

# A Prestack Three-parameter Inversion Method based on ATpV Regularization

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## Abstract

The Seismic pre-stack inversion technology can make full use of pre-stack information to accurately predict various parameters of reservoir media, which greatly enriches the means of describing reservoirs and is widely used in oil and gas exploration and development. However, seismic inversion is a typical ill-posed problem. Using regularization can overcome this problem and reduce the ill-posedness of the inversion problem. In this paper, a prestack three-parameter inversion method based on ATpV regularization is proposed. This method introduces ATpV regularization into prestack inversion, which effectively solves the ill-posed problem of inversion. The experimental results show that this method has the advantages of small inversion error and good sparsity.

## Keywords

ATpV Regularization; Prestack Inversion; Forward Modeling; Inversion Objective Function; Lq Norm.

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## 1. Introduction

Seismic inversion is widely used in oil and gas exploration and development, and is an important means to obtain parameters such as physical properties, elastic parameters, and anisotropy of underground reservoirs. The basis of pre-stack inversion is the Zoeppritz equation (1919), but because its equation is too complicated and inconvenient to use, many scholars have approximated it. In 1980, Aki & Richard simplified the Zoeppritz equation and proposed an approximate formula for linear reflection coefficient based on the velocity and density of longitudinal and shear waves. In 2003, Buland [1] realized the three-parameter inversion of AVO based on the Bayesian framework. In 2012, Zong Zhaoyun [2] derived the YPD approximation equation on the basis of the Aki & Richard approximation formula, and provided a method for direct inversion of Young's modulus and Poisson's ratio. In 2019, Zhang Rui [3] et al. proposed a two-term approximation equation through Young's modulus and the relationship between Poisson's ratio and density. In 2022, Ge et al. [4] derived a new three-attribute parameter reflection coefficient approximation equation based on Young's modulus and Poisson's ratio, and predicted the brittleness index.

Since seismic inversion is a typical ill-posed problem, it is ill-posed. In 1943, Tikhonov showed that the problem could be solved, and in 1963 proposed Tikhonov regularization [5], which advantageously eliminated the defects of nonlinearity and instability caused by the inversion of the Zoeppritz equation using the L2 norm. In 1992, Rudin et al. [6] proposed Total Variation (TV) regularization and applied it to image denoising, which can represent the sparse information of the difference through the L1 norm. In 2018, Wang et al. [7] introduced anisotropic total variation (ATV) regularization into seismic impedance inversion, which can obtain higher-precision inversion results. In 2018, Li et al. [8] introduced anisotropic total variation regularization (ATpV) based on Lp norm

into wave impedance inversion, which improved the inversion results. In 2021, Liu et al. [9] proposed Hybrid Total Variation (HTV) regularization, which improved the traditional TV regularization and achieved better results in inversion. At present, the regularization method is often used to approximate the solution in seismic inversion to reduce the ill-conditioned nature of the problem.

In this paper, a prestack three-parameter inversion method based on ATpV regularization is proposed. This method can effectively solve the ill-posed problem of seismic inversion and better restore the sparsity of the results. After that, the principle of the method is introduced, and the validity of the method is verified by experiments. The experimental results show that the inversion results of this method have small errors and strong stability.

## 2. Method Principle

### 2.1 Forward Modeling

First introduce the AKi & Richards approximation used this time:

$$R_{pp}(\theta) = (0.5 \sec^2 \theta) R_{v_p} + (-4 \frac{V_s^2}{V_p^2} \sin^2 \theta) R_{v_s} + (\frac{1}{2} (1 - 4 \frac{V_s^2}{V_p^2} \sin^2 \theta)) R_{\rho} \quad (1)$$

Among them,  $\theta$  is the incident angle;  $v_p$ ,  $v_s$  is the velocity of P-waves and S-waves;  $R_{v_p}$ ,  $R_{v_s}$ ,  $R_{\rho}$  are the reflection coefficients of the velocity of P-waves and S-waves and density.

Based on the inversion equation (1) and using the angle-dependent wavelet matrix, the angle-dependent seismic traces  $S(\theta)$  can be expressed as:

$$S(\theta) = \frac{1}{2} w(\theta) a_1 DL_{v_p} + \frac{1}{2} w(\theta) a_2 DL_{v_s} + \frac{1}{2} w(\theta) a_3 DL_{\rho} \quad (2)$$

Among them,  $L$  is the natural logarithm of the longitudinal wave velocity;  $D$  is the difference matrix. The above forward equation can be simplified to:

$$S = AL \quad (3)$$

### 2.2 Prestack Three-parameter Inversion based on ATpV Regularization

First, the concept of anisotropic total p-variation (ATpV) regularization is introduced, and the ATpV regularization with respect to the inversion parameters can be defined as:

$$ATpV(L) = \|LD_x\|_p^p + \|D_y L\|_p^p \quad (4)$$

Among them,  $\lambda$  is the regularization factor of the anisotropic total variation difference,  $\| \cdot \|_q$  is the  $L_q$  norm,  $D_x$  is the horizontal difference, and  $D_y$  is the vertical difference.

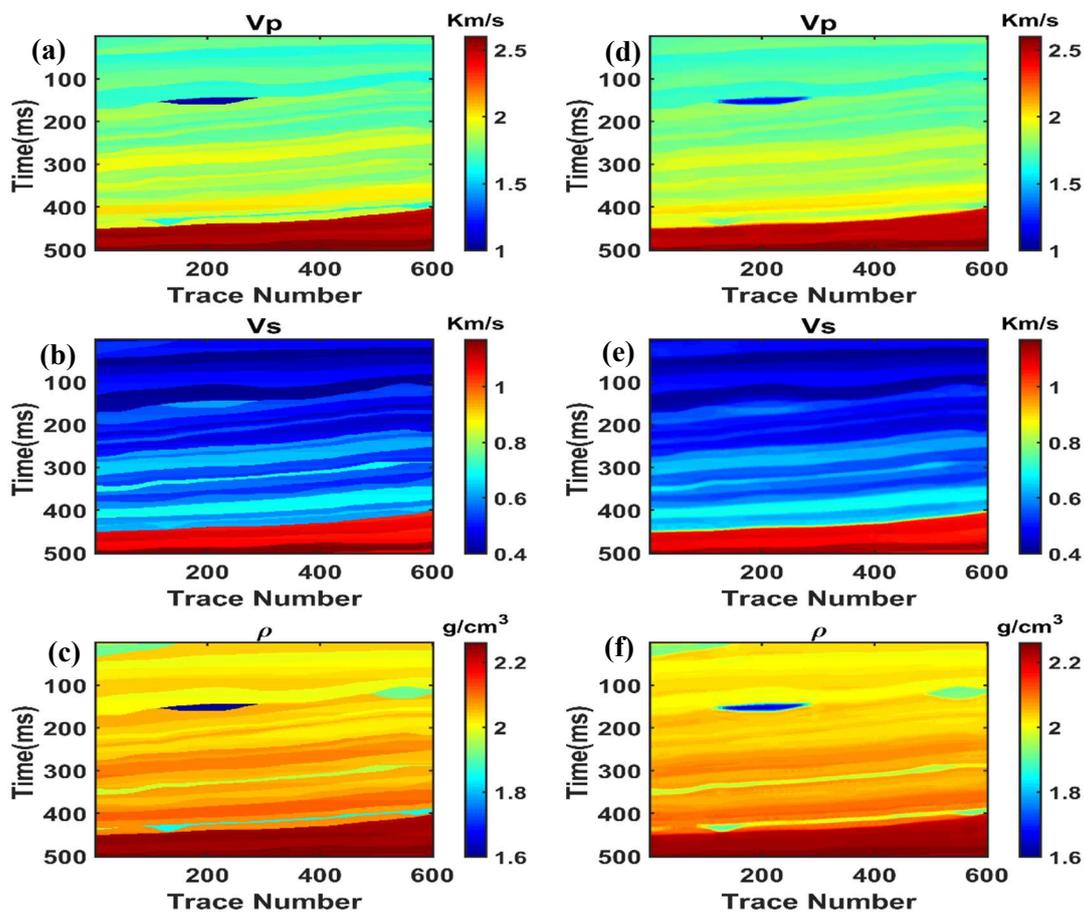
Based on the forward equation, the inversion objective function of the ATpV regularization constraint can be expressed as:

$$J(L) = \min_L \|AL - S\|_2^2 + \mu \|L - L_0\|_2^2 + \lambda (\|LD_x\|_p^p + \|D_y L\|_p^p) \quad (5)$$

Among them,  $L_0$  is the initial model,  $\mu$  is the regularization parameter of the initial model, and  $\lambda$  is the regularization factor of the anisotropic total variation difference.

### 3. Analysis of Results

The In order to verify the feasibility of the method, the theoretical model is tested and analyzed first. The theoretical model is shown in Fig. 1, which includes three parameters, Fig. 1(a) is  $V_p$ ; Fig. 1(b) is  $V_s$ ; Fig. 1(c) is density. The model has 600 horizontal tracks, and each track includes 500 sampling points. Fig. 1(d)-(f) are the inversion results. From the comparison between the inversion results and the exact model, it can be seen that the inversion results of this method are close to the real model, and have good continuity in the horizontal direction. The stratigraphic boundary is clear and can accurately reflect the geological structure. It can be seen that the inversion results of the method proposed in this paper have less error and higher accuracy.



**Fig. 1** (a)  $V_p$  real model; (b)  $V_s$  real model; (c) Density real model; (d)  $V_p$  inversion results; (e)  $V_s$  inversion results; (f) Density inversion results

The method is then verified by actual data. The actual data used this time is located in the southeast of Sichuan Province. The inversion result profiles of the three parameters are shown in Fig. 2-Fig. 4, and Fig. 5 is the single-track inversion result of the logging location. It can be seen from the profile of the inversion results that the vertical resolution is high, which can effectively describe the stratigraphic characteristics and accurately identify the reservoir location. The single-track results show that the inversion results of this method are in line with the real values as a whole, and the abrupt part of the values is the reservoir position. In this area, the reservoir characteristics can be accurately inverted, with small errors and high precision. The effectiveness of the method is further demonstrated.

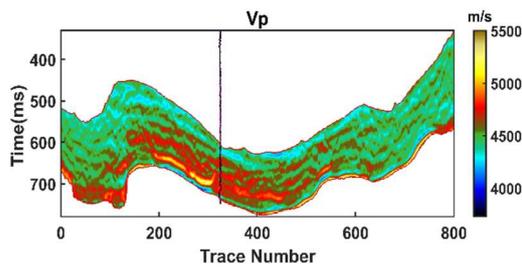


Fig. 2 Vp inversion results

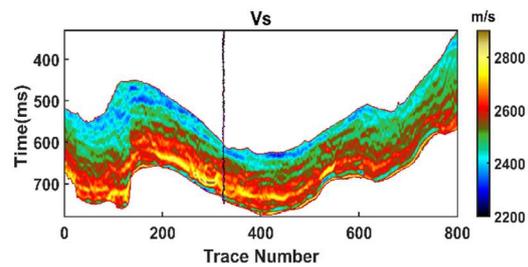


Fig. 3 Vs inversion results

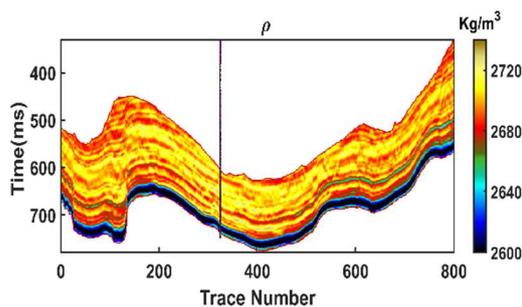


Fig. 4 Density inversion results

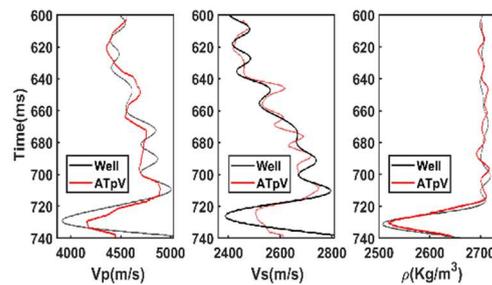


Fig. 5 Single-track inversion results

## 4. Conclusion

In this paper, we propose a pre-stack three-parameter inversion method based on ATpV regularization. Due to the ill-conditioned problem of seismic inversion, this method can reduce the ill-conditioned inversion problem and improve the inversion accuracy by constraining the inversion objective function through ApV regularization. The experimental results show that the inversion results of this method have high precision and practical significance.

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