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Research on M-θ Theoretical Relationship of Straight Mortise-tenon Joints in Chinese Traditional Timber Structures

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Abstract

There is great wisdom in the construction techniques of Chinese traditional timber structures. The use of tenon and mortise for beam-column joints is an important feature that distinguishes the Chinese traditional timber structures from modern building structures. The rotational performance of this semi-rigid connection also plays an important role in the study on seismic performance of timber structures. The straight mortise-tenon joint is a common timber structure joint. In this paper, the mechanical principle of the straight mortise-tenon joint of the timber structure is analyzed from the two aspects of the friction characteristics of the wood and the embedment mechanism, and the M-θ theoretical relationship of the straight mortise-tenon joint in elastic stage is deduced. This work also prepares for the next step to study the seismic performance of timber structural systems.

Keywords

Traditional Timber Structure; Straight Mortise-tenon Joint; M-Θ Relationship.

1. Introduction

The Chinese traditional timber structure architecture has its unique structural system, in which mortise and tenon connection is the most important, because the joint of beam and column of timber structure, the connection of paving layer and column top and the lap joint of the roof truss members all use mortise and tenon connection. For this reason, it can be said that mortise-tenon joint is the soul of traditional Chinses timber structure. The mortise and tenon connection method allows the connected beams and columns to have a certain relative slip and rotation, and so has obvious semi-rigidity. Therefore, the discussion on the rigidity of mortise-tenon joints is of far-reaching significance for the study of mechanical and seismic performance of traditional timber structure buildings, and is an indispensable theoretical support for accurate finite element simulation of timber structure buildings. At the same time, it also plays an extremely important role in the scientific restoration and protection of wooden structures. Among them, the special performance of M-θ relationship of semi-rigid connection of mortise-tenon joint can be a quantitative description of the load-deformation relationship of the joint under cyclic force, which can reflect the stiffness change law, deformation characteristics and energy consumption of mortise-tenon joint during the stress process, and is also the basis of determining the restoring force model and performing nonlinear seismic response analysis. In a word, this is a problem that must be solved in the research of traditional timber structures. The straight mortise-tenon joint is one of the most common connections in Chinese traditional timber structures. Many scholars have carried out experimental research and numerical simulation study on its mechanical properties, and some important results have been achieved. In this paper, the mechanical equilibrium of straight mortise-tenon joints is analyzed from the point of view of wood friction characteristics and embedment mechanism, and the theoretical expression of the relationship between bending moment and rotation angle is derived.

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2. Theory of Friction and Embedment in Wood

2.1 Geometry of Straight Mortise-tenon Joints

Straight mortise-tenon joints are commonly used in the connection between the beam and the column in the Chuan-dou timber structures. The construction method is to first open the rectangular mortise that penetrates the column body, and the same time, the beam is processed into a tenon with the same size as the mortise, and then the tenon is inserted into the mortise in the process of installation, and finally a relatively firm and stable wooden connection is formed. The geometric structure of the straight joint is shown in figure 1. As seen in figure 1, the subscript “L”, “W” and “H” denote the abbreviation of “length”, “width” and “height”, respectively; the “m” and “t” represent “mortise” and “tenon” in the straight mortise-tenon joint, respectively.

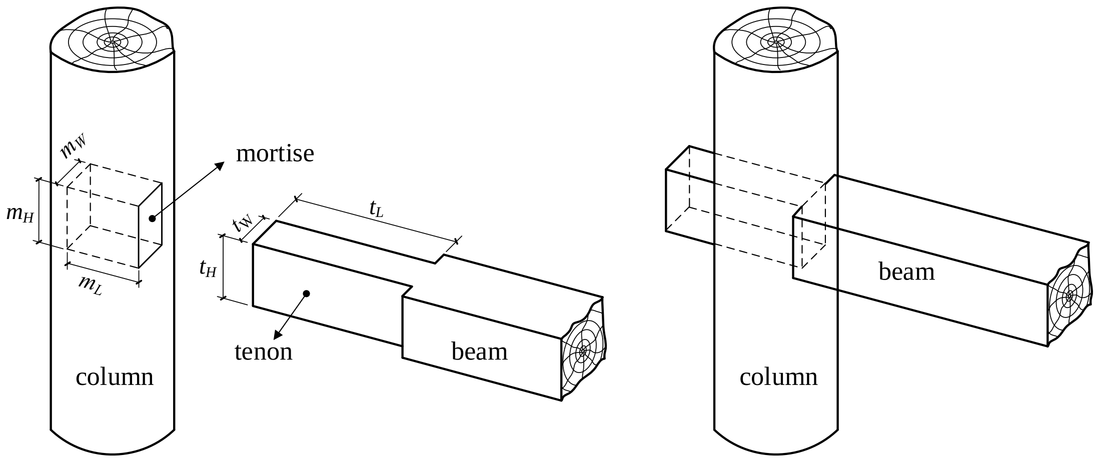

Figure 1. Straight mortise-tenon joint

2.2 Friction Properties of Wood

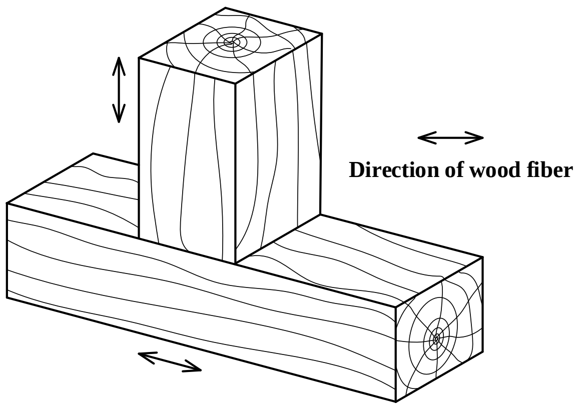

Figure 2. Straight mortise-tenon joint

As a typical organic material, due to the different type, composition and arrangements of wood growth cells, there are great differences in physical and mechanical properties of all kinds of wood species, including the friction between wood. The friction characteristics of wood are also related to its friction contact surface. The surface morphology of the same kind of wood is different and the friction characteristics of the contact surface are different because of tree age, dry shrinkage and swelling, processing error, defects and knots and so on. In addition, wood is a kind of anisotropic material, and the friction coefficient of the indirect contact surface of the transverse grain and the friction coefficient between the transverse grain the parallel grain is also different. In the straight mortise-tenon joint, the surface between the wood belongs to the case of transverse grain and parallel grain contact, as shown in figure 2. The static friction coefficient and dynamic friction coefficient of the

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interface between the transverse grain and the parallel grain of wood can generally be determined by the test method. In the process of rotation of the straight tenon joint, the contact surface between the mortise and the tenon will often produce a certain friction force and provide the corresponding rotational stiffness for the joint.

2.3 Embedding Properties of Wood

Embedment of wood refers to the compression and deformation of wood under pressure in the direction of the transverse grain, as shown in figure 3. In the mortise-tenon joints of the timber structure, because the parallel grain strength of the wood is much higher than the transverse grain strength of the wood, the wooden column will be locally compressed and embedded in the wooden beam in the process of joint rotation. When the wooden beam is locally deformed, mechanical behaviors such as yield and hardening will occur in this part of the wood, as shown in figure 4. This kind of deformation is much larger than common engineering materials, such as concrete, steel and so on. It is a typical large strain problem in mechanical analysis. Because of this, the straight mortise-tenon joint is often allowed to have a larger rotation angle, and the rotational stiffness of the joint is small, which is regarded as a semi-rigid connection in the structural mechanics analysis, which is also a great difference between the timber structure and the modern structure.

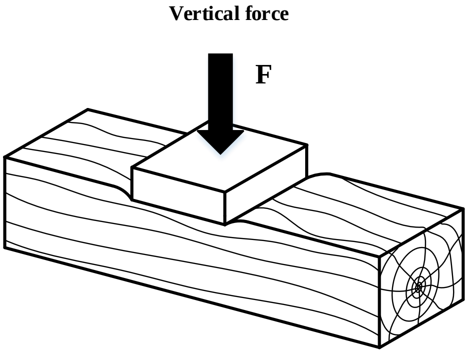

Figure 3. Straight mortise-tenon joint

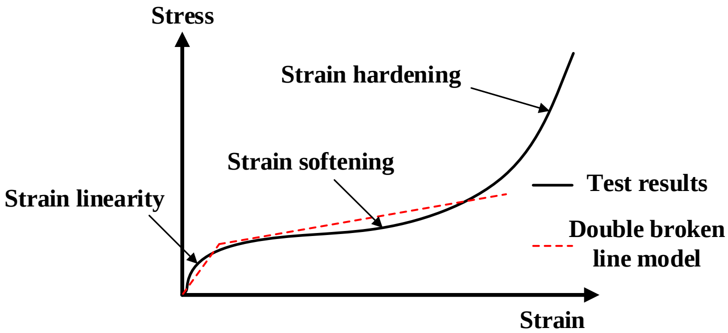

Figure 4. Stress-strain diagrams of compression of wood perpendicular to the grain

As shown in figure 5, it is assumed that the upper surface of the wood is squeezed and deformed by pressure perpendicular to grain, and the whole stress process is elastic, and the deformation curve is expressed by z=z(x), according to Hooke's theorem, the stress caused by deformation can be calculated by the following formula:

$$\sigma(x)=\varepsilon(x)E_w=\frac{z(x)}{z_0}E_w \tag{1}$$

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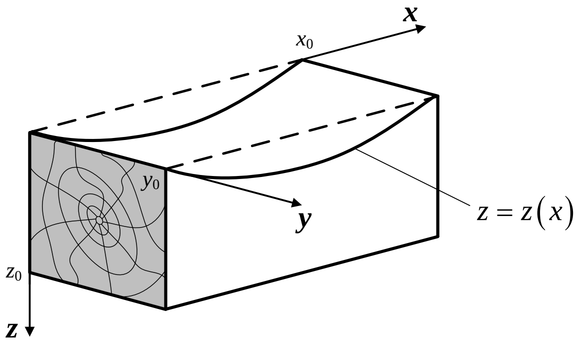

Figure 5. Schematic representation of the deformation for elastomer

Furthermore, the resistance force P caused by the compression deformation of wood perpendicular to grain is obtained as follows:

$$P = y_0 E_w \int_0^{x_0} \frac{z(x)}{z_0} dx = \frac{E_w}{z_0} V \tag{2}$$

3. M-θ Theoretical Relationship of Straight Mortise-tenon Joints

3.1 Basic Hypothesis

The actual stress process of the straight mortise-tenon joint of the timber structure is relatively complex, and the theoretical derivation is more complicated because of the wood anisotropy and the initial defects of the model. In order to facilitate the derivation, the factors that have little influence on the calculation results in the stress process of the joints are ignored or simplified, and the following basic assumptions are made.

(1) On the contact surface between the mortise and the tenon extrusion, it is assumed that the friction force is uniform, that is, the friction coefficient is the same and constant.

(2) From the previous experimental results, it is found that the stress-strain curve of woo under transverse compression generally goes through three stages: linear, softening yield and strain hardening. Considering that when the straight mortise-tenon joint rotates due to the overall deformation of the timber structure, the wood perpendicular to grain embedded pressure has not yet reached the strain strengthening stage, so when analyzing the theoretical relationship M-θ of the joint, the wood perpendicular to grain embedment constitutive relation is based on the double broken line model, as shown in the dotted line in figure 4.

(3) The deformation properties of wood in all directions are different from each other. In general, the compression elastic modulus of wood parallel to grain is 10-15 times of that perpendicular to grain, so when local embedment occurs, relative to the compression of perpendicular to grain on the contact surface of tenon, the deformation parallel to grain on the contact surface of mortise is very small and can be ignored. In other words, it is considered that the deformation occurs only in the embedded part of the upper and lower side of the tenon.

(4) The bending deformation of the tenon in the mortise is ignored. In the actual test, it is found that the bending deformation of the tenon in the mortise is very small, and it can be considered that the tenon only rotates in the plane, that is, the position of the tenon can be determined by the relative rotation angle between the mortise and the tenon.

(5) The influence of uneven force on the tenon is ignored. The influence of the roughness of the wood processing surface on the uneven force on the upper and lower surface of the tenon is not considered, that is, the uniform force on the embedded part of the tenon is considered.

(6) In the force analysis, it is considered that there is no gap between the tenon and the mortise. In fact, during the long-term use of timber structure, due to wood dry shrinkage cracking, moth eating,

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decay and other reasons, mortise-tenon joints will produce gaps and even loosening. In order to simplify the theoretical analysis, the gap between the mortise and tenon is ignored, so that in figure 1, the height and width of the tenon and the mortise are the same, that is, tH=mH, tW=mW.

3.2 M-θ Relationship of Straight Mortise-tenon Joints

3.2.1 Length of Embedment Zone

The deformation diagram of the embedded compression zone during the rotation of straight mortise-tenon joint is shown in figure 6.

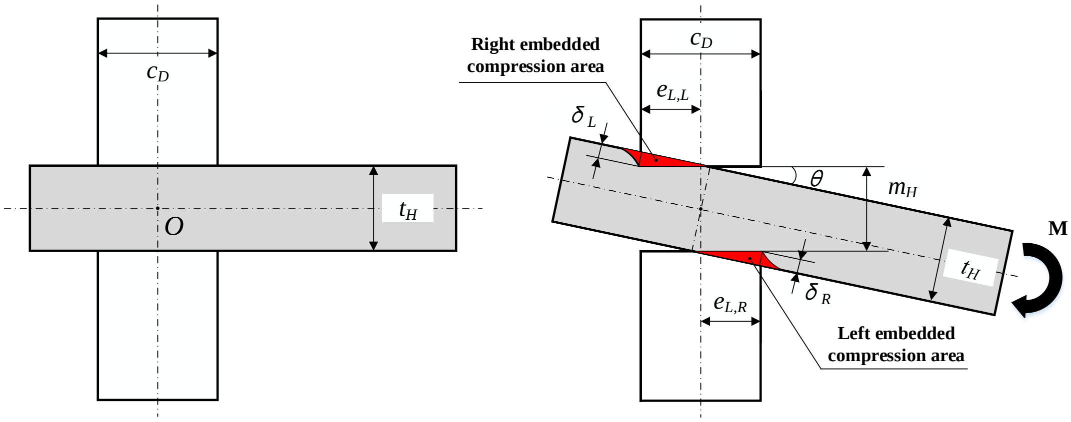

Figure 6. Deformation of the straight mortise-tenon joint model

At the initial stage of the rotation force, the joint rotates a certain angle θ clockwise, the upper and lower surface of the tenon squeeze the mortise to produce embedded pressure, the embedded tenon is deformed, and the embedded pressure on the left and right sides is equal in the opposite direction, forming a pair of force couples, therefore, a moment is generated to resist the external torque. The triangular embedded area on the left and right sides of the tenon is shown by the red shadow in figure 6. At this time, the expression of the length of the compressed area is expressed as:

$$e_{L,L} = e_{L,R} = \frac{1}{2}c_D \tag{3}$$

3.2.2 Force Analysis of Straight Mortise-tenon Joint

Figure 7 shows the force analysis of the straight mortise-tenon joint when the beam is rotated.

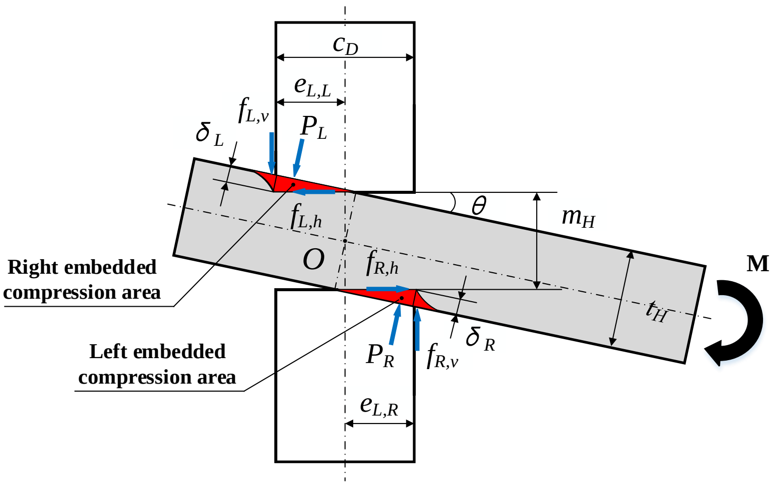

Figure 7. Force analysis of the straight mortise-tenon joint

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The stress process of straight mortise-tenon joint is divided into four stages, namely, the increase of forward bending moment, the decrease of forward bending moment, the increase of reverse bending moment and the decrease of reverse bending moment. It is uniformly defined that the beam end bending force is positive bending when clockwise, and the bending moment force is negative bending when counterclockwise. Because the joint is symmetrical, the stress of the joint is the same when the positive and negative bending moment increases and decreases, so only the force of the joint under positive bending is analyzed.

As shown in figure 7, when a clockwise bending moment is applied at the end of the beam, the embedded pressure behavior begins to occur on the left and right sides of the tenon, and the embedded pressure PL and PR are produced in the embedment area, and the direction is perpendicular to the compression surface. The tests found that the friction force between the mortise and tenon of the joint is composed of two parts: the horizontal friction force fL, h and fR, h caused by the relative sliding of the mortise and the tenon in the embedment area, and the vertical friction force fL, v and fR, v caused by the rotation and deformation of the tenon in the embedment area.

3.2.3 Embedded Compression Displacement of Wood Perpendicular to Grain

According to the reference [6], when the wood is elastic, the value of α can be deduced according to the following formula:

$$\alpha=\frac{3}{2z_1}=\frac{1.5}{t_H} \tag{4}$$

Here z1 represents the thickness of the wood perpendicular to grain in the compression direction, which is the tenon height tH in the straight mortise-tenon joints.

3.2.4 Volume Deformation of Wood in Embedment Area

In the early stage of the rotation of the joint, the wood in embedment areas on the upper and lower surfaces of the tenon are elastic, and the deformation of the triangular embedment areas are shown in figure 8. The wood embedment deformation consists of two parts: the deformation area under direct load action I and the deformation area under indirect load action II. In the system, the elastic deformation area is approximated as a trapezoid, and the total embedment depth is set as δ.

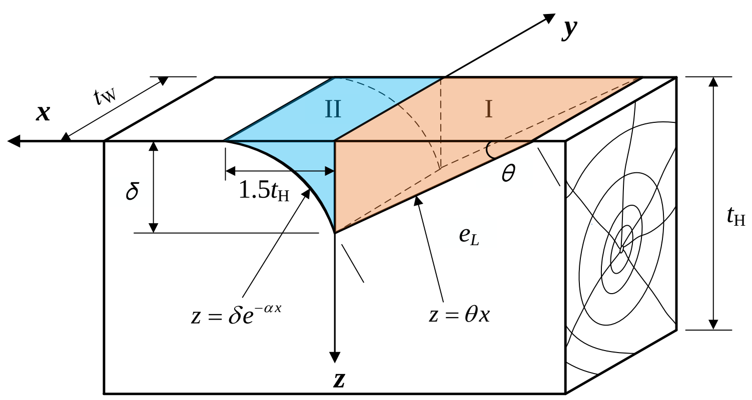

Figure 8. Embedding deformation volume

According to the geometric relationship in figure 8, it can be obtained:

$$\delta=e_L\sin\theta \tag{5}$$

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$$x_1=\frac{t_H}{1.5}\ln\frac{\delta_y}{\delta} \tag{6}$$

$$x_2=\frac{\delta\text{-}\delta_y}{\theta} \tag{7}$$

$$x_3=1.5t_H-\frac{t_H}{1.5}\ln\frac{\delta_y}{\delta} \tag{8}$$

$$x_4=\frac{\delta_y}{\theta} \tag{9}$$

From this, the deformation volumes VI and VII of the embedment zone in the elastic stage can be calculated as follows:

$$V_I=\frac{1}{2}t_W e_L\delta\cos\theta=\frac{1}{2}t_W e_L^2\sin\theta\cos\theta\approx 0.5e_L^2 t_W\theta \tag{10}$$

$$V_{II}=t_W\int_0^{1.5t_H}\delta e^{-\frac{1.5}{t_H}x}dx=0.6e_L t_W t_H\sin\theta\approx 0.6e_L t_W t_H\theta \tag{11}$$

3.2.5 Force of Embedment and Friction

In the elastic stage, the front pressure and friction force of the embedment compression area on the upper and lower surface of the tenon are PL and PR; fL, h and fR, h ; fL, v and fR, v, respectively, and the compressive elastic modulus perpendicular to grain of wood is EW. At the same time, because the straight mortise-tenon joint is symmetrical, PL = PR; fL, h = fR, h ; fL, v = fR, v. According to the material constitutive relation and basic assumption, the embedment pressure P caused by embedding deformation can be obtained by substituting the volume deformation of the tenon into the formula (2) as follows:

$$P=P_L=P_R=\frac{E_w}{t_H}(V_I+V_{II})=\frac{E_w}{t_H}\left(0.5e_L^2 t_W+0.6e_L t_W t_H\right)\theta \tag{12}$$

In order to facilitate the calculation, the friction coefficient is taken as a constant value, so the friction force in the embedment area on the upper and lower side of the tenon is:

$$f_h=f_{L,h}=f_{R,h}=\mu P_y=\mu P\cos\theta \tag{13}$$

$$f_v=f_{L,v}=f_{R,v}=\mu P_x=\mu P\sin\theta \tag{14}$$

3.2.6 M-θ Theoretical Relationship of Straight Mortise-tenon Joint

Taking the moment from the center point O of figure 7, the embedment pressure resistance bending moment MP and friction resistance bending moment Mf in the elastic stage are respectively as follows:

$$M_P=\left[P\cos\theta\left(\frac{c_D}{2}-\frac{e_L}{3}\right)+P\sin\theta\frac{m_H}{2}\right]\times 2=\frac{2E_w}{3t_H}\left(0.5e_L^2 t_W+0.6e_L t_W t_H\right)c_D\theta \tag{15}$$

$$
\begin{aligned}
M_f &= f_{L,h} \times \frac{m_H}{2} + f_{L,v} \times \frac{c_D}{2} + f_{R,h} \times \frac{m_H}{2} + f_{R,v} \times \frac{c_D}{2} = \mu P\left(m_H \cos\theta + c_D \sin\theta\right) \\
&= \frac{\mu E_w m_H}{t_H}\left(0.5e_L^2 t_W + 0.6 e_L t_W t_H\right)\theta
\end{aligned} \tag{16}
$$

Then the total resistance moment of the straight mortise-tenon joints in the elastic stage is:

$$
\begin{aligned}
M &= M_P + M_f \\
&= \frac{2E_w}{3t_H}\left(0.5e_L^2 t_W + 0.6 e_L t_W t_H\right)c_D\theta + \frac{\mu E_w m_H}{t_H}\left(0.5e_L^2 t_W + 0.6 e_L t_W t_H\right)\theta \\
&= K_P\theta + K_f\theta
\end{aligned} \tag{17}
$$

In formula (17), KP is the rotational stiffness provided by the embedment force of the tenon; and Kf is the rotational stiffness provided by the friction force of the tenon, and the calculation formula are as follows:

$$
K_P = \frac{2E_w}{3t_H}\left(0.5e_L^2 t_W + 0.6 e_L t_W t_H\right)c_D \tag{18}
$$

$$
K_f = \frac{\mu E_w m_H}{t_H}\left(0.5e_L^2 t_W + 0.6 e_L t_W t_H\right) \tag{19}
$$

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