ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

Portfolio Optimization Model based on Multi-objective Programming

Jing Liang[1], Zihang Chen[2], and Tengda Zhang[1]

[1] Yisheng innovative education base, North China University of Science and Technology, Tangshan 063210, China

[2] College of artificial intelligence, North China University of Science and Technology, Tangshan 063210, China

Abstract

When buying and selling volatile assets, traders tend to expect maximum returns and minimum risk. In this situation, it is important to develop the best trading strategy. In this paper, gold and Bitcoin as an example of two assets, with with $1000 as the initial capital, in each small cycle using gray prediction method to solve the expected exchange rate of gold and Bitcoin, and then establish a dual-objective programming model, and the ideal point method is introduced to construct evaluation function, the original model into a single objective optimization problem. Optimal trading decisions were simulated during the trading period from September 11, 2016 to September 10, 2021, with a final asset investment value of $231,620.

Keywords

Trading Decision Model; Grey Prediction; Ideal Point Method.

1. Introduction

Bitcoin is a digital virtual currency first officially launched by Nakamato in 2009. From 2016 to 2019, the currency market is developing rapidly, in 2017, a large number of investors flocked to the currency market, continue to surge in the prices this year, the currency has become the focus of all walks of life to discuss overnight, but later because the regulation, the currency heat fading, fell in 2018 to return to the rational price, more investors also return to rational.

Market trading is essentially a trade-off between benefits and risks. Investors are inevitably faced with a variety of risks in the market, especially when buying and selling volatile assets, how to minimize risks and maximize returns has become the most concerned issue for investors. Assuming that we start holding initial capital of $100 on September 11, 2016, the portfolio is determined to be bitcoin, GOLD and cash, using a five-year trading period with buying and selling commissions in mind, given pricing data for GOLD and Bitcoin (in files lBMA-gold. CSV and Bchaln-mkpru.csv), It is required to make trading decisions using only past asset prices up to now, given that day's price data, to determine how to change the assets in the portfolio to make the best trading strategy for today. We need to build a mathematical model to make investment decisions and analyze the following questions:

Use this model to find the value of the original $1,000 on September 10, 2021.

Verify with data that the model provides the best strategy.

2. Assumptions and Justifications

By adequate analysis of the problem, to simplify our model, we make the following well-justified assumptions.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

1) We assume that the outliers removed in the processing of daily price data correspond to dates that are not gold market open days.The data volume of the removed vacancy value and outlier value is very small compared with the total data volume, which will not affect the estab- lishment of the overall model. Therefore, for the convenience of discussion, the corresponding dates of these values are uniformly defined as non-gold market trading days.

2) We assume that the daily price of an asset is the closing price, the fluctuation of its daily price is negligible.We are trying to develop the best trading strategy for the day, and taking into account the volatility of the day complicates a simple problem. In general, the price of an asset does not fluctuate very much in a day, so this assumption is necessary and reasonable.

3) We assume that there is no financial crisis and other unexpected situations that cause significant changes in the trading market during the trading period.Unexpected events such as financial crises are so rare that good models adapt to most situations by following the principle of universality, so our models do not take such events into account.

4) We think that the best trading strategy in the market is to maximize the value of the assets held.In the market transaction, investors always hope to get as much profit as possible, assuming that they are completely for their own interests, the trading strategy is the best trading strategy.

3. Daily Price Forecasting Model

3.1 Establishment of Gray Prediction Model

Grey prediction is to predict the system which contains both known information and uncertain information. In this case, we only know the asset price before the day. If we want to develop the best trading strategy for today's trading, it is very important to predict the two asset prices of tomorrow based on past data. Grey prediction is to find the law of system change by processing the original data and generate the data series with strong regularity, and then establish the corresponding differential equation model, so as to realize the prediction of the development trend of asset prices. The specific steps are as follows: Assuming $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$ is the original non-negative data column, that is, the daily trading price of each asset given in the question, we sum it once to get the new generated data column $x^{(1)}=(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n))$.

Among them,

$$x^{(1)}(m)=\sum_{i=1}^{m}x^{(0)}(i),m=1,2,\cdots n \tag{1}$$

Let $z^{(1)}$ be the immediate mean value of the sequence $x^{(1)}$ to generate the sequence,which is $z^{(1)}=(z^{(1)}(1),z^{(1)}(2),\cdots,z^{(1)}(n))$,where,

$$z^{(1)}(m)=\delta x^{(1)}(m)+(1-\delta)x^{(1)}(m-1),m=2,3,\cdots n(\delta=0.5) \tag{2}$$

We call equation $x^{(0)}(k)+az^{(k)}=b$ the basic form of the GM (1,1) model (k=2,3... n),b represents ash action, and $-a$ represents development coefficient[1]. The matrix form is introduced below:

$$u=(a,b)^T,Y=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\\vdots\\x^{(0)}(n)\end{bmatrix},B=\begin{bmatrix}-z^{(0)}(2)\\-z^{(0)}(3)\\\vdots\\-z^{(0)}(n)\end{bmatrix} \tag{3}$$

Volume 8 Issue 7, 2022
DOI: 10.6919/ICJE.202207_8(7).0115

Thus,GM(1,1)model $x^{(0)}(k)+az^{(k)}=b$ can be expressed as $Y=Bu$ .The estimated values of parameters a and b can be obtained by using the least square method:

$$\hat{u}=\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}=(B^T B)^{-1}B^T Y \tag{4}$$

Among many prediction methods, gray prediction requires less samples and less computation, so it is suitable for time series prediction with strong exponential law. GM (1,1) model is more widely used in time series prediction, and often has good prediction accuracy.

3.2 Solution of the Model

When buying and selling of the volatile assets, we tend to use near the transaction date day forecast, in the ontology, the future we improve forecast accuracy, with 5 days prior to the data to predict gold and currency exchange rates, but this must lead to cannot use the data file to provide the large amount of data to make predictions, so we choose the grey prediction model, The gray prediction model, with GM (1,1) as its core, can be modeled and predicted according to a small amount of known information, which is exactly in line with our actual needs.

According to the daily price data files of gold and Bitcoin in this case, we removed the vacancy values and outliers, and the processed data passed the quasi-exponential law test. Considering the discreteness of daily price data and the negative, in 5 days for a cycle, through an accumulation generation weaken the randomness of regular new discrete data column, differential equation model, the discrete points the b-b generating original data of the approximate estimate, established GM (1, 1) model to predict the day after the price of gold and currency. The price of the day after the forecast is cycled every five days. The cooperation diagram between the predicted data and the original data is as follows:

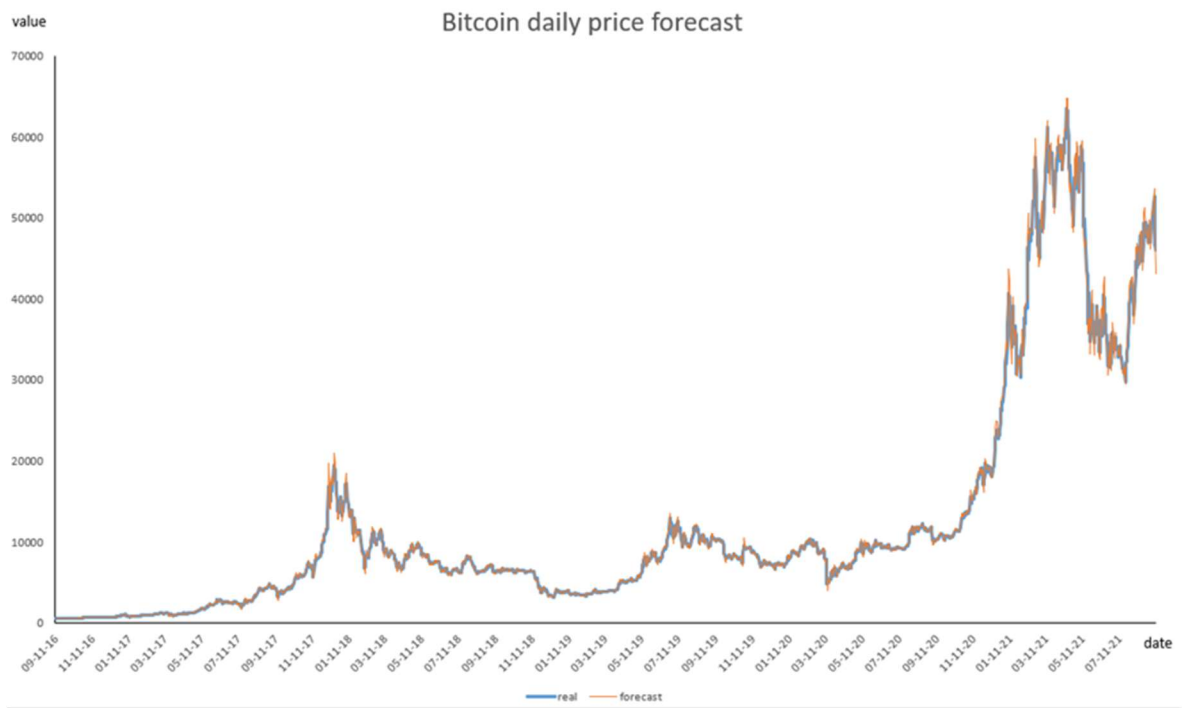

Fig. 1 Daily price forecast of bitcoin based on GM (1,1) model

Volume 8 Issue 7, 2022
DOI: 10.6919/ICJE.202207_8(7).0115

Fig. 2 Daily price forecast of gold based on GM (1,1) model

Next, We calculate the root mean square error, and the formula of the root mean square error is shown in Formula 5:

$$RMSE_i = \sqrt{\frac{\sum_{i=1}^{n}(real_i - fcst_i)^2}{n}} \tag{5}$$

Here $real_i$ represents the actual asset price, $fcst_i$ represents the predicted price, and n represents the amount of price data processed in the file.

The root mean square error of gold price prediction model is 8.74, and the root mean square error of bitcoin price prediction model is 12.97. It can be seen that the error is small and the overall fitting degree is good.

4. Best Trading Strategy of the Day Model

In order to maximize returns on the market, we decide whether to buy or sell today based on the price of gold and bitcoin predicted to rise or fall tomorrow. In addition, we need to consider the Open Day of the non-gold market.

For the convenience of subsequent discussion, we first define the expected rate of return. We believe that the lower the price of an asset today, the higher the price of the same asset tomorrow, and the higher the rate of return brought by the replacement of such asset today. Considering the impact of commission, the actual value changes during the replacement. According to the above analysis, the formula of return rate is as follows:

$$r_i = \frac{fcst_i(1 - a_i)}{real_i} - 1 \tag{6}$$

In the prediction model, we obtain fcst1 and fcst2 predicted values for gold and bitcoin respectively, where real represents today's price of an asset and αi represents the transaction commission for buying or selling such an asset. Buy an asset when its expected return is greater than zero; When an asset's expected return is less than zero, it should be sold.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

Based on the above analysis, the flow chart of the best daily trading strategy is as follows:

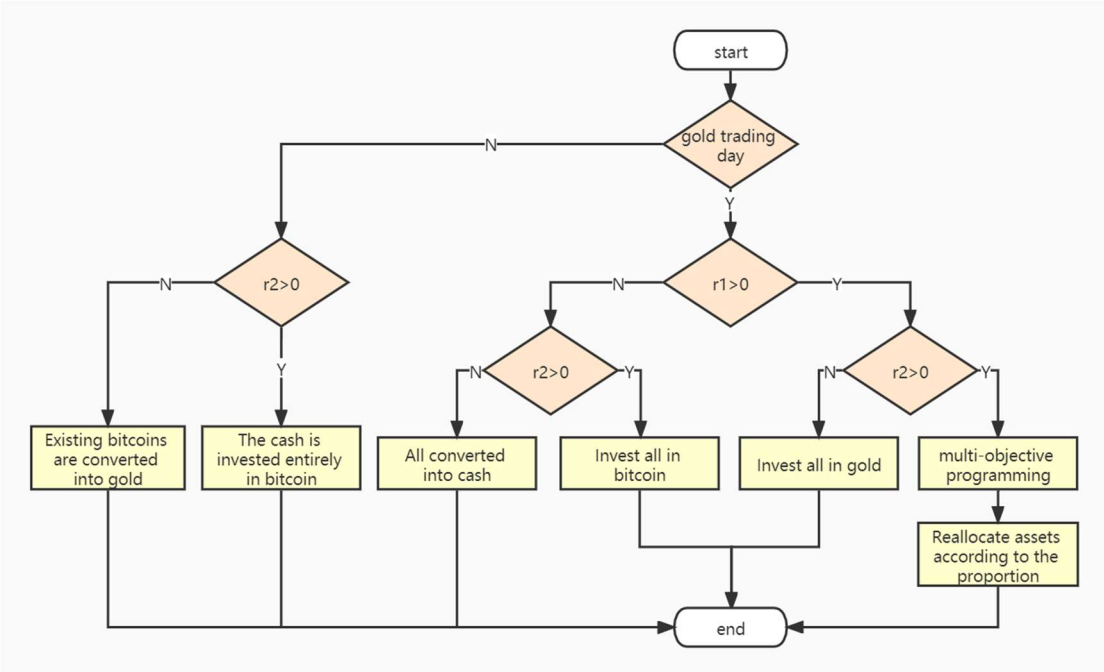

Fig. 3 Flow charts of today's best trading strategies

4.1 Portfolio Optimization Model

According to the above analysis, when the expected returns of gold and bitcoin are both greater than 0, both assets need to be purchased. At this time, a model needs to be established to determine the proportions of the two assets.

Our expectation is to maximize returns and minimize risks. According to the reference, in 1952, Markowitz introduced the theory of random variables. The return rate of securities investment is variable and can be regarded as a random variable. The fluctuation of the random variable, namely variance, is used to measure risk, and the average of asset return is used to measure investment return.

Assuming that there are n kinds of risk assets that can be combined, the proportion coefficient of the investment amount of a single risk asset in the total investment amount is $x_i, i=1,2\cdots n$, then the scale coefficient vector of venture investment is $x=(x_1 \ \ x_2 \ \ \cdots \ \ x_n)^T$,and must meet the sum of the scale coefficients of investment is 1, namely $\sum_{i=1}^{n} x_i=1$ .Remember the n-dimensional vector with all components of 1: $f=(1 \ \ 1 \ \ \cdots \ \ 1)^T$,then $\sum_{i=1}^{n} x_i=1$ can be expressed as $f^T x=1$ .Write down the expected return vector A of risky assets: $r=(r_1 \ \ r_2 \ \ \cdots \ \ r_n)^T$;The coveriance matris of the return on risky assets is $V=(\sigma_{ij})_{n\times n}$,where is covariance of r_i and r_j ,so the expected return of portfolio x is:$r(x)=\sum_{i=1}^{n} r_i x_i = r^T x$;The risk of the return rate of protfolio x is:$\sigma^2(x)=\sum_{i=1,j=1}^{n} \sigma_{ij} x_i x_j = x^T V x$,variance is actually the penalty for the deviation of the return rate of asset from the expected return return rate,which predicts the reliability coefficient of asset performance [2].

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

In the process of market transactions, investors tend to choose a in decision making can get as high as possible, avoid risk at the same time, however, the fact is often high-risk, high return and low risk and low return, how to balance the relationship between the benefits and risks to make decisions, and become the key problems of portfolio investment [3]. According to the above analysis, referring to the classical mean-variance model, we build a dual objective linear portfolio model.

When investors invest in gold and bitcoin, the expected rate of return, risk loss rate and investment proportion are respectively r_i, σ_i, $x_i=(1,2)$. The core problem of trading decision is the tradeoff between profit and risk, so the optimization goal is to maximize profit rate R and minimize risk σ. Then, the corresponding two-objective linear programming model is:

$$\begin{aligned} &\min \sigma = x^T V x \\ &\max R = r^T x \\ &s.t. \begin{cases} f^T x = 1 \\ x \geqslant 0 \end{cases} \end{aligned} \tag{7}$$

Where $x \geqslant 0$ is because the investment ratio is non-negative. When the investment ratio is positive, the proportion allocation of portfolio assets is meaningful.

Since there are only two risky assets to invest in, gold and bitcoin, the combined investment ratio of gold and bitcoin should be 1.

In solving the multi-objective programming problem, we first work out the optimal solution of each single objective programming problem, according to the actual problem background, construct the so-called evaluation function, by solving the optimal value of the evaluation function, so as to get the method of optimal solution of the multi-objective optimization problem. The ideal point method is a method to construct evaluation function.

In this case, we use the ideal point method to calculate the maximum expected return rate rmax and the minimum risk σmin, and get the ideal point (rmax, σmin). Because some traders like to take risks to make more money, while others want to take less risks and be cautious, they should choose according to their own preferences. In this case, it is required to maximize the return as far as possible, so we set the coefficient λ1=0.8 in front of the return. The coefficient λ1=0.2 in front of the risk needs to find the corresponding x in the feasible domain of the multi-objective function to make the Euclidean distance between the corresponding z and the ideal point as small as possible. In this way, a multi-objective programming model can be transformed into:

Objective fuction:

$$\min z = \sqrt{\lambda_1 (r - r_{\max})^2 + \lambda_2 (\sigma - \sigma_{\max})^2} \tag{8}$$

Constraints:

$$s.t. \begin{cases} f^T x = 1 \\ x \geqslant 0 \\ \sigma = x^T V x \\ r = r^T x \end{cases} \tag{9}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

According to the operation result of this trading model, the investment risk can be effectively reduced and the appropriate investment income can be obtained.

4.2 Points to Discuss the Allocation of Assets

1) Non-gold market Open Day, and bitcoin yield less than 0.

How to cope: Turn all bitcoins into cash.

Various assets held after the transaction:

Table 1. Case 1

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C = B_1 \times PDB \times 0.98 + C_1$	$G = G_1$	$B = 0$

2) Non-gold market Open Day, and bitcoin yield is greater than 0.

How to cope: Invest all in bitcoin.

Post-transaction asset holdings:

Table 2. Case 2

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C = 0$	$G = G_1$	$B = B_1 + C_1 \times 0.98/PDB$

3) Open Day of gold market, and gold yield and bitcoin yield are less than 0.

How to cope: Convert all your bitcoins and gold into cash.

Various assets held after the transaction:

Table 3. Case 3

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C = C_1 + G_1 \times PDG \times 0.99 + B_1$	$G = 0$	$B = B_1 + C_1 \times 0.98/PDB$

4) Open Day of gold market, and the yield of gold is less than 0, and the yield of bitcoin is greater than 0.

How to cope: Invest all in bitcoin.

Post-transaction asset holdings:

Table 4. Case 4

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C = 0$	$G = 0$	$B = B_1 + C_1 \times PDG \times 0.99 \times 0.98/PDB + C_1 \times 0.98/PDB$

5) Open Day of gold market, and gold yield is greater than 0, bitcoin yield is less than 0.

How to cope: Invest all in gold.

Volume 8 Issue 7, 2022
DOI: 10.6919/ICJE.202207_8(7).0115

Post-transaction asset holdings:

Table 5. Case 5

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C=0$	$G=G_1+B_1\times PDB\times 0.98\times 0.99/PDG+C_1\times 0.99/PDG$	$B=0$

6) Gold market Open Day, and gold and bitcoin yield is greater than 0.

Coping strategy: Dual-objective planning, according to the calculated proportion of investment in gold and bitcoin asset redistribution of gold investment.

Various assets held after the transaction:

Table 6. Case 6

Post-trade cash holdings	Post-trade gold holdings	Post-trade bitcoin holdings
$C=0$	$G=(G_1\times PDG/0.99+B_1\times PDB/0.98+C_1)/PDG\times(1/0.99+x_2/x_1\times 0.98)$	$B=G_1\times PDG\times x_2/PDB\times x_1$

Implement the above best trading strategy every day in the 5-year trading period, and program the cycle, and the initial investment value of 1000 on September 10, 2021 is $231620.

5. Asset Movements under Optimal Strategies

The change of asset value over time during the trading term is shown in the Fig 4:

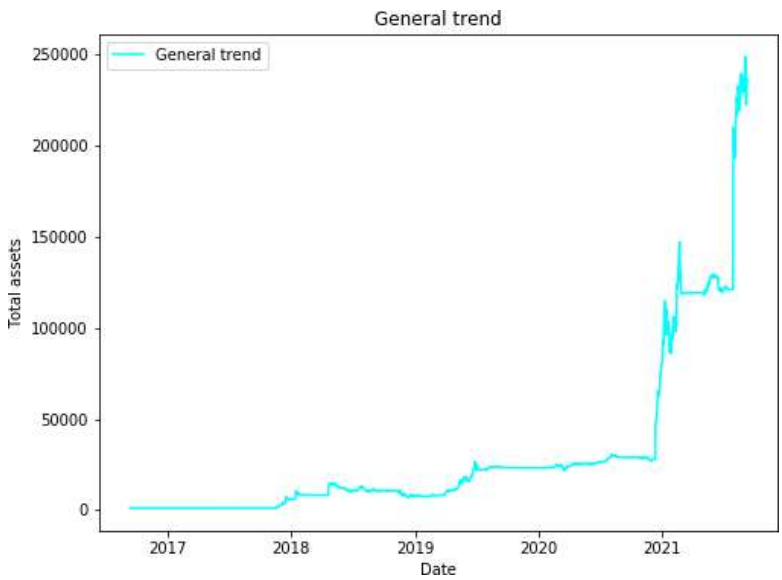

Fig. 4 Total asset trend

In addition, we weighed the risk and return, the linear programming model was established, The results of multiple solutions are stable at $231620, indicating to some extent that our model provides the best trading strategy.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

6. Model Evaluation and Further Discussion

6.1 Strengths

•Applies widely

This model is not only suitable for the investment of bitcoin and gold, but also can modify some parameters to be directly applied to other similar types of venture investment. It also plays an important role in stocks, bonds, funds and other fields[8].

•Strong discipline

The application of this trading strategy model to guide decision-making can overcome the weakness of human nature such as fear and greed, correct cognitive bias, procedually issue orders to achieve investment trading, and make traders more rational to make the most profitable choice[5].

•High probability of return

The model is accurate by computer calculation, high accuracy prediction, and auxiliary algorithm optimization, the application of ideal point method to give consideration to risk and profit, so that traders get as much profit as possible. The final result of this model is much higher than the original value of capital, again confirming the correctness of this model.

•High analysis efficiency

This model makes use of computer aided analysis of massive data and is obviously more efficient than manual analysis[10].

6.2 Weaknesses

•This model does not consider the investor's decision preference, but sets the weight of risk and return rate according to the question.

•The investment portfolio does not take into full consideration other practical factors in the market, such as market conditions, policy and cultural factors, etc. The model still needs to be updated and optimized to adapt to the increasingly complex investment market changes.

6.3 Further Discussion

In the subsequent discussion, we can select other prediction models for prediction comparison to reduce errors, such as time series model to predict future data.

In addition, when we use the ideal point method to solve the two-objective linear programming model, we do not consider the preference problem of decision makers, but directly assign high weight of returns according to the question. In the future, we plan to integrate preference interaction into this trading strategy model to help decision makers make decisions that meet their actual needs.

References

[1] Yang Guohua, Yan Yan, Yang Huizhong. Improved GM (1, 1) grey forecasting model and application [J]. Journal of nanjing university of science and technology, 2020, 44 (5) : 575-582. The DOI: 10.14177 / j.carol carroll nki. 32-1397 n. 2020.44.05.010.

[2] Deng Xue. Research on Portfolio Optimization Model [D]. South China University of Tech- nology,2010.

[3] Markowitz H. Portfolio Selection [J]. Journal of Finance, 1952, 7: 77-91.

[4] Huang WENlan. Research on portfolio optimization based on multi-objective evolution- ary algorithm [D]. Harbin commercial university, 2020. DOI: 10.27787 /, dc nki. GHRBS. 2020.000514.

[5] Meng X H. Application research of risk portfolio investment model based on multi-objective programming [D]. Wuhan University of Science and Technology,2013.

[6] Liu Lijun, LIANG Guopeng, Wang Haitao. Research on quantitative Trading Strategy System Model [J]. Shanghai Commerce,2021(09):158-159.

[7] Caferra Rocco,Tedeschi Gabriele,Morone Andrea. Bitcoin: Bubble that bursts or Gold that glitters? [J]. Journal of Economics, 2021205.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202207_8(7).0115

[8] Fan Zhuoran,Xu Jilong,Li Yuchen,Lu Chun. Multiobjective Programming Strategy of Small- and Medium-Sized Microenterprise Credit Based on Random Factors[J]. Journal of Function Spaces, 201,2021.

[9] Hu Shu month. The currency of the asset attributes research [D]. Southwest university of finance and economics, 2019, DOI: 10.27412 /, dc nki. Gxncu. 2019.001459.

[10] Wang Yahong. Bitcoin and gold: Who is a gold digger? [N]. China gold news, 2017-06-16 (008) DOI: 10.38298 / n.c. Nki NZGHJ. 2017.001023.