-suited for managing larger orders and is more applicable to real-world warehousing problems.

#### 4.2 Online Scheduling Experiment

To validate the efficacy of the proposed gray wolf online optimization algorithm based on insertion in addressing real-time online problems, it is compared with the unimproved gray wolf optimization algorithm.

In online scheduling scenarios, only a portion of the order information is available. To ensure comparability between algorithms, offline order information is shared between offline and online situations. A subset of the offline order information is designated as the known order information, while the remaining orders are treated as online order information. The online order information for Experiment 1 is shown in Tables 9-10. The last 2 offline inbound orders and the last 8 offline outbound orders were selected as online orders. For Experiment 2 and Experiment 3, similar selections were made, taking the last 5 inbound orders and the last 15 outbound orders for Experiment 2, and the last 10 inbound orders and the last 20 outbound orders for Experiment 3.

**Table 9.** Experiment 1: Online Inbound Orders

| Order number | Product number                   |
|--------------|----------------------------------|
| 6            | 39,31                            |
| 7            | 61,113,111,85,96,75,21,110,13,35 |

**Table 10.** Experiment 1: Online Outbound Orders

| Order number | Product number                   |
|--------------|----------------------------------|
| 11           | 33,77,5,37,107,55,134,48         |
| 12           | 7,8,139,127,90,60                |
| 13           | 102,112                          |
| 14           | 139,62,52                        |
| 15           | 143,76,60,33,138,107,114         |
| 16           | 56,143,36,37,139,16,66,42,27,150 |
| 17           | 116,72,115,85,33                 |
| 18           | 77,60                            |

**Table 11.** Pairing table for offline scheduling tasks in each experiment

| Offline scheduling |                |               |                |               |                |
|--------------------|----------------|---------------|----------------|---------------|----------------|
| Experiment 1       |                | Experiment 2  |                | Experiment 3  |                |
| Inbound tasks      | Outbound tasks | Inbound tasks | Outbound tasks | Inbound tasks | Outbound tasks |
| 1                  | 29             | 1             | 112            | 1             | 39             |
| 2                  | 96             | 2             | 124            | 2             | 64             |
| 3                  | 93             | 3             | 133            | 3             | 70             |
| 4                  | 23             | 4             | 159            | 4             | 80             |
| 5                  | 68             | 5             | 166            | 5             | 108            |
| ...                | ...            | ...           | ...            | ...           | ...            |
| 0                  | 4              | 0             | 144            | 0             | 299            |
| 0                  | 36             | 0             | 146            | 0             | 390            |
| 0                  | 101            | 0             | 172            | 0             | 391            |

Table 11 indicates that the "0" in the gray wolf optimization algorithm denotes scenarios where the number of inbound tasks in the order is less than the number of outbound tasks. Consequently, the outbound tasks that cannot be paired are executed individually.

In the insertion-based gray wolf online optimization algorithm employed to address the online scheduling problem, the known order information is initially optimized and solved. Subsequently, the online order information is inserted for joint optimization. The run-time results are presented in Table 12.

**Table 12.** Pairing table for online scheduling tasks in each experiment

| Online Scheduling |                |               |                |               |                |
|-------------------|----------------|---------------|----------------|---------------|----------------|
| Experiment 1      |                | Experiment 2  |                | Experiment 3  |                |
| Inbound tasks     | Outbound tasks | Inbound tasks | Outbound tasks | Inbound tasks | Outbound tasks |
| 1                 | 91             | 1             | 16             | 1             | 53             |
| 2                 | 41             | 2             | 209            | 2             | 109            |
| 3                 | 56             | 3             | 141            | 3             | 167            |
| 4                 | 103            | 4             | 244            | 4             | 216            |
| 5                 | 57             | 5             | 163            | 5             | 244            |
| ...               | ...            | ...           | ...            | ...           | ...            |
| 0                 | 21             | 0             | 151            | 0             | 217            |
| 0                 | 58             | 0             | 166            | 0             | 352            |
| 0                 | 99             | 0             | 99             | 0             | 361            |

Each algorithmic program was executed 10 times to calculate the average total job travel time. The results are summarized in Table 13.

**Table 13.** Results of shuttle scheduling for offline and online algorithms under different problem scales

| Order quantity | Scheduling type    | Mean job execution time |
|----------------|--------------------|-------------------------|
| 25             | Offline scheduling | 21200.4                 |
|                | Online scheduling  | 21902.6                 |
| 50             | Offline scheduling | 50330.7                 |
|                | Online scheduling  | 50283.3                 |
| 100            | Offline scheduling | 77028.7                 |
|                | Online scheduling  | 76957.6                 |

As observed from Table 13, the results of the offline gray wolf algorithm outperform those of the insertion-based gray wolf online optimization algorithm in the case of small batch orders. For medium-sized orders, the solutions from both approaches are comparable, with no significant differences. In the case of large orders, the insertion-based gray wolf online optimization algorithm yields better results compared to the unimproved gray wolf optimization algorithm, as indicated by shorter operation times.

The outcomes in the table demonstrate that, with an increase in order size, the performance of the gray wolf online optimization algorithm based on insertion also improves. This is attributed to the real-time nature of the insertion in small batch orders. In contrast, the offline algorithm, having knowledge of all tasks before execution, can achieve superior solutions in small batches due to the manageable scale of the problem. However, as the problem scale expands, it becomes challenging for the offline algorithm to find optimal solutions before reaching the maximum number of iterations, even with complete information about all tasks. On the other hand, the gray wolf online optimization algorithm based on insertion, dealing with online order arrivals, consistently optimizes the overall order with each iteration, demonstrating improved solution quality as the task scale increases. In real-world warehousing scheduling scenarios, decision-makers typically do not have knowledge of all tasks initially, and tasks arrive online. Therefore, the gray wolf online optimization algorithm based on insertion proves to be more suitable for practical warehousing operations.

## 5. Conclusion

In this paper, we investigated the scheduling optimization problem of a four-way shuttle dense storage system. We analyzed and established the travel time model for both single and compound operations, considering scenarios involving the same and different layers of compound operations. Three scheduling experiments, each with varying order sizes, were designed. The gray wolf optimization algorithm and genetic algorithm were applied to solve the offline scheduling problem for tasks involving 25, 50, and 100 orders, demonstrating the superiority of the gray wolf optimization algorithm in addressing such problems.

Considering real-world warehouse scheduling, we proposed a gray-wolf online optimization algorithm based on insertion. An online scheduling experiment was designed to address the problem, and the solution scheme was compared with that of the offline scheduling experiment. The experimental results indicate that the gray wolf online optimization algorithm based on insertion, presented in this paper, performs better in solving the online scheduling problem of a four-way shuttle dense storage system and is more suitable for practical applications.

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