

Comparison of Different Theoretical Calculation Methods of Soil Squeezing Effect

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Abstract

The calculation of soil squeezing effect in the construction process of soil squeezing pile has very important engineering practical value. Through the theoretical analysis method, the commonly used soil squeezing effect calculation theory, circular hole expansion theory, column hole expansion theory and spherical hole expansion theory are compared and analyzed, and the engineering applicability of each theory is clarified. The results show that the construction of soil compaction pile can be abstracted as the expansion process of column hole on the side of pile and the expansion process of ball hole at the end of pile in semi infinite space. Compared with the circular hole expansion theory, the column hole expansion theory is more in line with the engineering practice of soil compaction pile construction. The modified Cambridge model and Mohr Coulomb model commonly used in geotechnical engineering have been successfully applied to circular hole expansion theory, cylindrical hole expansion theory and spherical hole expansion theory. More advanced constitutive models need to be introduced into relevant expansion theory. The column hole expansion theory considering soil stratification, displacement boundary in hole, deterioration of soil mechanical properties and soil plug effect is a very valuable research direction.

Keywords

Soil Squeezing Effect; Calculation Method; Round Hole Expansion; Column Hole Expansion; Spherical Pore Expansion.

1. Introduction

Inserting vertical reinforcement into natural foundation (such as mixing pile, static pressure pile sinking, etc.) is a typical engineering practice of soil reinforcement with a wide range of applications. However, this kind of method will cause great disturbance to the surrounding soil. When there are buildings, tunnels and other structures around, it will have an adverse impact on the nearby structures. Therefore, clarifying the theoretical calculation method of soil squeezing effect has very important research value and engineering application value for controlling the adverse impact on the surrounding environment caused by this kind of project.

Researchers at home and abroad have done a lot of research work on soil squeezing effect and related theories, and the most commonly used calculation method is still the finite element calculation method. However, this method has high requirements for the depth of theoretical knowledge and practical operation level of engineering technicians, resulting in great differences in the results calculated by the same finite element software even for different personnel of the same project. Considering that engineering technicians prefer analytical theoretical calculation methods in engineering applications, in order to facilitate the application of engineering technicians, this paper summarizes the theoretical calculation methods of soil squeezing effect by using the theoretical

review method, and defines the advantages, disadvantages and application scope of various methods. On this basis, the direction of further research is pointed out.

The construction of soil compaction pile can be abstracted as the expansion process of column hole on the side of pile and the expansion process of ball hole at the end of pile in layered semi infinite space. The column hole expansion theory and spherical hole expansion theory are based on the circular hole expansion theory, and they are first applied to the stress analysis of metal materials, and then introduced into the analysis of geotechnical problems. Therefore, this paper briefly summarizes the circular hole expansion theory, and introduces the cylindrical hole expansion theory and spherical hole expansion theory.

2. Cavity Expansion Method

2.1 Fundamental Theory

It is assumed that the soil as shown in Figure 1 is an ideal elastic-plastic material with uniform isotropy. The linear elastic relationship is satisfied before plastic yield, and the Mohr Coulomb yield criterion is satisfied after plastic yield. Ignoring the influence of physical force, the circular hole expands under the action of uniform pressure from p_0 to p_u .

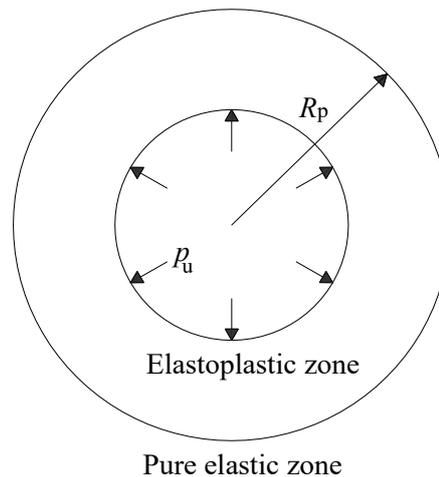


Figure 1. Circular hole expansion model

The circular hole expansion problem can be simplified as a plane strain axisymmetric problem, so only the displacement along the radius direction $u(r)$ can establish the stress balance equation [1].

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

Geometric equation:

$$\begin{cases} \varepsilon_r = \frac{du_r}{dr} \\ \varepsilon_\theta = \frac{u_r}{r} \end{cases} \quad (2)$$

The elastic-plastic constitutive equation satisfies the linear elastic relationship when the soil is in the elastic stage:

$$\begin{cases} \varepsilon_r = \frac{1-\mu^2}{E} \left(\sigma_r - \frac{\mu}{1-\mu} \sigma_\theta \right) \\ \varepsilon_\theta = \frac{1-\mu^2}{E} \left(\sigma_\theta - \frac{\mu}{1-\mu} \sigma_r \right) \end{cases} \quad (3)$$

When the soil enters the elastic-plastic deformation stage, the stress meets the Mohr Coulomb yield criterion, that is, it meets the:

$$\sigma_r - \sigma_\theta = (\sigma_r + \sigma_\theta) \sin \varphi + 2c \cos \varphi \quad (4)$$

Where, σ_r and σ_θ represent radial stress and tangential stress respectively; ε_r and ε_θ denote radial strain and tangential strain respectively; E and μ represent elastic modulus and Poisson's ratio respectively; c and φ represent cohesion and internal friction angle respectively.

The problem of circular hole expansion also satisfies certain boundary conditions. When the radius r is the initial radius of the circular hole R_0 :

$$\sigma_r|_{r=R_0} = p_0 \quad (5)$$

When the soil enters the plastic yield and the radius r is the final radius of the circular hole R_u , it meets the requirements:

$$\sigma_r|_{r=R_u} = p_u \quad (6)$$

And the stress state meets the yield condition:

$$\sigma_r|_{r=R_u} - \sigma_\theta|_{r=R_u} = (\sigma_r|_{r=R_u} + \sigma_\theta|_{r=R_u}) \sin \varphi + 2c \cos \varphi \quad (7)$$

At the boundary between pure elastic zone and elastic-plastic zone, the radius is R_p . When the radius r is R_p , the stress state meets the yield condition:

$$\sigma_r|_{r=R_p} - \sigma_\theta|_{r=R_p} = (\sigma_r|_{r=R_p} + \sigma_\theta|_{r=R_p}) \sin \varphi + 2c \cos \varphi \quad (8)$$

The expression of displacement $u(r)$ can be obtained by selecting the appropriate stress function and solving the equations shown in equation (1) to equation (8). In practical calculation, in order to

determine the unknown parameters in the stress function, it is usually necessary to introduce some additional constraints, such as assuming that the volume change of soil after expansion is equal to the sum of the volume changes of elastic zone and plastic zone.

2.2 Theoretical Deepening

Based on the basic circular hole expansion theory, the basic solution equations are established by using the non associated flow law [2], modified Cambridge model or improved modified Cambridge model [3, 4], and the basic equations are solved by analytical method or finite difference method [3], so as to obtain the displacement and stress expressions under various corresponding conditions. Although many scholars have improved the basic circular hole expansion from the constitutive model, from the basic equation to the solution, the basic governing equation is still formula (1) ~ formula (8), which has not changed essentially.

3. Column Hole Expansion Theory

3.1 Fundamental Theory

According to the circular hole expansion theory, the pile sinking and soil squeezing process is treated as an axisymmetric problem of plane strain. However, the pile sinking and soil squeezing process in practical engineering is a three-dimensional process, and the circular hole expansion theory can not well reflect the change process of soil displacement or stress around the pile with the vertical direction. Therefore, it is more reasonable to solve the problem of cylindrical hole expansion in theory. As shown in Figure 2, most of the soil within the pile length of the cylinder after ignoring the pile top and pile bottom, the cylinder expands under the action of trapezoidal pressure and lateral resistance.

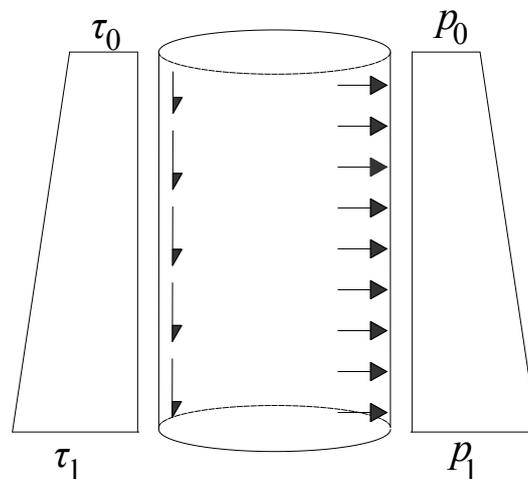


Figure 2. Column hole expansion model

The problem shown in Figure 2 can be regarded as a spatial axisymmetric problem, and its corresponding stress balance equation [5].

$$\begin{cases} \frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \\ \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = \gamma \end{cases} \quad (9)$$

The geometric equation is:

$$\left\{ \begin{array}{l} \varepsilon_r = \frac{\partial u_r}{\partial r} \\ \varepsilon_\theta = \frac{u_r}{r} \\ \varepsilon_z = \frac{\partial w}{\partial z} \\ \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial w}{\partial r} \end{array} \right. \quad (10)$$

According to the elastoplastic constitutive equation, when the soil is in the stage of complete elastic deformation, the stress-strain relationship satisfies:

$$\left\{ \begin{array}{l} \varepsilon_r = \frac{1}{E} [\sigma_r - \mu(\sigma_\theta + \sigma_z)] \\ \varepsilon_\theta = \frac{1}{E} [\sigma_\theta - \mu(\sigma_z + \sigma_r)] \\ \varepsilon_z = \frac{1}{E} [\sigma_z - \mu(\sigma_r + \sigma_\theta)] \end{array} \right. \quad (11)$$

When the soil enters plasticity, the stress meets the Mohr Coulomb yield criterion formula (4). Where, τ_{rz} is shear stress; ε_z and σ_z represent the strain and stress along the axial direction respectively; w and u_r represent displacements along the axis z and along the radius, respectively.

The boundary conditions corresponding to the cylindrical hole expansion problem are similar to those corresponding to the circular hole expansion problem. When the radius r is the initial radius of the circular hole R_0 , it is satisfied:

$$\left\{ \begin{array}{l} \sigma_r|_{r=R_0} = p_0 + \frac{p_l - p_0}{l} z \\ \tau_{rz}|_{r=R_0} = \tau_0 + \frac{\tau_l - \tau_0}{l} z \end{array} \right. \quad (12)$$

Where, p_l and p_0 respectively represent the internal pressure of the column hole when the depth z is equal to the pile length l and the depth z is equal to 0; τ_l and τ_0 respectively represent the friction force in the column hole when the depth z is equal to the pile length l and the depth z is equal to 0; The boundary between pure elastic zone and elastic-plastic zone meets $r = R_p$, meets the stress continuity condition and the Mohr Coulomb yield criterion of stress yield condition:

$$\sigma_r|_{r=R_p} - \sigma_\theta|_{r=R_p} = (\sigma_r|_{r=R_p} + \sigma_\theta|_{r=R_p}) \sin \varphi + 2c \cos \varphi \quad (13)$$

When plasticity is not considered, the spatial axisymmetric problem described in equations (9) to (11) can be solved by the initial parameter method even in the case of layered soil [6]. However, when considering plastic yield, the initial parameter method is no longer applicable. The stress function method can be used in combination with the boundary condition equations (12) to (13), so as to obtain the displacement u_r and w . Comparing the column hole expansion theory with the circular hole expansion theory, it can be seen that although the mechanism of the column hole expansion theory is more similar to the actual pile sinking and soil squeezing process, the column hole expansion theory considers the degree of freedom along the depth direction, which makes the solution of the column hole expansion theory more complex.

3.2 Theoretical Deepening

The traditional column hole expansion theory is based on the Mohr Coulomb yield condition. However, the modified Cambridge model is more reasonable for soft clay. Therefore, the modified Cambridge model [7,8,9] is introduced into the column hole expansion model, and the corresponding governing equations are established, and then solved by semi analytical method [10,11]. The Mohr Coulomb considering shear expansion is more in line with the actual deformation of sand. Therefore, Mantaras et al. [12] took the Mohr Coulomb criterion as the yield criterion and the Rowe shear expansion model as the plastic potential function, deduced the column hole expansion model and solved it. The traditional theory of cylindrical cavity expansion is mainly improved from two aspects. The first aspect is to use a more accurate yield function instead of Mohr Coulomb yield criterion, and the second aspect is to improve the solution method of the theoretical equation of cylindrical cavity expansion. At present, analytical method, semi analytical method or finite difference method are still mainly used. In any case, the basic theoretical framework corresponding to the column hole expansion theory is unchanged, including stress balance equation, geometric equation, stress-strain relationship and boundary conditions.

4. Spherical Cavity Expansion Theory

4.1 Fundamental Theory

When the pile penetrates into the soil, the soil around the pile expands, which can be regarded as cylindrical expansion; The expansion of soil at the pile end can be regarded as spherical expansion. For the spherical expansion problem, as shown in Figure 3, with the increase of the pressure p in the hole, the spherical region will gradually enter the elastic-plastic state from the fully elastic state.

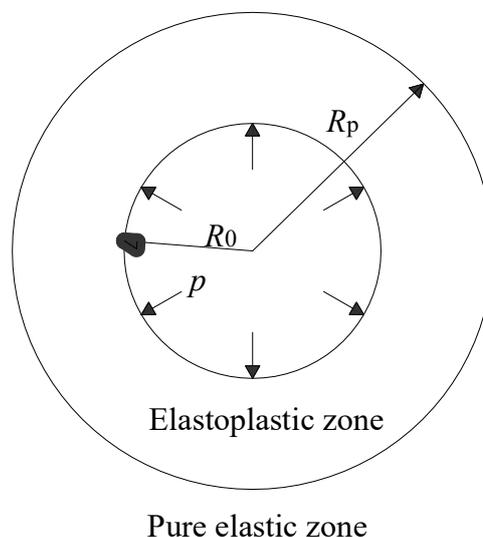


Figure 3. Section of spherical hole expansion model

It is assumed that the soil is a homogeneous isotropic ideal elastic-plastic material, which meets the Mohr Coulomb strength yield criterion. Then the spherical hole expansion belongs to the problem of spherical symmetry and the stress balance equation in polar coordinates:

$$\frac{d\sigma_r}{dr} + 2\frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (14)$$

Geometric equation:

$$\begin{cases} \varepsilon_r = \frac{du_r}{dr} \\ \varepsilon_\theta = \frac{u_r}{r} \end{cases} \quad (15)$$

According to the elastic-plastic constitutive equation, when the soil is in the stage of elastic deformation, the relationship between stress and strain is satisfied:

$$\begin{cases} \varepsilon_r = \frac{1}{E}(\sigma_r - 2\mu\sigma_\theta) \\ \varepsilon_\theta = \frac{1}{E}[(1-\mu)\sigma_\theta - \mu\sigma_r] \end{cases} \quad (16)$$

When the soil enters plasticity, the stress meets the Mohr Coulomb yield criterion formula (4).

The boundary condition is similar to the expansion boundary condition of circular hole. When the radius r is the initial radius of circular hole R_0 , it satisfies:

$$\sigma_r|_{r=R_0} = p \quad (17)$$

When the radius r is infinity, it is satisfied:

$$\sigma_r|_{r=\infty} = 0 \quad (18)$$

The boundary between the pure elastic zone and the elastic-plastic zone satisfies $r = R_p$, satisfies the stress continuity condition and the yield condition equation (13).

Assuming that the volume change of soil after expansion is equal to the sum of the volume changes of elastic zone and plastic zone, the expression of displacement can be obtained by selecting an appropriate stress function and solving the equations shown in equation (14) to equation (18).

4.2 Theoretical Deepening

The improvement of spherical cavity expansion model is also mainly carried out from two aspects: yield function and solution method. Modified Cambridge model and Mohr Coulomb model are

commonly used constitutive models in geotechnical engineering calculation. On the basis of basic spherical cavity expansion theory, using modified Cambridge model instead of Mohr Coulomb model can also establish corresponding governing equations [13], and then it can be solved by analytical method, semi analytical method and finite difference method [14].

5. Directions and Suggestions for Further Research

(1) Both circular hole expansion theory, column hole expansion theory and spherical hole expansion theory assume that the soil is homogeneous and isotropic material. However, in practical engineering, the soil is layered, and sometimes the mechanical properties of layered soil are quite different. How to equivalent layered soil to homogeneous isotropic soil in order to minimize the error caused by considering soil stratification is a direction worthy of research. In addition, how to establish the theory of column hole expansion and spherical hole expansion in layered soil considering soil yield is the key to clarify the soil compaction effect. It is suggested that relevant research work can be done in the future.

(2) The calculation models refined in the construction of reamed pile and steel casing cast-in-place pile are mostly to specify the hole diameter before and after reaming, and the displacement boundary conditions are known in the hole. However, the existing circular hole expansion theory, column hole expansion theory and spherical hole expansion theory are established according to the known force boundary conditions in the hole, and the corresponding solution method can adopt the stress function method. It is suggested that in the future, the circular hole expansion theory, column hole expansion theory and spherical hole expansion theory considering the displacement boundary in the hole can be established, and the corresponding equations can be solved by analytical method, semi analytical method or finite difference method, so as to solve the theoretical calculation problem of the disturbance of steel casing pressure on the surrounding stratum.

(3) Existing studies show that with the progress of soil disturbance, the soil properties will generally deteriorate, typically structural soft soil [15,16]. How to apply the constitutive model considering the deterioration of soil properties to the theory of column hole expansion and spherical hole expansion, so as to more truly reflect the disturbance effect of pressed pile on the surrounding stratum, which is also a direction worthy of research.

(4) In the process of steel casing down pressing, there is soil plug effect in the casing. Under different degrees of soil plug effect, the disturbance of steel casing down pressing to the surrounding stratum is obviously different. The establishment of column hole expansion theory considering soil compaction effect and soil plug effect is a direction worthy of study.

6. Conclusion

In order to clarify the common calculation methods of soil squeezing effect in geotechnical engineering, the relevant theories are divided into circular hole expansion theory, column hole expansion theory and spherical hole expansion theory. The results show that:

(1) The construction of soil compaction pile can be abstracted as the expansion process of column hole on the side of pile and the expansion process of ball hole at the end of pile in semi infinite space. Compared with the circular hole expansion theory, the column hole expansion theory is more in line with the engineering practice of soil compaction pile construction.

(2) The modified Cambridge model and Mohr Coulomb model commonly used in geotechnical engineering have been successfully applied to circular hole expansion theory, cylindrical hole expansion theory and spherical hole expansion theory. More advanced constitutive models need to be introduced into relevant expansion theory.

(3) The column hole expansion theory considering soil stratification, displacement boundary in hole, deterioration of soil mechanical properties and soil plug effect is a very valuable research direction.

Acknowledgements

The authors are grateful to funding received from the Project of China Construction 4th Engineering Bureau under grant numbers CSCEC4B-2021-KTB-27, SCEC4B-2021-KTB-31 and CSCEC4B-2022-KTA-10.

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