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Free Vibration of Elastic Beam in Magnetic Field

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Abstract

Based on Maxwell's equation, this paper considers the quasi-static field and adopts the electrodynamic equation suitable for elastic beam. The differential equation of motion of elastic beam in electromagnetic field is deduced by D'Alembert's principle.On this basis, according to the theoretical analysis of the elastic beam, the differential equation of the free vibration of the elastic beam under the action of the longitudinal uniform magnetic field represented by the deflection w is obtained. The mode shape function W and the time function T in three different cases are obtained by the separation of variables method. Under the given boundary conditions and initial conditions, the deflection expressions of simply supported beams at both ends are obtained. Finally, through specific numerical analysis, it is concluded that under a certain magnetic induction intensity, the vibration amplitude and vibration time of the simply supported beam decrease with the increase of the magnetic induction intensity, and the convergence speed becomes faster. When the magnetic induction intensity reaches a certain critical value, with the enhancement of the magnetic induction, the vibration of the beam intensifies.

Keywords

Elastic Beam; Uniform Magnetic Field; Free Vibration.

1. Introduction

With the development of science and technology, elastic beams are widely used in new energy vehicles, medical robots, artificial intelligence and other fields. Considering the complexity of the working environment in these fields, scholars are very interested in the interaction of electromagnetic and elastic fields. Therefore, many scholars have done a lot of research on some dynamic interactions that may occur between the electromagnetic field and the elastic field in the elastic body. Dynamic interactions including Lorentz force, elastic deformation to the current density constitutive equation and the correction of the differential equation of motion, which is very important.

Guan-YuanWu[1] studied the calculation formulas for transient vibration and dynamic instability of bi-material magneto-elastic cantilever beams under alternating magnetic field and thermal loads. QianZhao, YinanLiu et al. [2] used the finite element analysis software ANSYS to calculate the relationship between the size of the piezoelectric cantilever beam and the natural frequency. Jacek M. Bajkowski, Bartłomiej Dyniewicz [3] investigated an experimental study of a layered steel beam with a pneumatically controlled core of pressurized particulate matter. The simulation results show that this method can effectively reduce the vibration frequency. Luděk Pešek, Petr Šulc [4] studied the anti-resonance phenomenon of a forced vibration beam used for vibration suppression under additional parameter excitation. Feiyun Zhao and Jinyang Liu [5] studied the influence of nonlinear equations on the stability of elastic beams moving in a wide range. Jean-Jacques Marigo and Kim Pham [6] analyzed the influence of an array of plates or beams on a semi-infinite elastic ground on the propagation of elastic waves hitting the interface. JunZhang,XuebinZhang et al. [7] proposed a bending wave control technology based on u-shaped phonon beam and found that the element surface

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made of the same material as the motherboard, in which the bending wave can be controlled. Ambrosini, RD et al. [8] proposed a method for simulating stationary random processes in the frequency domain, which can be advantageously applied to the dynamic analysis of thin-walled open-section beams. Aand Ogden, RW [9] studied the governing equations of solid materials capable of producing large electro-elastic deformations in the static case. Otténio, M., Destrade, M. & Ogden, R.W.[10] gives the deformation control equation of infinitesimal incremental disturbance superimposed on a finite static deformation field with magnetic and elastic interaction.

Although scholars have done a lot of research, there is little research on the free vibration analysis of elastic beams. Therefore, this paper mainly studies the free vibration of elastic beam in longitudinal uniform magnetic field. Analyzing the convergence degree of the free vibration of the elastic beam under different magnetic induction strengths, so as to control the free vibration of the elastic beam, is also of great significance in practical engineering problems.

2. Differential Equations of Vibration for Beams

In the analytical calculations in this section, electromagnetic elastic solids will be treated in a linear elastic manner, assuming that the material and electromagnetic field are in an isotropically homogeneous medium. In the motion of the material, the current density constitutive equation of the electromagnetic field will take a nonlinear form, the basic electrodynamic equation and the basic elastic mechanics equation will be coupled together, and the elastic field and the electromagnetic field will interact and influence each other.

2.1 Basic Equations of Electromagnetic Fields

In the case of electromagnetic field interactions, all electromagnetic field vectors satisfy Maxwell's equations. Maxwell's equations are composed of four equations: Gauss's theorem, Gauss's law of magnetism, Faraday's law of electromagnetic induction and Maxwell-Ampere's law.

For many practical problems, although the electromagnetic field varies with time, it is slow enough. Such an electromagnetic field is called a quasi-stationary field, and is also called a quasi-static field. The problem can be solved under the pseudo-steady field conditions, provided that the pseudo-steady field conditions are satisfied. The first condition for the establishment of a seemingly stable field is that the change frequency ω of the electromagnetic field is much smaller than the characteristic frequency ω_σ of the metal:

$$\omega << \omega_\sigma \tag{1}$$

For most metals, the characteristic frequency is much larger than the changing frequency of the electromagnetic field.

The second condition for the establishment of the seemingly stable field is when the linearity of the investigated area is much smaller than the wavelength:

$$R << \frac{\lambda}{2\pi} \tag{2}$$

The influence of displacement current can be ignored under the condition that the two quasi-stable conditions are satisfied. At the same time, the sheet through which the current flows is uniform in all directions, so there is no accumulation of charges, and the charge density is zero. So Maxwell's equations can be rewritten as:

$$\nabla \bullet \boldsymbol{D} = 0 \tag{3}$$

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$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{4}$$

$$\nabla \bullet \boldsymbol{B} = 0 \tag{5}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{6}$$

In the formula, E represents the electric field, H represents the magnetic field, B represents the magnetic induction intensity, and D represents the electrical displacement. J and ρe are current density and charge density respectively.

Lorentz force density expression:

$$\boldsymbol{f}^{e} = \rho_e \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B} \tag{7}$$

When the rectangular beam generates an induced current, formula (7) is transformed into:

$$\boldsymbol{f}^{e} = \boldsymbol{J} \times \boldsymbol{B} \tag{8}$$

The Lorentz force expression and Maxwell's equations together form the fundamental equations of electrodynamics.

2.2 Differential Equations of Vibration for Rectangular Beams

When analyzing the motion of electromagnetic solids in the electromagnetic field, such as the radial motion of the material in the electromagnetic field, the vibration or rotation of the material in the electromagnetic field, the nonlinear constitutive equation is usually used for calculation. However, when the moving speed of the material is much smaller than the speed of light, the constitutive equation of the electromagnetic material in the isotropic homogeneous medium is linear, and the constitutive relationship of the current density is nonlinear, and the specific form is as follows:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{9}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{10}$$

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \rho_e \boldsymbol{v} \tag{11}$$

In the formula, ε,μ and σ represent permittivity, permeability and conductivity, respectively.

In the rectangular beam problem discussed in this paper, the charge density is zero, and the velocity is represented by displacement, Equation (11) can be rewritten as:

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{B}) \tag{12}$$

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When the rectangular beam is in the constant magnetic field of B0x, and the thin plate is in the equilibrium position under the transverse load q in the y direction, as shown in Fig. 1, the thin plate will vibrate slightly after the disturbance is removed and the magnetic field line will be cut to generate y The direction of the Lorentz force f.

According to formula (8) and formula (12), the specific expression of the Lorentz force per unit volume can be obtained as follows:

$$f_y = -\sigma \frac{\partial w}{\partial t} B_{0x}^2 \tag{13}$$

According to Eq. (13), the Lorentz force on the cross-section of the elastic beam can be expressed as follows:

$$f_y = -\sigma A \frac{\partial w}{\partial t} B_{0x}^2 \tag{14}$$

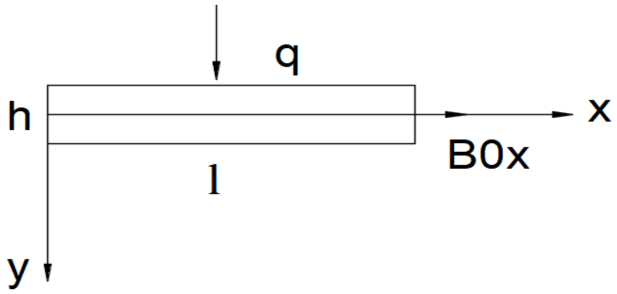

Fig. 1 Longitudinal magnetic field rectangular beam

The static equilibrium differential equation of the elastic rectangular beam is:

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 w}{\partial x^2} \right) = q \tag{15}$$

where E is the elastic modulus and I is the moment of inertia. In a rectangular beam, the moment of inertia of the beam section is expressed as:

$$I = \frac{1 \times h^3}{12} \tag{16}$$

The balance equation shown in the above formula only discusses the statics problem. In the dynamical problem, the assumption of an ideal elastic body and the assumption of small displacement are still used. The physical and geometric equations are still applicable to any instant in the dynamical problem and do not need to be changed. But the equation of balance must be replaced by the equation of motion.

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When establishing the differential equation of motion of the object, considering the inertial force exerted by the elastic body due to the acceleration and the Lorentz force generated by the vibration in the electromagnetic field, the differential equation of motion of the elastic beam can be obtained according to D'Alembert's principle:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right)=q+f_y-\rho A\frac{\partial^2 w}{\partial t^2} \tag{17}$$

Substituting Equation (14) into Equation (17), the vibration differential equation of the elastic beam in the magnetic field is obtained as follows:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right)+\sigma A\frac{\partial w}{\partial t}B_{0x}^2+\rho A\frac{\partial^2 w}{\partial t^2}=q \tag{18}$$

3. Vibration Analysis of Rectangular Beam in Magnetic Field

In the previous section, this paper obtained the differential equation of motion of the elastic beam in the magnetic field. In this section, the influence of the magnetic field on the beam after the elastic beam is disturbed will be analyzed in detail. First the deflection of the beam at equilibrium position we(x):

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w_e}{\partial x^2}\right)=q \tag{19}$$

Subtracting equation (19) and equation (18) can get the following expression:

$$EI\frac{\partial^4}{\partial x^4}(w_t-w_e)+\rho A\frac{\partial^2 w_t}{\partial t^2}+\mu^2\sigma AH_{0x}^2\frac{\partial w_t}{\partial t}=0 \tag{20}$$

Because we is a function of x, it does not change with time, and equation (20) can be transformed into the following form:

$$EI\frac{\partial^4}{\partial x^4}(w_t-w_e)+\rho A\frac{\partial^2 (w_t-w_e)}{\partial t^2}+\mu^2\sigma AH_{0x}^2\frac{\partial (w_t-w_e)}{\partial t}=0 \tag{21}$$

When the deflection of the elastic beam during free vibration is measured from the equilibrium position, the deflection w at any time is:

$$w=w_t-w_e \tag{22}$$

Equation (21) is transformed into the following form by Equation (22):

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$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}+\mu^2\sigma AH_{0x}^2\frac{\partial w}{\partial t}=0 \tag{23}$$

At this time, solving the vibration problem of a rectangular beam in a magnetic field boils down to solving w(x, t) by formula (23) under the given boundary and initial conditions.

In this section, the method of separating variables is used to decompose w(x, t) into a function T of time t and a mode shape function W of x. The specific form is as follows:

$$w(x,t)=W(x)T(t) \tag{24}$$

Substituting equation (24) into equation (23) yields an equation of the form:

$$\frac{EI\partial^4 W}{\rho hW\partial^4 x}=-\frac{\partial^2 T}{T\partial t^2}-\frac{\mu^2\sigma H_{0x}^2\partial T}{\rho T\partial t} \tag{25}$$

The left side of the above formula is a function of x, and the right side is a function of time t. If the above formula can be established, it can only be equal to a constant, and the constant is taken as ω2, and the following two formulas are obtained:

$$\frac{d^4 W}{dx^4}-\gamma^4 W=0 \tag{26}$$

$$\frac{d^2 T}{dt^2}+\beta^2\frac{dT}{dt}+\omega^2 T=0 \tag{27}$$

Among them,

$$\begin{cases}\gamma^4=\omega^2\dfrac{\rho h}{EI}\\ \beta^2=\dfrac{\mu^2\sigma H_{0x}^2}{\rho}=\dfrac{\sigma B_{0x}^2}{\rho}\end{cases} \tag{28}$$

Formula (27) is a second order homogeneous linear differential equation with constant coefficients, which is solved.

$$r^2+\beta^2 r+\omega^2=0 \tag{29}$$

The solution form of the above formula can be divided into three cases:

The first case is that at $\beta^4-4\omega^2=0$, $r=-\beta^2/2$, the equation has two identical real roots, and the time function T is:

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$$T(t)=(A+Bt)\exp\left(-\frac{\beta^2}{2}t\right) \tag{30}$$

In the second case, when $\beta^4-4\omega^2<0$ and $r=-\beta^2\pm i\sqrt{4\omega^2-\beta^4}/2$, the equation has a pair of conjugate complex roots, and the time function T is:

$$T(t)=\exp\left(-\frac{\beta^2}{2}t\right)\left(A\cos\frac{\sqrt{4\omega^2-\beta^4}}{2}t+B\sin\frac{\sqrt{4\omega^2-\beta^4}}{2}t\right) \tag{31}$$

The third case is that when $\beta^4-4\omega^2>0$ and $r=-\beta^2\pm\sqrt{\beta^4-4\omega^2}/2$, the equation has two different real roots, and the time function T is:

$$T(t)=A\exp\left(\frac{-\beta^2+\sqrt{\beta^4-4\omega^2}}{2}t\right)+B\exp\left(\frac{-\beta^2-\sqrt{\beta^4-4\omega^2}}{2}t\right) \tag{32}$$

In case 2, there is a pair of conjugate complex roots. When the magnetic field does not exist, that is, when the magnetic field is 0, the time function $T(t)=A\cos\omega t+B\sin\omega t$ is consistent with the time function of the free vibration of the elastic beam.

According to the boundary conditions, the mode function of the simply supported beam at both ends can be taken as follows:

$$W=\sin\frac{\pi}{l}x \tag{33}$$

The lowest natural frequency of simply supported beams at both ends is:

$$\omega=\frac{\pi^2}{l^2}\sqrt{\frac{EI}{\rho h}} \tag{34}$$

The initial conditions for a simply supported beam at both ends are as follows:

$$\begin{cases}(W)_{t=0}=W_0=\sin\frac{\pi}{l}x\\ \left(\frac{\partial W}{\partial t}\right)_{t=0}=v_0=0\end{cases} \tag{35}$$

When the time function T has a pair of the same real roots, the formula (35) is substituted into the equation (30) to solve the values of the constants An and B, and the deflection expression is obtained.

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$$w=\left(1+\frac{\beta^2}{2}t\right)\exp\left(-\frac{\beta^2}{2}t\right)\sin\frac{\pi}{l}x \tag{36}$$

When the time function T has a pair of conjugate complex roots, the formula (35) is substituted into the equation (31) to solve the values of the constants An and B, and the deflection expression is obtained.

$$w=\exp\left(-\frac{\beta^2}{2}t\right)\left(\cos\frac{\sqrt{4\omega^2-\beta^4}}{2}t+\frac{\beta^2}{\sqrt{4\omega^2-\beta^4}}\sin\frac{\sqrt{4\omega^2-\beta^4}}{2}t\right)\sin\frac{\pi}{l}x \tag{37}$$

When the time function T has a pair of different real roots, the formula (35) is substituted into the equation (32) to solve the values of the constants An and B, and the deflection expression is obtained.

$$w=\left(\frac{\beta^2}{2\sqrt{\beta^4-4\omega^2}}+\frac{1}{2}\right)\exp\left(\frac{-\beta^2+\sqrt{\beta^4-4\omega^2}}{2}t\right)+\left(\frac{1}{2}-\frac{\beta^2}{2\sqrt{\beta^4-4\omega^2}}\right)\exp\left(\frac{-\beta^2-\sqrt{\beta^4-4\omega^2}}{2}t\right)\sin\frac{\pi}{l}x \tag{38}$$

Under the boundary conditions and given initial conditions of the simply supported beam at both ends, the mode function W and amplitude function T of the simply supported beam at both ends are analyzed and calculated, on the basis of which the deflection equation w (x,t) under different conditions is derived. In the last section, the specific parameters will be selected for numerical calculation, so as to analyze the influence of different magnetic field on the free vibration of simply supported beam.

4. Numerical Analysis

A simply supported beam made of copper material is selected and the specific parameters are given as follows: Elastic modulus E=119Gpa, Poisson's ratioν=0.326, Material density ρ=8200kg/m3, Electric conductivity σ=5.65×107(Ω·m)-1, The height of the simply supported beam is 0.1m, the side length of the beam is 1m, and the width of the beam is calculated as 1.

The moment of inertia and free vibration frequency of simply supported beam are as follows:

$$I=\frac{1\times h^3}{12}=8.3\times10^{-5} \tag{39}$$

$$\omega=\frac{10\pi^2}{l^2}\sqrt{\frac{EI}{\rho A}}=1.08 \tag{40}$$

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Take the magnetic induction intensities B1=1×10-2T, B2=1.3×10-2T and B3=1.5×10-2T. At this time, the equation has a pair of conjugate complex roots corresponding to case 2, and the time functions T are respectively:

$$T_1(t) = \exp(-0.35t)(\cos 1.02t + 0.34\sin 1.02t)$$
$$T_2(t) = \exp(-0.58t)(\cos 0.91t + 0.64\sin 0.91t)$$
$$T_3(t) = \exp(-0.78t)(\cos 0.75t + 1.03\sin 0.75t)$$

The function image is shown in fig. 2:

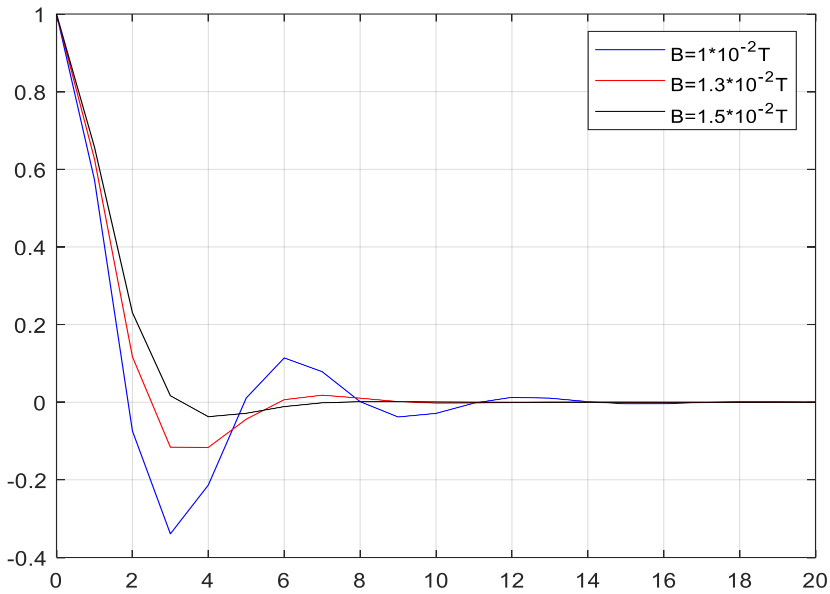

Fig. 2 Time function under critical magnetic induction

As can be seen from image 2, the greater the magnetic induction intensity, the faster the vibration attenuates, and at the same time, the amplitude of the vibration becomes smaller and smaller, indicating that the enhanced magnetic induction intensity can quickly restore the beam to the equilibrium position and stop the vibration after the free vibration occurs.

Take the magnetic induction intensities B4=1.5×10-2T, B5=1.77×10-2T and B6=2×10-2T. At this time, the equation has a pair of conjugate complex roots, the same real roots and different real roots respectively, corresponding to the three cases in the previous section. The time functions T are:

$$T_3(t) = \exp(-0.78t)(\cos 0.75t + 1.03\sin 0.75t)$$
$$T_5(t) = (1 + 1.08t)\exp(-1.08t)$$
$$T_6(t) = 1.31\exp(-0.52t) - 0.31\exp(-2.24t)$$

The function image is shown in fig. 3, fig. 4:

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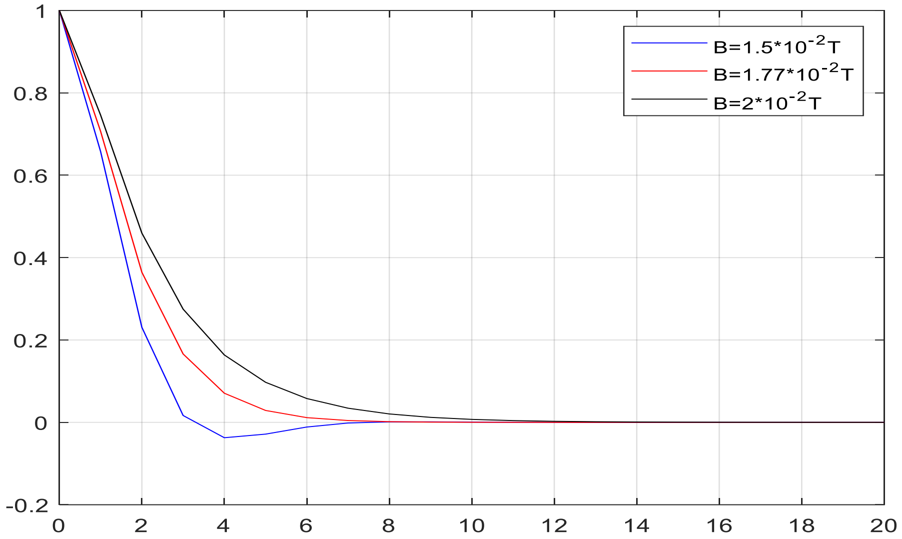

Fig. 3 Comparison of different magnetic induction intensities

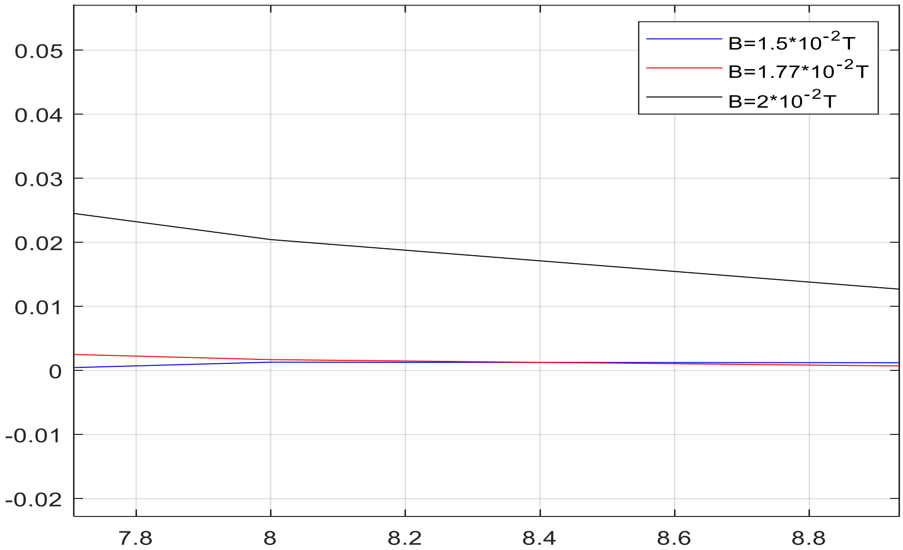

Fig. 4 Local contrast diagram of different magnetic induction intensity

As can be seen from figs. 3 and 4, when the magnetic induction intensity increases to an equilibrium value, that is, when the magnetic induction intensity is B5, the time to stop vibration is the shortest. When the magnetic induction intensity exceeds this value, the free vibration time of the beam begins to increase gradually, and the vibration amplitude becomes larger, which is not conducive to the control of the free vibration of the elastic beam.

5. Conclusion

In this paper, the differential equation of free vibration of elastic beam in magnetic field is derived by D'Alembert principle, and some conclusions are obtained by numerical analysis. When a beam with transverse load is in a state of bending equilibrium, when a beam vibrates slightly at the equilibrium position due to interference, by increasing the magnetic field, and in a certain range, the strength of the magnetic field can reduce the free vibration time of the beam and reduce the vibration amplitude

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of the beam. If the increased magnetic field is too large, it will aggravate the vibration. Therefore, in the engineering problems, it is particularly important to reasonably increase the strength of the magnetic field to control the free vibration of the beam.

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