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Dynamic Program Glen Canyon-Hoover Dam Cascade Reservoir Summary

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Abstract

Because of the drought condition in the Colorado River basin, officials from five states are negotiating to determine the best way to manage water and electricity production from the Glen Canyon Dam and the Hoover Dam. In this paper, we analyze the Glen Canyon Dam-Hoover Dam cascade reservoir operation and propose the corresponding water allocation plan. Specifically, we conduct nonlinear regression analysis on the water level and storage capacity of the two reservoirs, and the functional relationship between the water level and storage capacity was obtainedit with correlation coefficients R2 >0.99. Furthermore, we establish the Operate Model of Cascade Reservoir (OMoCR) under constraint conditions. The Particle Swarm Optimization (PSO) is used to simulate the variation of the water storage of different reservoirs. Finally, we establish a multi-objective optimization model based on water supply and power generation competition to achieve maximum economic and social benefits. The constraint method and interactive decision preference method are used to transform the multi-objective optimization model into single objective optimization model. Take the case of drought as an example for further analysis, and use Dynamic Programming (DP) to solve the model.

Keywords

Water Allocation; Multi-objective Optimization Model; Dynamic Programming.

1. Introduction

1.1 Problem Background

Since the formation of the Colorado River Water Project system, river development has brought a steady flow of water and power to the riparian states, giving rise to large agricultural and metropolitan areas in the desert of the American Southwest. However, as economic and social development continues, water use in some states has increased, and the agreements made hundreds of years ago no longer meet the basic water and power needs of the states today. Naturally, the question arises as to how to develop a reasonable water allocation plan is critical. The purpose of this paper is to closely coordinate the operation of Glen Canyon Dam (Lake Powell) and Hoover Dam (Lake Mead), balancing the allocation of water resources between the interests of each party and between supply (water availability) and demand (electricity requirements).

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement to address the following problems:

Problem 1. Develop and analyze a mathematical model to help negotiators cope with a fixed set of water supply and demand conditions. Use the model to inform the operation of a dam under different conditions.

Problem 2. Develop a model to describe the process of competition for water resources between general use and power production, with the goal of maximizing the benefits of competition to recommend the best method of water allocation.

Problem 3. Use the model to address what should be done to meet all water and power demands when water resources are constrained.

Problem 4. Represent the model in multiple scenarios and analyze and evaluate our model.

1.3 Our Work

To solve this problem, we need to balance water and hydroelectric power, allocate water rationally and address competing interests. We inform the dam work according to the target above.

In this paper, we will focus on the operation process of the cascade reservoir and analyze it in detail. A operate model of cascade reservoir based on dynamic weight is proposed. The model describes the operation mechanism of two closely related dams by compensating the storage capacity of upstream reservoir to downstream reservoir. By considering our model, we can recommend the best means to resolve the competing interests of water availability for general usage and electricity production.

In Section 2, we state several basic assumptions. Section 3 contains the notations used in model statement. Section 4 provides sufficient details about our model and applications to solve general problems. Section 5 presents criteria and detailed recommended strategies for addressing competing interests. Section 6 further studies our model and provides sensitivity analysis. Finally, we evaluate the models in Section 7 to identify their strengths and weaknesses. At the end of the article, we provide an article about our findings for Drought and Thirst magazine.

2. Assumptions and Justifications

To simplify the problem, we make the following assumptions to complete our model:

The value of the loss of water upward evaporation as well as downward infiltration in the reservoir exists anytime and is approximately constant. The effects caused by evaporation and infiltration are small and can be replaced by average values.

Ignore the expansion and contraction of water. Temperature-induced changes in water volume have a limited effect on the results. Therefore, it does not make sense to introduce expansion and contraction in our system.

The impact of other existing lakes and dams in the Colorado River Basin on the model is ignored. Without intercepting other dams, it affects only the input data of the river and the study can be simplified.

The data we use in our models are reliable and accurate. All data sources are reliable, and the optimization of the problem and the strategies developed are based on actual data.

3. OMoCR: Operate Model of Cascade Reservoir

3.1 The Preparation of Model

3.1.1 Relationship between Reservoir Capacity and Level

In our model, we need to address the volume of water taken from Glen Canyon Dam and Hoover Dam when the community's demand reaches a specified level and the water in the two reservoirs reaches a specified height. Therefore, the relationship between the water level in the reservoirs and the volume of water needs to be considered.

Due to the geographical location of the reservoir, the inflow and outflow of the reservoir and other factors such as precipitation, the relationship between the reservoir level and the reservoir capacity usually shows a highly nonlinear relationship. Gaussian function [1] is selected as the nonlinear regression model for the relationship between reservoir level and reservoir capacity of the two reservoirs, and the regression analysis is carried out using the data of reservoir level and reservoir capacity of the two reservoirs, and the specific model is as follows:

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$$y = ae^{-\left(\frac{x-b}{c}\right)^2} \tag{1}$$

where:

x is water height in the reservoirs;

y is the volume of water in the reservoirs;

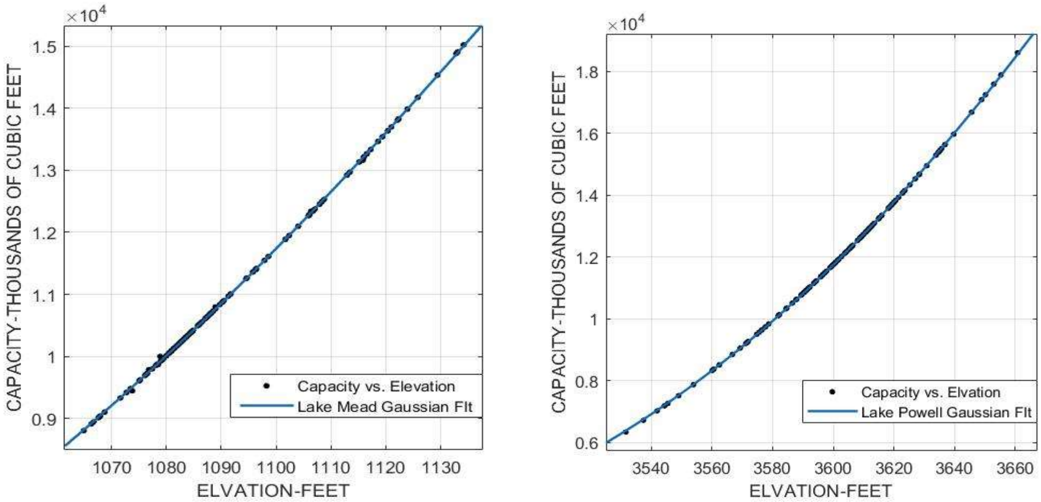
a, b, c is three parameters of the Gauss equation.

The Gaussian model can fit the relationship between reservoir level and reservoir capacity well and develop an accurate model of the relationship between reservoir capacity and reservoir level. The simulated results can be used as an important reference basis for the optimization and operational scheduling of the Colorado River reservoir.

As can be seen from Table 1 and Figure 1, the correlation coefficients R^2 between water level and reservoir capacity of the two reservoirs are all higher than 0.99 when the significance level α=0.05, and the significance probability R< 0.05 for the three models, indicating that the Gaussian function model can fit the relationship between reservoir level and reservoir capacity very well.

Table 1. Calculated results of nonlinear regression analysis between reservoir level and reservoir capacity

Parameters	Lake Mead		Lake Powell	
	Calculated values	Confidence interval	Calculated values	Confidence interval
a	3.32×10^4	$[3.167\times10^4, 3.473\times10^4]$	5.484×10^4	$[5.355\times10^4, 5.613\times10^4]$
b	1371	[1359,1383]	3977	[3971,3983]
c	265.9	[260,271.8]	303.6	[301.2,306]
R^2		0.9999		1
SSE		1.786×10^4		1.318×10^4
RMSE		11.77		10.07



a.Lake Mead b.Lake Powell

Figure 1. Nonlinear regression analysis model between reservoir level and storage capacity

3.1.2 Analysis of Reservoir Operation Mechanism

In order to study water resources management issues and to gain insight into how the reservoir operates, the inflow and outflow models within the reservoir are to be studied. For convenience, we describe as in Figure 2:

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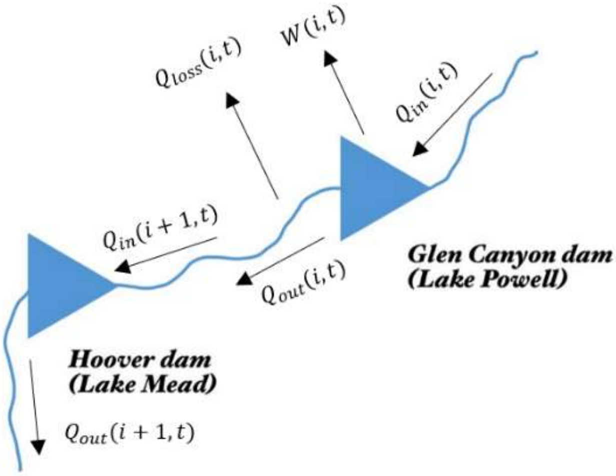

Figure 2. Illustration to water cycle in reservoirs

Water can be accumulated and reserved in reservoirs created by dams. By establishing the water balance equation of the reservoir, the change of water volume during the water transfer can be calculated. Then, the water balance [2] equation can be expressed as:

$$V_{i,t+1} = V_{i,t} + Q_{i,t}^{in} - Q_{i,t}^{out} - Q_{i,t}^{loss} - W_{i,t} \tag{2}$$

where:

$V_{i,t+1}, V_{i,t}$ are the storage capacity of reservoir i at the beginning and end of time period t, respectively;

$Q_{i,t}^{in}, Q_{i,t}^{out}, Q_{i,t}^{loss}$ are the incoming water, outgoing water, and lost water of reservoir in time period t;

$W_{i,t}$ is the part of water for external use, including agriculture water supply, industry water supply, residential water supply and water for hydropower generation.

This paper examines not just the operation of a single reservoir, but two cascade reservoirs. Their operations are closely related, and the water flowing from Glen Canyon Dam supplies part of the water input from Hoover Dam.

$$Q_{Powell}^{out} = cQ_{Mead}^{in} \tag{3}$$

where:

c is the proportion of water outflows from the Glen Canyon dam supply part of the water input to the Hoover dam, between (0,1).

When no additional water is provided in the system, the output of the Lake Powell reservoir is the entire input of the Lake Mead reservoir, i.e., c = 1.

In terms of the water balance equation on the macro system, Q_t^{out} is the amount of water flowing out of the entire reservoir system, i.e., how much more water is eventually allowed to flow from the Colorado River into the Gulf of California, after the water allocation scheme is implemented, for the remaining water resources available for Mexico to have. When in a dry period, as the amount of water entering the reservoir is reduced, the amount of water leaving the reservoir is correspondingly reduced. But the $Q_t^{out} = 0$ situation is not allowed, which would violate Mexico's water rights.

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When Mexico's rights are not met, it needs to be financially compensated [8]. According to the market value p of Mexican water and the water quantity q not obtained in Mexico's equity, the compensation amount m that Mexico should receive is determined.

$$m = \sum_{i=1}^{N} p_i \times q_i \tag{4}$$

Here is p_i because different kinds of water have different prices, for example industrial water and domestic water. This method is the most commonly used method in the world at present. For example, water resources compensation in the Mekong River basin firstly determines the allocation of water rights in the Mekong River, and then each country determines the compensation amount of water transferred between them according to its own water consumption and the market value of local water.

3.2 The Establish of Model

The geographical location of Glen Canyon Dam and Hoover Dam shows that the two reservoirs operate in close coordination and that the reservoirs form a tandem reservoir complex due to hydraulic and hydrological linkages between them.

Figure 3. Location of Glen Canyon and Hoover Dam

By reviewing the information [3,4], the water supply between reservoir clusters makes full use of water and hydro energy resources by means of reservoir capacity compensation to satisfy water supply (agriculture, industry, and residences) and water for power generation. In order to realize the simultaneous water supply of each member reservoir in the reservoir group in the actual problem, a Operate Model of Cascade Reservoir (OMoCR) based on dynamic weights is established.

Scheduling principles/instructions:

When supplying water, Lake Mead, which has a small regulating capacity downstream, is used to supply water to its maximum capacity, and the remaining water supply task is undertaken by Lake Powell, which has a large regulating capacity upstream. In this way, the storage capacity of large reservoirs can be fully utilized and the abandonment of reservoirs can be reduced.

In order to ensure the power generation needs of the downstream reservoirs, the release and storage of water from Lake Powell must take into account the compensation of the storage capacity of Lake Mead downstream.

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Regarding the compensation timing, when the water level of upstream Lake Powell is lower than the compensation water supply line, in order to avoid the reduction of its own water supply due to the compensation water supply, no compensation water supply is provided to Lake Mead at this time; when the water level of upstream Lake Powell is higher than the compensation water supply line and the water level of downstream Lake Mead is lower than the compensation water supply line, that is, the water supply capacity of the downstream reservoir itself cannot meet the demand for power generation water supply, compensation water supply is required at this time.

Regarding the determination of compensation water, dynamic weighting is used to determine the reservoir replenishment water, and the weighting coefficient is determined according to the reservoir's current time available water supply and reservoir capacity coefficient, which is calculated as follows:

$$\omega_i = \frac{\frac{VP_{i,t}}{\beta_i^2}}{\frac{VP_{1,t}}{\beta_1^2} + \frac{VP_{2,t}}{\beta_2^2}} \tag{5}$$

where:

$i = 1$,Lake Powell Reservoir;$i = 2$,Lake Mead Reservoir;

$VP_{i,t}$ is the amount of water available in time t;

β_i is reservoir capacity factor.

In the planning and design stage as well as in the actual water supply scheduling operation, the available reservoir capacity and reservoir capacity factor at a certain time are determined.

The Objective function:

The objectives set are different for problems with different specific optimization objectives. In the subsequent model, the objective function is set individually corresponding to each problem, which is done in the solution of the model.

The Decision Variables:

The decision variables of the model are the joint reservoir scheduling process, and the number of decision variables should be kept as small as possible in order to optimize the solution for practical operation. The key node of a combined variable controlling the input for decision making, which describes the two reservoir scheduling processes, can be generalized to the following equation:

$$X = \begin{pmatrix} X_{1,t_1}, & X_{1,t_2}, & \cdots & X_{1,t_n} \\ X_{2,t_1}, & X_{2,t_2}, & \dots & X_{2,t_n} \end{pmatrix} \tag{6}$$

where:

n is number of scheduling nodes;

t_n is the nth scheduling node corresponds to the time period;

$X_{i,t}$ is reservoir i, water level at time t.

Owing to the limit in reservoir's scale and serviceability, the restraints are listed as follows:

The Constraints:

(1) Water balance constraint.

(2) Reservoir water level constraint.

$$Z_{i,t}^{min} \le Z_{i,t} \le Z_{i,t}^{max} \tag{7}$$

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where:

$Z_{i,t}^{min}$ is i reservoir t time can reach the minimum operating water level, the general situation is the reservoir dead water level;

$Z_{i,t}$ is water level in reservoir i at time t;

$Z_{i,t}^{max}$ is i reservoir t time can reach the highest operating water level (flood stage for the flood limit, non-flood stage for the normal reservoir storage level).

(3) Reservoir water level - reservoir capacity curve constraint.

$$V_i = f_i(Z_i) \tag{8}$$

where:

$f_i(Z_i)$ is i reservoir h-v (water level - reservoir capacity function).

(4) Non-negative constraint: All the above variables are non-negative.

3.3 The Solution of Model

Data from five continents and two dams were collected, and the month-by-month information on incoming runoff and water demand in 2021 was used as input information for the model to carry out the reservoir simulation model. Based on the geographical location, the water supply required by the five continents was reasonably allocated to Lake Powell and Lake Mead, which were divided into Lake Powell reservoir area and Lake Mead reservoir area from top to bottom according to the direction of water supply allocation. Following the top-down allocation direction, a water balance analysis was performed for Lake Powell, and the incoming water from the Lake Powell reservoir area was combined to derive the pumping process for this reservoir area as part of the input for the Lake Mead reservoir area.

Table 2. Characteristics of the two reservoirs (feet)

	Dead pool	Minimum power pool	Capacity
Lake Powell	3370.0	3490.0	3700.0
Lake Mead	895.0	1050.0	1229.0

The proportion of water allocated to each state and use is shown in Figure 4:

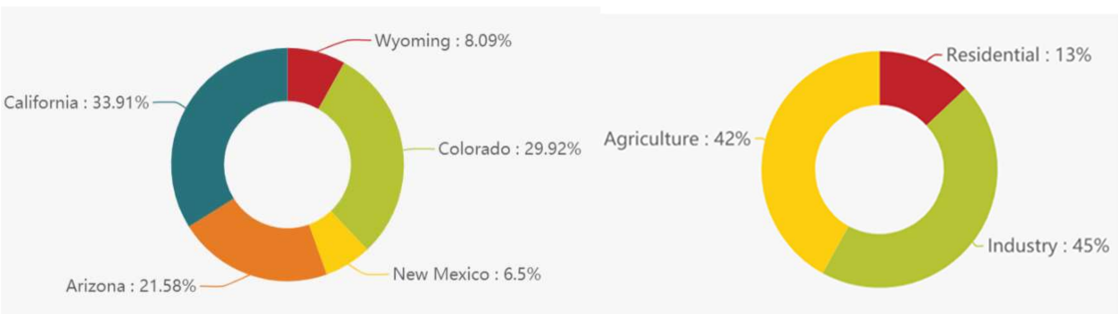

Figure 4. Colorado River Apportionments: U.S. States

The water supply from these two dams needs to be allocated to agriculture, industry, and the residents of the five states, in addition to power generation. Thus water resources need to be allocated in four ways. In addition to this, the water supply to Lake Powell also needs to take into account the

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compensatory water supply to Lake Mead, as the outflow from Glen Canyon Dam supplies part of the water input from Hoover Dam.

For a given set of water levels, what needs to be solved for is the amount of water withdrawn from each lake. We describe the volume of water pumped, S, as the change in reservoir capacity, or net water discharge, before and after reservoir dispatch. According to the water balance equation, the discharge from each dam should be regulated at a constant level under normal water cycle conditions. This is consistent with the historical data on the change in reservoir capacity over time for Lake Powell and Lake Mead reservoirs.

For this purpose, we established the pumping variance R^2 to describe the level of reservoir stability. the definition and programming model of R^2 is as follows.

$$min\, R^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left[S_{i,t} - \bar{S_i}\right]^2 \tag{9}$$

$$\begin{cases} V_{i,t+1} = V_{i,t} + Q_{i,t}^{in} - Q_{i,t}^{out} - Q_{i,t}^{loss} - W_{i,t} \\ Q_i^{outmin} \le Q_{i,t}^{out} \le Q_i^{outmax} \\ Z_{i,t}^{min} \le Z_{i,t} \le Z_{i,t}^{max} \\ V_i = f_i(Z_i) \\ S_{i,t} = V_{i,t+1} - V_{i,t} \end{cases} \tag{10}$$

Where:

$S_{i,t}$stands for the water discharge of the reservoir i in month k.

n refers to the month to be studied.

$\bar{S_i}$is the mean discharge in month n.

In order to find the optimal solution, Particle Swarm Optimization (PSO) [5] was used to simulate the variation of water storage in different reservoirs. A better pumping scheme was successfully implemented by several calculations compiled by MATLAB.

The monthly storage variance in the reservoirs is represented by the figures in Table 3:

Table 3. Water Withdrawals Scheme of the Reservoirs in Normal Years(10^6km^3)

Month	**Jan**	**Feb**	**Mar**	**April**	**May**	**Jun**
Lake Powell	-8.7632	7.6513	7.6643	6.2821	4.6332	6.2381
Lake Mead	-7.6654	6.6754	6.4864	5.6727	3.9825	5.2439
Month	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Lake Powell	-9.6724	-8.6345	-9.4282	-0.3566	-7.8568	-10.0563
Lake Mead	-0.6253	-1.2787	-0.8794	-0.2413	-0.7324	-0.7348

For example, -8.7632 means that in January, the Lake Powell must reduce water storage by 8.7632. To put it another way, the net water discharge is $8.7632 \times 10^6 km^3$. It's worth noting that the data for quantitative calculations is gathered in regular years. Even though no extreme circumstances have been proven, we cannot immediately apply the figures presented in Table 4 when dealing with a specific year. Under regular water cycles, however, a preliminary treatment is given. In the months of Jan, Jul, Aug, Sep, Oct, Nov, and Dec, the reservoir should hold water. Water storage should be lowered in the remaining months.

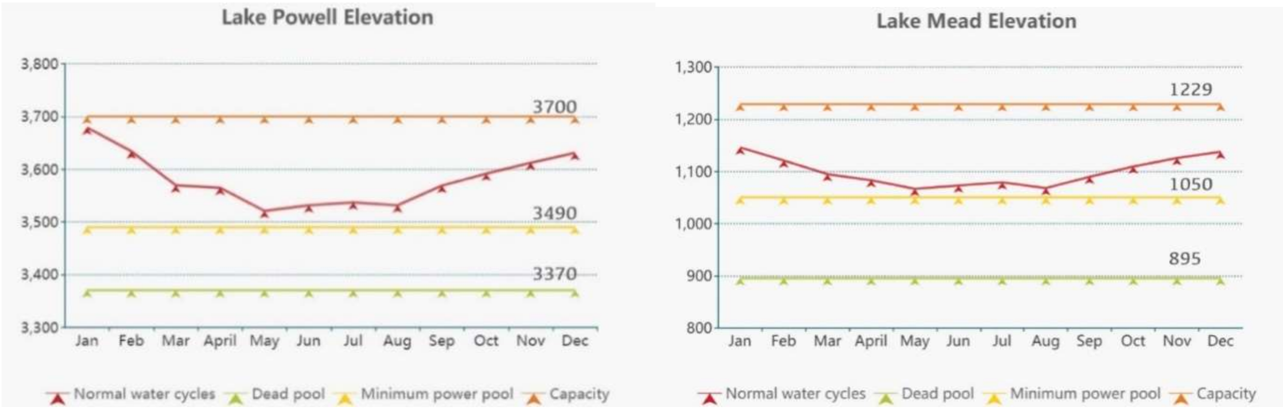

Figure 5. Lake Powell and Lake Mead Elevation

If no additional water is provided (through rainfall, etc.) and the monthly demand is fixed, what needs to be solved for is the number of days consumed to meet the demand. This case will be viewed from a macroscopic point of view, the two reservoirs as a dynamically balanced aggregated reservoir. The water supply in this case is actually provided by the input flowing into the whole reservoir system. As a result, we build the programming objective as follows:

$$min\,T \tag{11}$$

$$\begin{cases} V_{t+1} = V_t + Q_t^{in} - Q_t^{out} - Q_t^{loss} - W_t \\ Q_t^{outmin} \le Q_t^{out} \le Q_t^{outmax} \\ V^{min} \le V_t \le V^{max} \\ \sum_{t=1}^{T} W_t = Need \end{cases} \tag{12}$$

Solve for the number of days per month that need to be pumped (28 days are taken in April) as shown in Table 4:

Table 4. Number of days of water withdrawal from the reservoir per month

Month	**Jan**	**Feb**	**Mar**	**April**	**May**	**Jun**
Days	7.238	7.673	6.529	8.874	12.375	14.027
Month	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Days	15.276	16.725	12.647	10.371	6.785	6.941

Over time, the amount of water in the reservoir is diminishing and the monthly water supply cannot meet the fixed demand. When the number of days needed to complete the monthly water supply exceeds the number of days in each month, additional water needs to be provided to ensure that the fixed demand is met. We have set the number of days here to 30 days. The simulation is still performed using the model from the previous question. The results are shown in Table 5.

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Table 5. Result of optimal operation of two serial feeding reservoirs

Month	Lake Powell				Lake Mead			
	Water requirements $/km^3$	Additional water supply$/km^3$	Reservoir water supply$/km^3$	Water supply rate/%	Water requirements $/km^3$	Additional water supply$/km^3$	Reservoir water supply$/km^3$	Water supply rate/%
Jan	27029.57	640.97	24661.78	91.24%	7722.73	173.59	7085.60	91.75%
Feb	26250.31	723.77	23725.03	90.38%	7500.08	217.15	6843.83	91.25%
Mar	27115.74	867.79	24894.96	91.81%	7747.35	381.12	7022.97	90.65%
April	27021.24	1082.46	24629.86	91.15%	7720.35	461.25	7084.19	91.76%
May	27195.54	1421.37	24701.71	90.83%	7770.15	505.68	7134.55	91.82%
Jun	27672.44	1739.77	25082.3	90.64%	7906.40	573.57	7162.40	90.59%
Jul	28566.77	1973.58	26184.3	91.66%	8161.93	604.86	7441.23	91.17%
Aug	29278.51	2093.83	26514.62	90.56%	8365.28	639.33	7665.95	91.64%
Sep	28963.04	2279.27	26521.46	91.57%	8275.15	672.57	7516.32	90.83%
Oct	27758.57	2341.25	25340.80	91.29%	7931.28	684.86	7274.31	91.72%
Nov	26225.58	2590.14	23684.32	90.31%	7493.02	725.99	6808.16	90.86%
Dec	26301.96	2763.89	24160.98	91.86%	7514.84	758.83	6842.26	91.05%

4. Water Supply - Power Generation Competition Model

4.1 The Establish of Model

The main task is to allocate water to the power stations on the dam and other users. Therefore, we could restate the task with two points:

Adjust the water in reservoirs to meet the requirement for general usage.

Transfer the water discharge capacity to meet electricity production request.

We can find that there is a competition between these two tasks. This requires planning the allocation of water resources based on model 1, when water resources for general (agricultural, industrial, residential) use and power production are in competition. We summarize these two tasks as the economic and social benefits generated by dams. Among them, the marker of economic benefit is the generation of electricity by dams, and the marker of social benefit is meeting the need for water. We should make full use of water resources so as to make the economic and social benefits as large as possible.

The Power Generation Benefit:

$$I_e = \sum_{i=1}^{n} \sum_{t=1}^{T} B_{i,t} \tag{13}$$

Where:

$B_{i,t}$ is the power generation benefit of reservoir i at time t. The month is taken as the calculation period.

Let $E_{i,t}$ be the power generation capacity of reservoir i at time t, then the power generation efficiency function can be written as:

$$B_{i,t} = cE_{i,t} \tag{14}$$

Where:

c is the price of power supply.

Since the current electricity supply in the United States uses the electricity tariff, c is a constant, so $B_{i,t}$ can be directly expressed as $E_{i,t}$ in this paper.

In order to create huge economic benefit, reservoirs meet a large number of agricultural, industrial, and domestic needs by supplying water to water users, and bring huge social benefits.

$$I_s = min \sum_{i=1}^{n} W_{i,t} \tag{15}$$

The minimum water supply in the study period is taken as the water supply quantity, mainly because the trade-off between power generation and social benefits is only needed when there is water shortage. Water shortage often occurs during the dry period, when the reservoir discharge is low, and the water supply quantity is the most valuable supply quantity.

Total benefits as the objective function:

$$I = max(I_e + I_s) \tag{16}$$

In summary, this paper uses Multi-Objective Optimization approach [6] to model the water allocation problem.

The formulas of this model is:

$$\begin{gathered} max \sum_{i=1}^{n} \sum_{t=1}^{T} E_{i,t} \\ max \left[min \sum_{i=1}^{n} W_{i,t} \right] \end{gathered} \tag{17}$$

$$\begin{cases} V_{i,t+1} = V_{i,t} + Q_{i,t}^{in} - Q_{i,t}^{out} - Q_{i,t}^{loss} - W_{i,t} \\ V^{min} \le V_{i,t} \le V^{max} \\ NN \le N_t \le NT \\ W_{i,t} \le WT \end{cases} \tag{18}$$

Where:

N_t is the power output of the turbine at time t;

NN is the technical minimum output of the turbine;

NT is the expected output of the hydraulic turbine;

WT is the maximum overwater flow of the dam turbine.

To facilitate solving this multi-objective planning problem, we need to perform a transformation of the model by first using the constraint method to find the set of non-inferior solutions of the multi-objective model, and then applying the interactive decision preference method to find the best trade-off solution from the set of inferior solutions. Turning the objective function of maximizing water supply into a constraint, i.e., setting:

$$min \sum_{i=1}^{n} W_{i,t} \ge WF \tag{19}$$

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The equation is equivalent to:

$$\sum_{i=1}^{n} W_{i,t} \geq \mathrm{W}F \tag{20}$$

Where:

WF is the amount of water supply to be determined and is a parameter to be found.

After the above transformation, the multi-objective optimization model becomes a single-objective optimization model with the following model form:

$$max \sum_{i=1}^{n} \sum_{t=1}^{T} E_{i,t} \tag{21}$$

$$\begin{cases} V_{i,t+1} = V_{i,t} + Q_{i,t}^{in} - Q_{i,t}^{out} - Q_{i,t}^{loss} - W_{i,t} \\ V^{min} \leq V_{i,t} \leq V^{max} \\ NN \leq N_t \leq NT \\ W_{i,t} \leq WT \\ \sum_{i=1}^{n} W_{i,t} \geq WF \end{cases} \tag{22}$$

The above single-objective optimization model is a nonlinear optimization model, and we choose the Dynamic Programming (DP) [6] method to solve it.

In practice, water supply cannot be fully met. In the case of dry water, a certain degree of unsatisfied water supply demand should be allowed, i.e., water supply destruction, especially in the case of high water supply requirements (e.g., drought), which makes it difficult to make the water supply demand be completely satisfied. To solve this problem, the objective function is combined with the water supply constraint to obtain the following objective function.

$$max \sum_{t=1}^{T} \left[\sum_{i=1}^{n} E_{i,t} - w \times A \times \sigma_t \times \left(WF - \sum_{i=1}^{n} W_{i,t} \right)^b \right] \tag{23}$$

After the above transformation, the water supply constraint is weakened and a negative component: $w \times A \times \sigma_t \times (WF - \sum_{i=1}^{n} W_{i,t})^b$ is added to the objective function as a penalty term. The effect of this term is that when the actual water supply is less than the agreed water supply WF, it will be penalized and the total efficiency will be reduced. Thus, the reservoir is compulsorily forced to increase the water supply, and σ_t is the variable in the penalty term, i.e:

$$\sigma_t = \begin{cases} 0 & \sum_{i=1}^{n} W_{i,t} \geq WF \\ 1 & \sum_{i=1}^{n} W_{i,t} < WF \end{cases} \tag{24}$$

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Where:

w, b are the model parameters, and by adjusting the magnitude of w and b, the degree of satisfaction of the water supply WF can be adjusted.

A is the scale adjustment factor.

With the above transformations, the final single-objective optimal scheduling model is of the following form:

$$max \sum_{t=1}^{T} \left[\sum_{i=1}^{n} E_{i,t} - w \times A \times \sigma_t \times \left(WF - \sum_{i=1}^{n} W_{i,t} \right)^b \right] \tag{25}$$

$$\begin{cases} V_{i,t+1} = V_{i,t} + Q_{i,t}^{in} - Q_{i,t}^{out} - Q_{i,t}^{loss} - W_{i,t} \\ V^{min} \leq V_{i,t} \leq V^{max} \\ NN \leq N_t \leq NT \\ W_{i,t} \leq WT \\ \sum_{i=1}^{n} W_{i,t} \geq WF \end{cases} \tag{26}$$

This single-objective optimal scheduling mathematical model is solved by the dynamic programming method. The solution of the single-objective optimization model is a non-inferior solution of the multi-objective optimization model. By taking several water supply solutions within the upper and lower limits of the WF, several single-objective optimization models are obtained, and the calculation of the single-objective optimization model is performed to obtain the set of non-inferior solutions of the multi-objective optimal scheduling of the reservoir.

To select the best solution from the set of inferior solutions, we construct a decision preference coefficient g, which is calculated as follows:

$$g = -\frac{dB}{dWF} \tag{27}$$

The physical meaning of g is the decrease of reservoir generation efficiency caused by the increase of unit water supply. In the physical sense, g can be regarded as the shadow price of reservoir water supply, which can be used as the reference price of reservoir water supply. Generally, when WF increases, g will gradually increase, and the rate of increase will be faster and faster.

The optimal reservoir scheduling plan can be based on the magnitude of g value and other factors to determine a suitable water supply scale from the non-inferiority solution, i.e. to choose a reservoir scheduling plan that has a certain water supply scale and does not reduce the power generation efficiency too much.

4.2 Strategies for Emergency Drought Situation

For a given water supply target WF, the reservoir does not have enough water supply during dry periods to meet all water and power demands. In this case, the reservoir will be penalized, thus affecting the overall total benefit. By adjusting the magnitude of w and b, we find the right combination of water transfer and adjust the scheduling method so that the total benefit in this case is the highest. Here we study the case when the reservoir's water supply cannot meet the demand, i.e., the penalty must be present:

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$$\sum_{i=1}^{n} W_{i,t} < WF \tag{28}$$

The model parameters b and w have a certain influence on the calculation results. By adjusting the magnitude of the model parameters b and w, the water supply guarantee rate and the annual power generation capacity of the hydropower plant can be adjusted.

We adjust the model parameters b, w to regulate the guaranteed rate of water supply and the annual power generation capacity of the hydropower plant. When the water resources are in shortage, the guaranteed rate of water supply is smaller.

In the calculation, by adjusting the magnitudes of w and b in model, we can adjust the reservoir scheduling method to have relatively high guaranteed rate of water supply and annual power generation during the dry period with the same WF. By using the Dynamic Programming Method, the indicators of each scenario can be obtained for the multi-objective inferior solution set. The results of each scenario are listed in Table 6, and the values of decision preference coefficient g for each scenario are also listed in Table 6.

Table 6. Water supply-generation multi-objective scheduling non-inferiority solution set

Solutions	Water Supply /(m^3/s)	Water supply guarantee rate/%	Annual power Generation/(10^6kWh)	Decision preference coefficient g
1	30	64.28%	24454.13	34.8134
2	30	71.19%	24446.78	38.8967
3	30	79.87%	24315.95	17.6144
4	30	73.56%	24286.99	19.3735
5	30	68.76%	24226.49	27.8134

From the perspective of the decision preference factor, the best solution is the one with a water supply quantity $\boldsymbol{WF}$ of 30 and a water supply guarantee rate of 79.87%.

5. Model Evaluation

5.1 Strengths

● We analyze the operation mechanism of cascade reservoir from the principle of water balance. Focus on macro deployment and avoid tedious micro analysis.

● We develop corresponding modulating strategy based on the situation and make a relative comprehensive plan. This can inform specifically how the dam is operating.

● The structure of the whole model is rigorous and reasonable. It can also be applied to other dams and has good universality.

● We do sensitivity analysis to test the accuracy of our model. It shows that our models are fairly robust to changes in parameter value.

● Our data comes from the official website such as United States Bureau of Reclamation (USBR), which is believable.

5.2 Weaknesses

● Some actual water conditions are omitted. The real dam operations are much more complicated. So actually the solution of our equation may be not fully consistent with actual data numerically.

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- We paid an emphasis on the balance between economic and social benefits, while lack details of weighing competing relationships. For example, power supply may have other benefits to dam construction.
- We mainly focus on the current benefits, while overlook the situation in the future. But in fact, future prospect is important for sustainable development.
- We are limited to obtain part of electricity and water requirement data. Therefore, the analysis of requirements may not be comprehensive and may have some deviation.

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