Bicycle Race from the Perspective of Physics

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Abstract

Developing a Road cycling race strategy is a significant but complex process for riders. Without such a decision model, riders would not be able to prepare and race scientifically. We address the problem of optimizing total time for the player to cover the entire distance through the linear programming, physical kinetics, and differential equation which ideally make individual and team achieve relatively good results. Based on queried literature we establish 3-Parameter Critical Power Model and define the power profiles. Using various types of contestants' power profiles. To analyze the power principle of the bicycle on the slope we formulate the Road Bike Power Model. We divide the competition into three phases: up, down and flat. In addition to Individual Time trials, we also discuss the team strategy model. Our suggested solution which is reliable and practicable under the influence of the external environment and expected target deviation through sensitivity analysis, includes detailed solutions and team strategies according to different roads and types of contestants.

Keywords

Linear Programming; Differential Equation; Power Curve; Physical Kinetics.

1. Introduction

1.1 Problem Background

Races that take place on marked courses on open roads and highways are known as cycling road races. The type of event, the route and the ability of the riders are the key factors for its success. In an individual time-trial, each cyclist should ride a fixed route individually and the winner is the rider who completes the specified distance in the shortest time. Each type of rider has a different power curve [1]. In general, the more power a rider generates, the less time the rider can maintain without decreasing and reverting. When a rider chooses to exceed the limits of his power curve, the rider needs more time to recover.

1.2 Research Overview

It's widely known that riders have a limit on the total energy, and limits that accumulate from past aggressiveness and for exceeding the power curve limits. Therefore, we should develop models that can be applied to each type of rider that decides the relationship between the rider's position on the course and the power the rider applies to solve the following problems.

• Defining the power profiles for time trial specialist and the different type rider, taking into account gender differences.

• Apply the constructed model to the self-designed track, where the self-designed track has at least four sharp turns and at least one road slope, and the end of the route should be at its starting point nearby.

• Consider the potential impact of weather factors such as wind direction and wind strength on the model, and determine the sensitivity of the results to weather effects.

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• It is impossible for a rider to ride exactly to the model our team has built, so in order to give riders and sporting directors an idea of the possible range of key sections of a particular track, our team has developed a sensitivity to rider deviations from the target power distribution.

• Expand the model to find the optimal power distribution scheme for the team time trial.

• Provides a simple, easy-to-understand rider competition guide with rider guidance and model summaries.

1.3 Our Approach

This topic required us to develop a model that could be applied to any type of rider to determine the relationship between the rider's position on the track and the intensity of the rider's application. Our work consisted mainly of the following elements as well as a general overview of the model(Figure 1).

• Based on the differential equation model of bicycle physical movement, the corresponding relation between position and power is optimized.

• The influence of the slight change in environment and individual factors of players on the model results is analyzed.

• For the team competition, the strategies that can be taken are discussed and the model is extended.

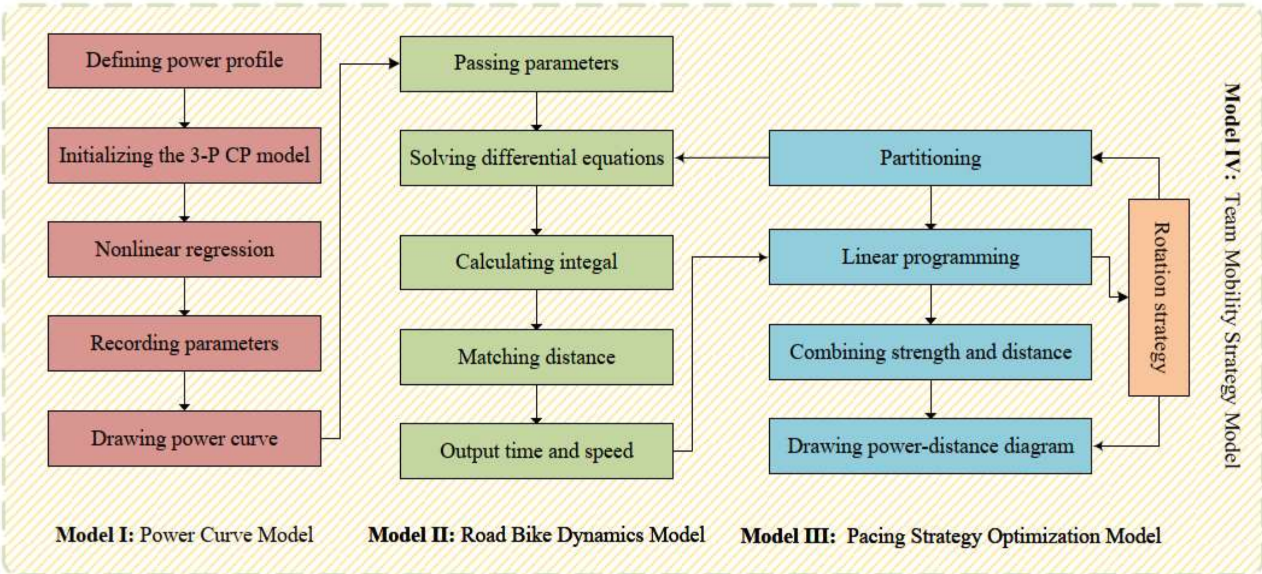

Figure 1. Model Overview

2. Model Construction and Solving

2.1 Power Curve Model

It is well known that different types of cyclists have very different power curves. Intuitively, sprinters should have higher explosive power, and rouloeur should have higher power for long periods of time. There are well-established models on the market for predicting a rider's power curve based on the power output achieved by a cyclist at several specific TTEs, which we have described in our literature review[2].

2.1.1 Profiles for Different Riders

Power Profile is telling us more about the abilities in cycling. Four very important values telling us about: sprint abilities (5 s), anaerobic capacity described by 1 minute maximum power, 5 minutes to tell us about VO2max capability and 20 min to describe our FTP. Those four numbers divided by weight are telling us about your talent in cycling. All of them can be trained and improved by specific training[3].

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In the actual competition, in order to correctly evaluate the characteristics of the players, it is necessary to measure the data according to a specific step and compare with the table. After synthesizing various materials, we will give the profiles of time trial experts and printers in this paper. Maximal Power Output is shown in

Table **1**.

Table 1. Maximal Power Output (in W/kg)

Type	Men				Women			
	5s	1min	5min	FT	5s	1min	5min	FT
Time Trial Specialist	15.88	8.4	5.53	5.15	15.11	7.03	5.41	4.7
Sprinter	19.96	8.28	4.08	3.73	12.95	6.66	3.83	3.14

2.1.2 3-Parameter Critical Power Model

The 3-P CP model expresses certain characteristics of a player with three parameters through data analysis. CP is the abbreviation of critical power, which means that a player can ride almost "infinitely" under this power. AWC represents the total energy a runner can consume when riding at a power that exceeds the CP.

$$t = \frac{AWC}{P - CP} + \frac{AWC}{CP - P_{\max}} \tag{1}$$

In which CP is critical power, that is to say, under this power, the rider can ride almost indefinitely. AWC refers to the total energy consumed by the rider when riding with power exceeding CP. P is the rider’s current output power.

2.1.3 Result

Fitting was performed using the nonlinear fitting tool in SPSS. Note that AW C, CP, Pmax should be greater than zero. The result is shown in Table 2.

Table 2. Three parameters of different players (in W/kg)

	Male TTS	Male sprinter	Female TTS	Female sprinter
AWC	322.728	442.88	272.178	344.844
CP	4.532	3.16	4.562	2.782
Pmax	18.301	17.646	17.645	14.708

The data used and Formula1 the power curves that can be drawn for different types and different players are shown in Figure 2.

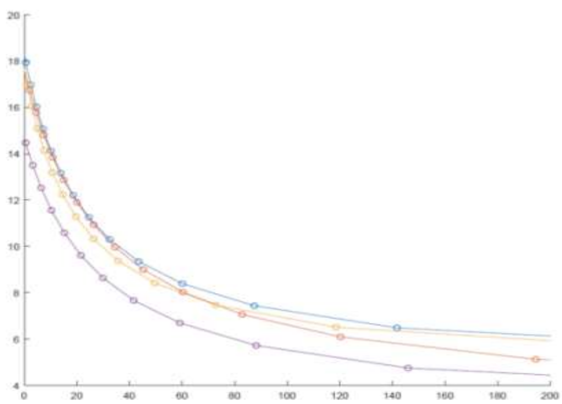

Figure 2. Power curves of different players

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In addition, the abscissa represents the duration and the ordinate represents the output power. Power Curve shows us how the power available to the rider changes over time. The left half of the curve usually means the amount the rider can choose during the burst or charge phase. It can be seen that sprinters have a significant advantage. And the second half of the curve usually means the amount a rider can choose on a long ride. In this section, time trial experts have an advantage over sprinters.

2.2 Road Bike Power Model

Cyclists usually provide power by pedaling hard. This power is transmitted through the chain to drive the rear wheels. During the entire ride, some of this power will be used to overcome the effects of wind, friction and gravity. Next, we will discuss the impact of these three parts separately.

2.2.1 The Effect of Gravity

Suppose the mass of the person is M, the mass of the bicycle is m, and the slope of the ground is (x). According to basic trigonometry, the power required by a person to overcome gravity at x can be expressed as.

$$\Delta_{\text{gravity}} = (M+m)g\sin(\theta(x))v \tag{2}$$

Wherein, g is expressed as 9.81 m/s2.

2.2.2 The Effect of Frictional Resistance

When we examine the influence of this part of the resistance, we usually focus on the friction between the transmission mechanism, the friction between the tire and the road surface, etc. For the sake of simplicity, we think that all these frictions can be represented by a friction parameter R(x) about x. According to Di Pampero et al. (1979) and Gordon (2005), the force required to overcome rolling resistance can be expressed as.

$$\Delta_{\text{rolling}} = (M+m)gR(x)v \tag{3}$$

2.2.3 The Effect of Wind Resistance

It is assumed that the force exerted by the wind resistance increases with the speed of the rider relative to the square of the prevailing wind. Suppose w(x, t) represents the strength of the wind measured relative to a stationary object and represents the direction of movement relative to the cyclist. Therefore, the power required to overcome wind resistance is.

$$\Delta_{\text{wind}} = A(x)(v-\omega\cos\varphi)^2 v \tag{4}$$

where A(x) represents the drag coefficient, which can vary with air density and is therefore a parameter related to x. If P(x,t) represents the power transmitted by the cyclist to the rear wheel, the rider and the rate of change of kinetic energy of the bicycle is the difference between the power applied and the power required to overcome the resistance.

2.2.4 Combination

The riders and the rate of change of kinetic energy for a cyclist is the difference between the power applied and the power required to overcome resistance.

$$\frac{d}{dt}\left(\frac{1}{2}(M+m)v^2\right) = P - \Delta_{\text{gravity}} - \Delta_{\text{rolling}} - \Delta_{\text{wind}} \tag{5}$$

The equation of motion that can be further obtained from this is as follows.

$$\frac{dx}{dt} = v \tag{6}$$

$$(M+m)v\frac{dv}{dt} = P(x,t) - [\sin(\theta(x)) + R(x)] \times (M+m)gv - A(x)(v - \omega\cos\varphi)^2 v \tag{7}$$

2.3 Pacing Strategy Optimization Model

The road cycling time trial is primarily a measure of athleticism, and the rider's only strategy is to spend as little time as possible. In real competitions, players often use rhythm strategies, that is, alternating power output to shorten the time as much as possible[4].

Before the actual build, we need to tease out a few physiological constraints.

2.3.1 Physiological Constraints

For the riders, they can relieve the fatigue caused by the sprint in the previous stage by taking short breaks, but the total energy that the rider can use is limited. Of course, if the rest period is long enough, the player can recover all the energy, just like in our daily life. In short, the AWC in the 3C−CP model limits how long a player can ride at a power that exceeds the CP.

$$E_{total} <= AWC \tag{8}$$

In actual competitions, riders face the ever-changing track and conditions and need to choose their own different power configurations. After riding with the power of the charge for a while, the rider will get tired, so for the next period, he needs to reduce the power to recover his strength. If we look at the entire charge-rest process, we will find that the player's statistical average power should be consistent with the power output in the equilibrium state.

Therefore, in the rhythm decision of each charge-rest period, the constraint of average power needs to be satisfied, which limits the athlete's charge power to meet the requirements of fatigue recovery. The average power constraint can be expressed as.

$$\frac{1}{L}\int_0^L P(x)dx = \overline{P} \tag{9}$$

2.3.2 Pacing Strategy

Facing different road conditions, riders need to ride at different rhythms. Current research shows that under constant conditions, maintaining a uniform operating rate is the best option. But in the case of uphill and downhill, if the riders increase the output power on the uphill and reduce the output power on the downhill, the total time saved is even more. Therefore, the player's power usually has the following three states.

$$P = \begin{cases} P_1 & \text{uphillpower} \\ P_2 & \text{recoverypower} \\ \overline{P} & \text{averagepower} \end{cases} \tag{10}$$

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P1 corresponds to the climbing power, P2 corresponds to the downhill resting power, and P¯ represents the stable power on a straight road. In this way, players can rhythmically change their output power according to different road conditions.

In addition to driving in a straight line, we also need to be aware of the speed limit brought about by the existence of sharp turns, so in the process of entering the corner from the straight, it may be necessary to adjust the power in advance to achieve the best speed of the corner. In summary, the rider's rhythm strategy can be summed up as.

- On straight roads, drive with steady power.
- On uphill and downhill roads, increase uphill power as much as possible.
- On sharp curves, control the entry speed in advance.

Therefore, under the guidance of such a strategy, we can establish the following non-planning model.

$$\begin{cases} \min T = \sum_{i=1}^{n} t_i \\ \text{s.t.} \begin{cases} \frac{1}{L_i}\int_0^{L_i} P(x)dx = \overline{P} \\ \sum_{i=1}^{n} E_i \le AWC \\ P(x) = (M+m)v\frac{dv}{dt} + \Delta_{\text{gravity}} + \Delta_{\text{rolling}} + \Delta_{\text{wind}} \end{cases} \end{cases} \tag{11}$$

Among them, Li and Eirepresent the length of the i-th segment and the energy consumed, respectively. Pi(x)represents the power that needs to be allocated at x for the i-th segment. Due to the limitations of differential equations, we need to sort out the x-t relationship indirectly through the v − t relationship, which will be detailed in the next subsection.

2.3.3 Results

On this simple track, players only need to use the pacing strategy in one up and down section, and use the balanced strategy in other places. Track details are shown in Figure 3.

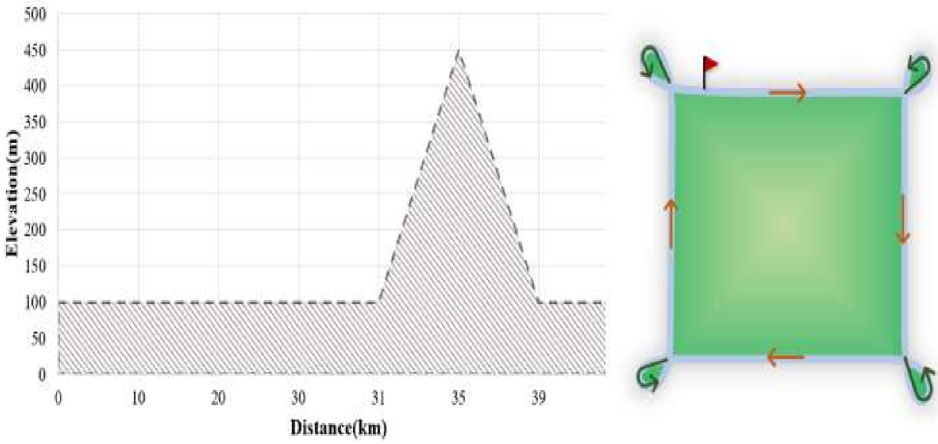

Figure 3. Track details

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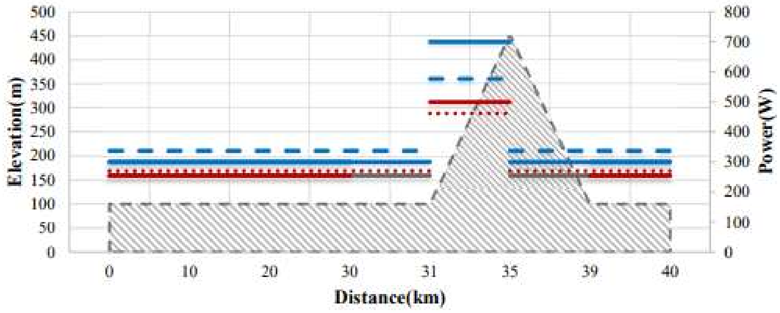

Figure 4. Application on custom circuit

On this simple track, you only need to use the pacing strategy on one up and down section, and the balance strategy in other places. The rider's power distribution strategy is shown in Figure 4.

2.4 Team Mobility Strategy Model

2.4.1 Wind Resistance Analysis

A cyclist can greatly reduce his or her air resistance by "hiding" behind others. At certain critical moments in a team race, such as late-race charges, mid-race overtakes and disengagements, cyclists need to be properly allocated. This strategy shows us how important it is to conserve energy and use it at the right time[5]! We can use the 9 to make corrections to the wind-related part of the dynamics. For the ath person in a team, the wind effect on him is.

$$\Delta_{\text{wind}}^{a} = \alpha_a A(x)\left(v_a - \omega\cos\varphi\right)^2 v_a \tag{12}$$

Then the dynamic model for the ath person can be rewritten as.

$$\frac{d}{dt}\left(\frac{1}{2}\left(M_a + m_a\right)v_a^2\right) = P_a - \Delta_{\text{gravity}}^{a} - \Delta_{\text{rolling}}^{a} - \Delta_{\text{wind}}^{a} \tag{13}$$

2.4.2 Mobile Rotation

In the official game, the rational application of the rotation skills plays a decisive role in the victory of the team game. Rotation requires the lead driver to move to the outside of the track to follow at the end of the line, while leaving a starting window for teammates behind.

2.4.3 The Team Model

Under normal circumstances, the speed of the six members of the team should be consistent. Furthermore, the overall goal of the model is to reduce the overall time spent by the team.

$$\begin{cases} \min T = \sum_{a=1}^{4}\sum_{i=1}^{n} t_i \\ \text{s.t.} \begin{cases} \frac{1}{L_i}\int_0^{L_i} P_a(x)dx = \overline{P}_a \\ \sum_{i=1}^{n} E_i \le AWC \\ v_1 = v_2 = v_3 = v_4 = v_5 = v_6 \\ P(x) = (M+m)v\frac{dv}{dt} + \Delta_g \text{ ravity } + \Delta_{\text{rolling}} + \Delta_{\text{wind}} \end{cases} \end{cases} \tag{14}$$

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we believe that players with higher AWC are suitable as boosters to resist the influence of wind resistance as much as possible. In this case, if there are two teammates as boosters, the speed limit after separating teammates will be rewritten as follows.

$$v_1 = v_2 = v_3 = v_4 \tag{15}$$

Note that in the official game, the rational application of rotation skills plays a decisive role in the victory of the team game. Rotation requires the lead driver to move to the outside of the track to follow at the end of the line, while leaving a starting window for teammates behind.

3. Sensitivity Analysis

Note that the description of wind resistance in the dynamic model is as follows.

$$\Delta_{\text{wind}} = A(x)(v - \omega\cos\varphi)^2 v \tag{16}$$

Obviously, the power expended in overcoming wind resistance will vary with the wind strength w(t) and φ the wind direction. This will affect the player's overall time and fitness allocation strategy.

On the 10km straight test, we assumed that the strength and magnitude of the wind would not change with x at this point. Take w from 0 to 0.4 and φ from 0 to 180°, respectively, and influence of wind direction intensity is shown in Figure 5.

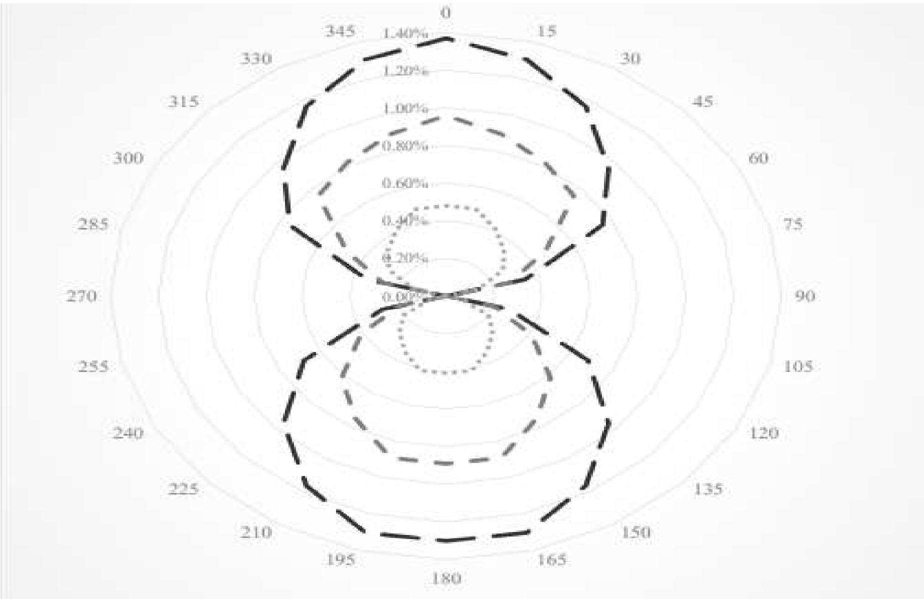

Figure 5. Influence of wind direction intensity

4. Conclusion

Road bicycle dynamic model based on physical model has better interpretability than statistical fitting model. Sensitivity analysis of the model proves the effectiveness and robustness of the model under different parameter combinations. Read a lot of professional literature about bicycle time trial extensively, and absorb some of the essence of research results, so as to help cyclists and Directeur Sportif make reasonable decisions.

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The power of the model is planned by road sections. If some overall perspectives are brought into the model, it will help to do more intelligent processing at the junction of each road section.

References

[1] Di Prampero, P. E., Cortili, G., Mognoni, P., Saibene, F. (1979). Equation of motion of a cyclist. Journal of Applied Physiology, 47, 201–206.

[2] Gordon, S. (2005). Optimising distribution of power during a cycling time trial. Sports Engineering, 8, 81–90.

[3] Bon, M. V. , & Vroemen, G. . (2018). Power Speed Profile: Performance model for road cycling (1).

[4] QIAO Jie2021.Application of Power-duration Curve and Critical Power in Predicting Cycling Performance.

[5] Marco van Bon & Guido Vroemen(2018).Power Speed Profile: Performance model for road cycling (1).