

# Analysis of City Rainfall Trends based on M-K Test and Grey Prediction

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## Abstract

**This paper mainly studies the rainfall patterns and cycles of some cities, predicts their future rainfall and conducts quantitative analysis. Firstly, the daily precipitation data of three meteorological stations in Zhengzhou for the past years were pre-processed, and the data of the past 42 years were selected to summarize the annual average precipitation of Zhengzhou for the past years. Then the M-K test program based on MATLAB platform was written to analyse the sudden change of annual average precipitation, calculate the wavelet coefficients of annual average precipitation and plot the wavelet variance. The real contour plot of wavelet coefficients is plotted using SURFER, and the results of the abrupt variation are combined to derive the prediction of annual rainfall trends and the law of periodic variation, which provides some reference significance for the prediction and analysis of precipitation trends in other cities.**

## Keywords

**M-K Test; Grey Prediction; Annual Rainfall Prediction; Rainfall Cycle.**

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## 1. Introduction

With the rapid development of modern society and technology, global climate anomalies occur frequently. In recent years, extreme weather phenomena have occurred frequently in some parts of China [1]. Disasters such as heavy rainfall, snowstorms and floods have caused serious damage to local areas and affected people's safety and livelihoods, especially in the northern regions. This paper focuses on studying the rainfall patterns and cycles of some cities, predicting their future rainfall and conducting quantitative analysis, establishing time series models, and combining M-K test analysis, wavelet analysis and grey prediction for comprehensive research.

Firstly, the conceptual daily precipitation data of the city's meteorological stations were pre-processed and the data of the last 42 years were selected to summarise the annual average precipitation of the city in the past years [2]. Then the M-K test program based on MATLAB platform was written to analyse the sudden change of annual average precipitation, calculate the wavelet coefficients of annual average precipitation and plot the wavelet variance. Using SURFER to plot the true contours of wavelet coefficients, combine the results of abrupt variation and draw conclusions to build a grey prediction program to evaluate different hazards and obtain the annual rainfall trend and cyclical change pattern of the city.

## 2. Data Selection

The data in this paper comes from the Mathematical Modelling Competition B of the 2021 Digital Dimension Cup. Through the observation and analysis of the daily precipitation observation data of Zhengzhou in the past 70 years, we can know that the occurrence of precipitation is often

accompanied by changes in natural conditions such as temperature and wind speed. This paper focuses on the study and analysis of the annual variation characteristics of precipitation in Zhengzhou, considering only the average annual precipitation of the past few years. After sifting through the precipitation data that have been provided, missing data and erroneous records for earlier years are more serious. In addition, in the context of global warming, the earlier data are not sufficiently informative [3]. Therefore, data from the past 42 years were selected and pre-processed in this paper.

### 3. Rainfall Analysis based on the M-K Test Method

In this paper, visualising the annual average rainfall line graph for the past 42 years for three stations in Zhengzhou. From it, it can be seen more visually that the precipitation measured at these three stations in 1980, 1982, 1984, 1992, 1994, 1996, 2003, 2011, 2013 and 2016 show obvious peaks. Among them, 1982, 2003, 2016 and 2021 are particularly prominent, which can reflect the relatively high rainfall in Zhengzhou in the aforementioned years.

The M-K test was then used to analyse the variation. The advantage of this method is that the sample does not need to have strict distribution conditions and the disturbance of a small number of outliers has little effect.

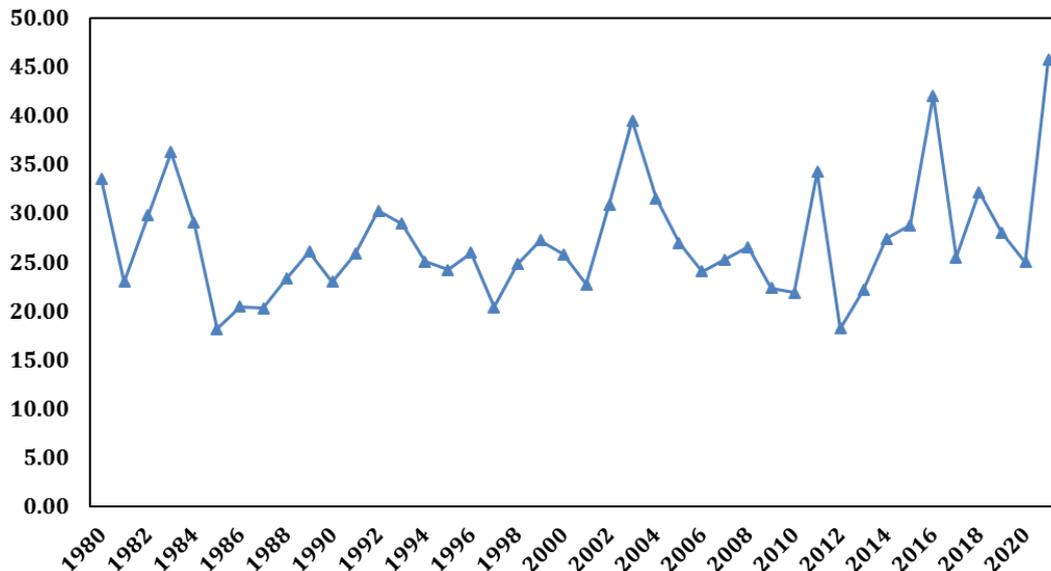


Figure 1. Zhengzhou Average annual precipitation of stations.

First define a set of sequences:

$$S_k = \sum_{i=1}^n \sum_j^{i-1} \delta_{ij}, \quad (k = 2, 3 \dots, n), \quad (1 \leq n \leq 42) \tag{1}$$

$$\delta_{ij} = \begin{cases} 1 & X_i > X_j \\ 0 & X_i < X_j \end{cases}, \quad 1 \ll j \ll i \tag{2}$$

$X_i, X_j$  is time series data, that is, the rainfall data is sorted in time series.

Then construct the statistic  $UF_k$  according to the disintegration idea of the M-K test method.

$$UF_k = [S_k - E(S_k)] / \sqrt{V(S_k)}, \quad (k = 1, 2, \dots, n) \quad (3)$$

In this formula,

$$E(S_k) = n(n - 1) / 4 \quad (4)$$

$$V(S_k) = n(n - 1)(2n + 5) / 72 \quad (5)$$

Turn the time series X upside down and repeat the above steps. The statistics UB<sub>k</sub> should take the opposite number of UF<sub>k</sub> to represent the correct reverse sequence trend [4]. Then give the significance level 'α=±1.96', and then plot the Figure 2.

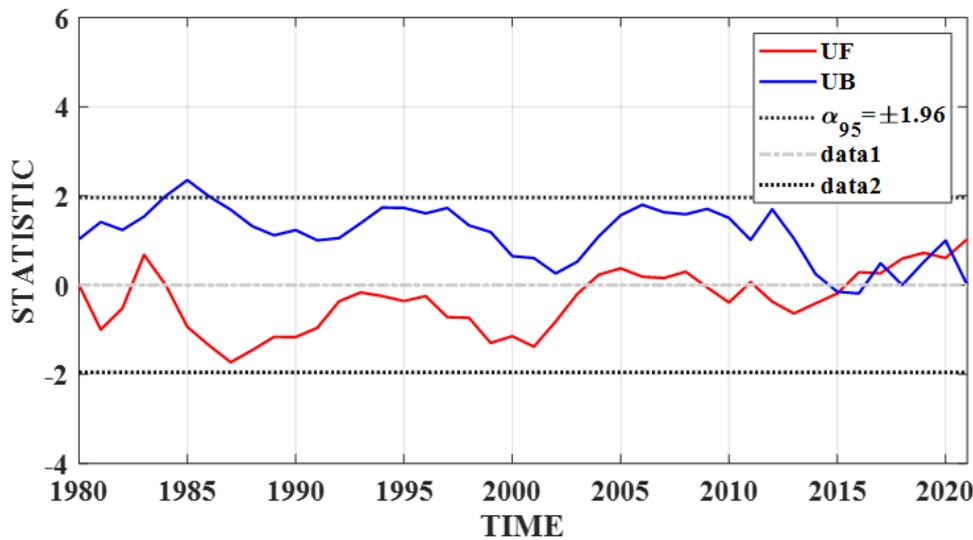
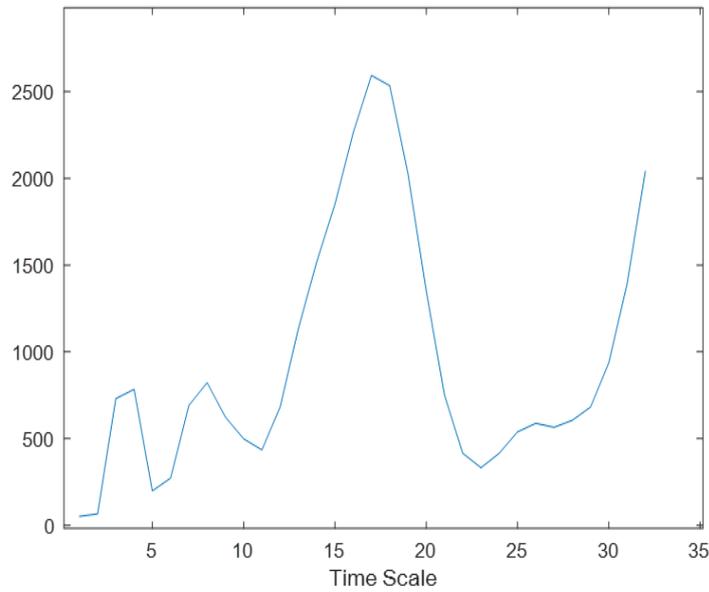


Figure 2. Zhengzhou M-K Mutation Analysis Chart.

It is clear from the graph that the annual precipitation from 1984 to 2003 shows an overall decreasing but not significant trend, with the trend from 2003 to 2015 fluctuating up and down. there are several abrupt changes in 2015 and beyond, indicating more frequent fluctuations in rainfall in these years, and an overall increasing trend after 2015.

The annual precipitation data for Zhengzhou were sorted by time series, and the dataset was created in MATLAB and processed using Wavelet Analyzer in MATLAB. To avoid boundary effects on this problem, the signal expansion function in the toolbox was used to expand the data symmetrically by 11 data on each side. The wavelet coefficients were then calculated and saved using the complex continuous wavelet 1-D function [5]. The real part and variance of the wavelet coefficients were calculated in MATLAB using various functions and a wavelet variance plot was drawn.

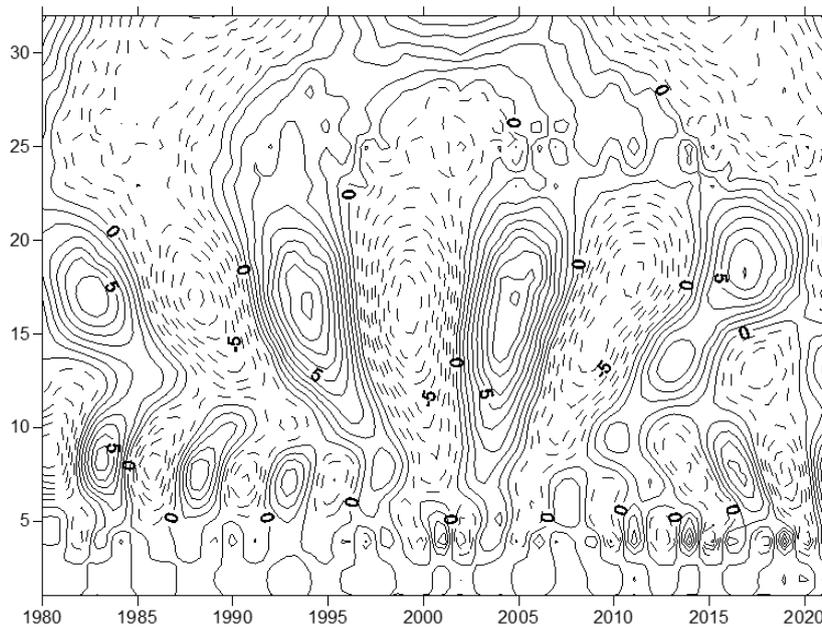
Contour map of the real part of the wavelet using SURFUR software. The real part of the wavelet coefficients was first meshed using the 'Kriging' meshing method, and then the contour map function was used to plot the contour map of the real part of the wavelet coefficients [6].



**Figure 3.** Zhengzhou Wavelet Variance Chart.

By analysing Figure 3, it can be concluded that the maximum peak corresponds to a time scale of 17 years, which belongs to the first main cycle, and the second and third peaks correspond to time scales of 8 and 4 years, which belong to the second main cycle. The multi-timescale characteristics of annual precipitation can be visualised through an analysis of Figure 4. There are seven main shocks on the 13-21 year timescale and multiple shocks on the 6-7 year timescale.

However, 2021 is near the peak of each cycle and is in the midst of an oscillatory cycle of increasing rainfall. Therefore, this extreme rainfall is partly influenced by global warming and other factors, and partly caused by the historical rainfall cycle. Zhengzhou is at a point in the historical cycle of increasing rainfall at this time.



**Figure 4.** Zhengzhou Contour Map of The Real Part of Wavelet Coefficients.

### 4. Example Validation

Based on the rainfall situation in recent years around the world, Fuyang, Anhui Province was selected for example validation in this paper. Through its city statistical yearbook, the National Meteorological Data Centre and the WheatA meteorological software platform, the annual average rainfall data of Fuyang city for the past 42 years were collected with no missing values or anomalies that required further processing [7].

The rainfall data from Fuyang were analysed by M-K test, wavelet coefficients were calculated and the associated periodograms were plotted to obtain the following result.

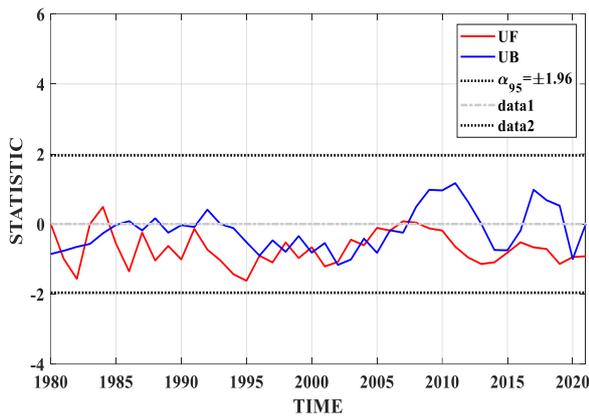


Figure 5. Fuyang M-K Mutation Analysis Chart.

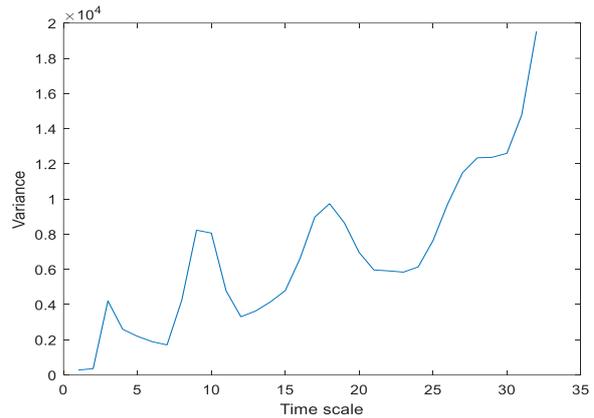


Figure 6. Fuyang Wavelet Variance Chart.

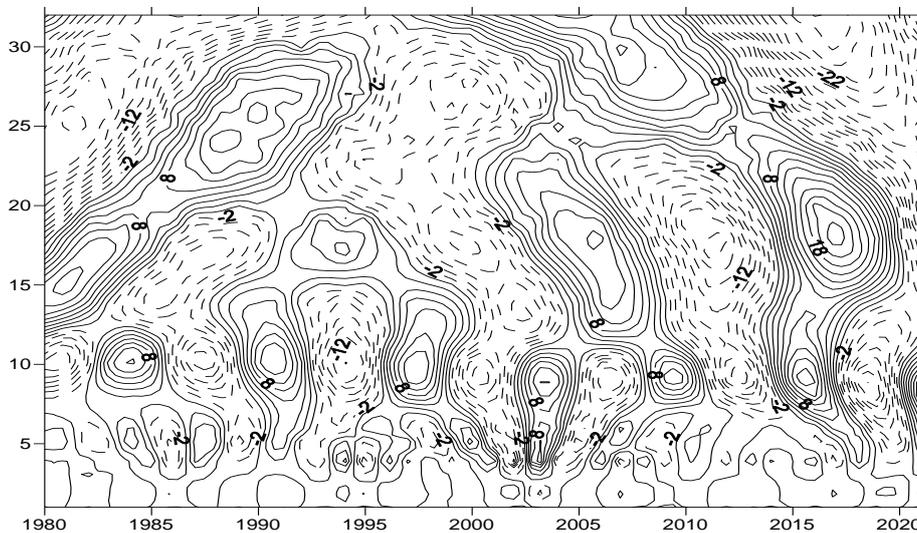


Figure 7. Fuyang Contour Map of The Real Part of Wavelet Coefficients.

As can be seen from Figure 5, there are many abrupt change points in Fuyang, and the interannual variability of precipitation fluctuates relatively frequently, with an insignificant overall trend. As can be seen in Figure 6, the maximum peak corresponds to a time scale of 18 years, the first major cycle. The second largest cycle is 9 years and the third largest cycle is 3 years. Combined with Figure 7, it can be seen that there are 13 oscillations on the time scale of 5 to 11 years, 8 oscillations on the time scale of 13 to 20 years and 5 oscillations on the time scale of 23 to 31 years. Of these, the cyclical variations on the 5 to 11 year time scale are relatively stable and global in nature.

In summary, Fuyang is currently experiencing an increase in precipitation, but not significant precipitation between now and the next three to five years.

## 5. Grey Prediction Model

### 5.1 Model Establishment

This paper has obtained the relevant data of precipitation in Fuyang in the first 10 years. In order to predict the annual precipitation in the place from 2022 to 2026, a GM (1, 1) model has been constructed for prediction and analysis.

First introduce the relevant principles of GM(1,1) model prediction.

(1) Define the data type

This paper records the annual precipitation data of Fuyang in the past ten years as. In order to eliminate the randomness of the data and make the model predict the data more regular, the original data is accumulated to generate the accumulated data [8]. At the same time, define a series of adjacent mean values of the precipitation data of the place, denoted as, the specific formula is as follows:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}, \dots, x^{(0)}(n)) \quad (6)$$

$$x^{(1)} = (x^{(1)}(1), x^{(1)}, \dots, x^{(1)}(n)) \quad (7)$$

$$z^{(1)}(m) = [x^{(1)}(m) + x^{(1)}(m-1)]/2, \quad m = 2, 3, \dots, n \quad (8)$$

(2) Quasi-exponential law test

The premise of using the GM(1,1) model to predict precipitation is that the precipitation data must meet the quasi-index test. The smoothing ratio of the accumulated data  $x^{(1)}$  is defined as:

$$P(k) = \frac{x^{(1)}(k)}{x^{(1)}(k-1)} - 1 = \frac{x^{(0)}(k) + x^{(1)}(k-1)}{x^{(1)}(k-1)} - 1 = \frac{x^{(0)}(k)}{x^{(1)}(k-1)}, \quad k = 2, 3, \dots, n \quad (9)$$

When the smoothing ratio  $P$  is  $P(k) < 0.5$ , the precipitation data conforms to the test of the quasi-exponential law. Here we define that when the proportion of  $P(k)$  is greater than 80%, the annual precipitation data of the two places comply with the test of the quasi-exponential law.

(3) Grey differential equation

In order to solve the parameters  $a$  and  $b$ , use least square method to calibrate parameters  $a$  and  $b$ :

$$\hat{U} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y \quad (10)$$

(4) Whitening equation

the general solution formula of the whitening equation is as follows:

$$\hat{x}^{(1)}(t+1) = \left[ x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right] e^{-\hat{a}m} + \frac{\hat{b}}{\hat{a}}, t = 1, 2, 3, \dots, n-1 \quad (11)$$

The accumulated data is obtained by accumulating the original data, so the fitted and predicted values of precipitation data over the years can be obtained.

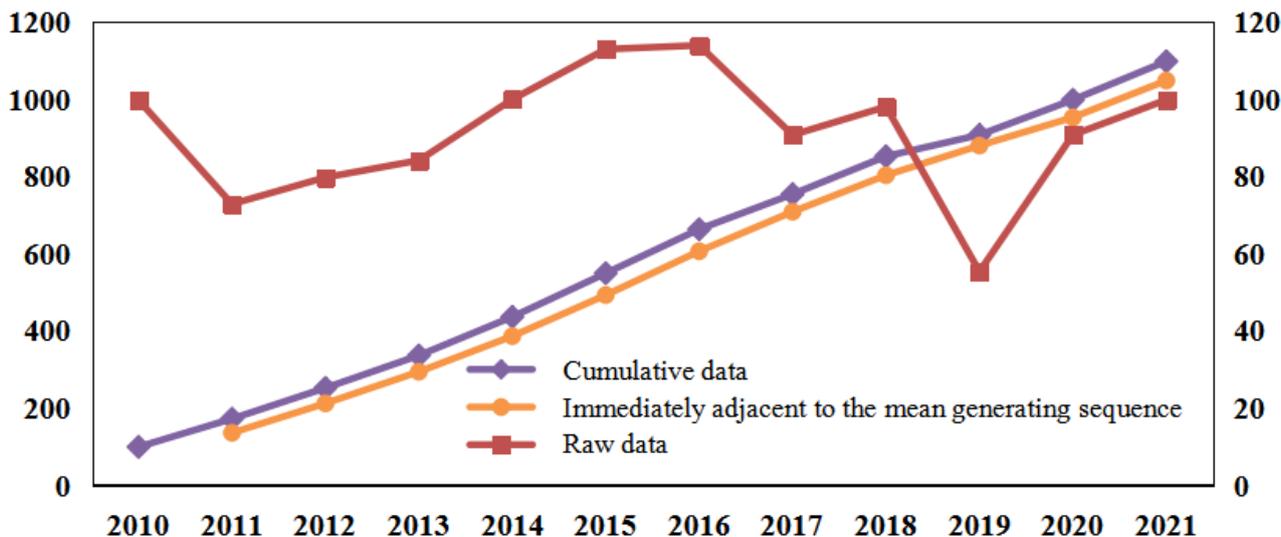
(5) Residual error test

In order to quantitatively test the fitting effect of the GM(1,1) model on the annual precipitation data, the absolute residual, the relative residual, and the average relative residual are defined. When the average relative residual < 20%, the GM(1,1) model has a better fitting effect.

**5.2 Model Solving**

(1) In order to better describe the solution process of GM(1,1), a flow chart of the model for predicting annual precipitation is given.

(2) Input the annual precipitation data  $X^{(0)}$  of Fuyang into the MATLAB program. Matlab obtains the accumulated data  $X^{(1)}$  of the place according to the code operation, and the adjacent mean value generation data  $Z^{(1)}$ .



**Figure 8.** Fuyang annual precipitation data chart.

The smooth ratio of the precipitation data in Fuyang is less than 0.5, accounting for 90%, so the annual precipitation conforms to the test of the quasi-exponential law.

Obtain the values of model parameters a and b according to the least square method. Fuyang model's parameters a=-0.0062, b=87.122.

(3) Substitute the obtained parameters a and b into the whitening equations of the precipitation models in the place respectively. According to matlab, the fitting value of the precipitation in the place and the predicted precipitation from 2022 to 2026 are constructed as a schematic diagram, as follows.

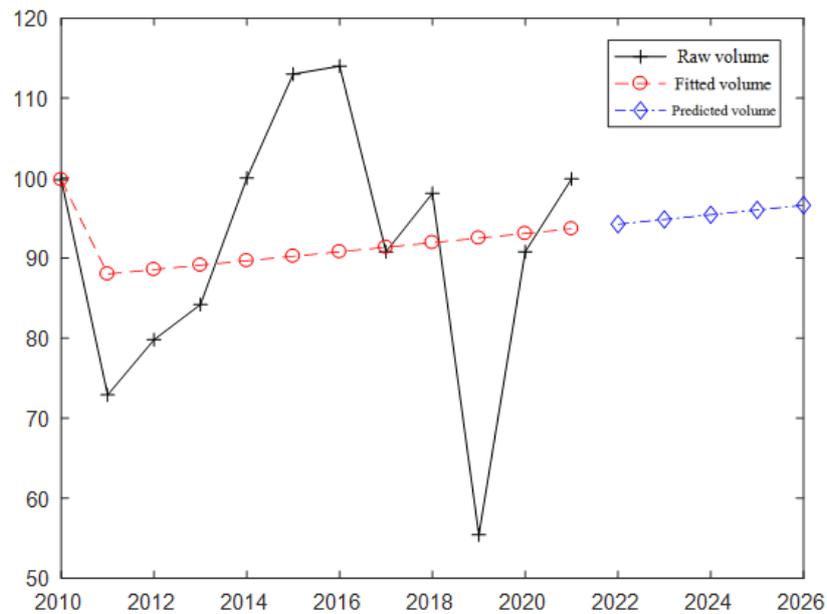


Figure 9. Fitted map of annual precipitation forecasts for Fuyang.

According to matlab, obtain the average relative residual of the annual precipitation data predicted by the model in Fuyang is 1.3% ,having a good fitting effect.

## 6. Conclusion

This paper has carried out a quantitative analysis of the average annual precipitation of two cities through the use of mutation analysis and cycle analysis, and combined with the gray forecast model, we have a good forecast of the rainfall trend of each city in the next three to five years. The prediction and prevention of extreme weather is of great significance to human production and life.

In the event of extreme weather, the city's disaster prevention projects cannot be completely relied on. What is more important is to improve the entire society's awareness of disaster prevention and the ability to adapt to extreme weather. Meanwhile, the government should perfect emergency measures to deal with extreme weather. The government should increase investment in the establishment of early warnings of regional climate change to be able to respond to the arrival of heavy rainfall in advance, thereby reducing casualties and economic losses. A more complete underground drainage system should be built to cope with more intense rainfall.

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