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Analysis of Chua's Chaotic Circuit based on Fractional Capacitance

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Abstract

The actual capacitance contains the characteristics of fractional calculus. Fractional capacitor has higher flexibility in circuit design and practical application. Combined with RC integer approximation and chaotic circuit theory, a fractional Chua's chaotic circuit based on Fractal CPE capacitor model is proposed in this paper. In this paper, Multisim software is used to build the circuit simulation of the system and analyze its chaotic characteristics.

Keywords

Fractional Capacitor; Chaotic Circuit; RC Integer Approximation.

1. Introduction

At the beginning of the 20th century, it was generally believed that the basic circuit elements were resistance, capacitance and inductance. In 1964, Carlson G [2] first expounded the concept of fractional capacitance and gave the realization of 1 / 3 order capacitance; Adhikary [5] introduced how to use RC ladder circuit to realize fractional order system; Oustaloup [3] uses the recursive distribution of zeros and poles to describe 0 Approaching form of 5th order circuit; Valsa [1] describes the RC model of a constant phase element, so that the required fractional capacitor can be realized with a small number of RC elements in the optional frequency band. Fractional order circuits have better performance than many integer order circuits. At present, a large number of articles focus on the practical application of fractional order circuits. In 1983, Professor L. O. Chua[4] proposed the first circuit to produce chaotic circuit phenomenon, which expanded people's understanding of the application of chaotic circuit. The fractional order theory can better describe the genetic characteristics, and the use of fractional order operator can better objectively reflect the actual characteristics of chaotic system circuits. Therefore, based on the integer order approximation of Valsa [1], a 0.5-capacitor consist of 6 resistors and 6 capacitors is established in this paper. The CPE capacitor model is applied to the standard Chua's chaotic circuit. Finally, the different characteristics of fractional Chua's chaotic circuit are analyzed by Multisim Simulation.

2. Integer Order Approximation of Fractional Capacitance

In this paper, using the integer order approximation method proposed by Valsa[1], a finite number of resistive and capacitive elements are used to achieve some selectable frequency bands 0.1~0. 9-order fractional capacitor circuit.Valsa[1] deduced a simple mathematical network model of constant phase element CPE.The impedance formula of the ideal constant phase element is:

$$Z(j\omega) = \psi\omega^{\alpha} j^{\alpha} = \Psi\omega^{\alpha}(\cos\varphi + j\sin\varphi) \tag{1}$$

$$\varphi=\frac{\pi}{2}\alpha or \varphi=90\alpha \tag{2}$$

The standard fractal CPE model refers to the parallel connection of RC branches with m-order parameter recursion, as shown in Figure 1.

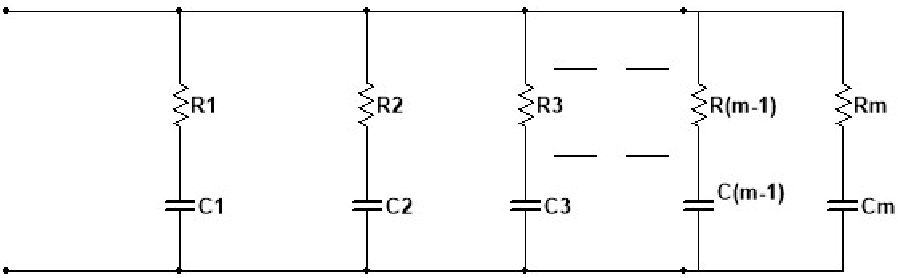

Figure 1. Fractal CPE model

Parameter recursive satisfaction:

$$R_{k+1} = aR_k, C_{k+1} = bC_k, k = 1,\ldots,m-1, 0 < a < 1, 0 < b < 1 \tag{3}$$

The choice of R_1 and C_1 can help us determine the starting point of the phase band we want to achieve:

$$\omega_{min} \approx \frac{1}{R_1C_1}, \omega_{max} \approx \frac{\omega_{min}}{(ab)^{m-1}} \tag{4}$$

The values of a and b can help us determine the fractional capacitance phase we want to achieve:

$$\varphi = 90\frac{log\ a}{log\ (ab)} \tag{5}$$

When m tends to infinity, the model can well simulate the constant phase fractional capacitor element. However, for practical reasons, the value of m should be as small as possible, which will make the response ineffective. In order to solve this problem, some additional components can be used to replace the missing branches to meet the preset conditions as much as possible.

Therefore, under low frequency conditions, selecting an equivalent resistance in parallel helps to improve the characteristics:

$$R_p = \frac{R_1}{a} + \frac{R_1}{a^2} + \frac{R_1}{a^3} + \ldots \frac{R_m}{a^m} = R_1\frac{1-a}{a} \tag{6}$$

At high frequency, selecting an equivalent capacitor in parallel helps to improve the characteristics:

$$C_p = \frac{C_1b^m}{1-b} \tag{7}$$

On this basis, a fractal CPE capacitor model with 6 resistors and 6 capacitors is established, which can approximately meet the requirements of fractional capacitor phase in the frequency band of 1K

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~ 1000K.If R_1 =10kΩ, C_1 =0.01μF,m=5,a=0.4,b=0.4,then we can achieve 0.5-order fractional capacitance circuit (as shown in Figure 2).

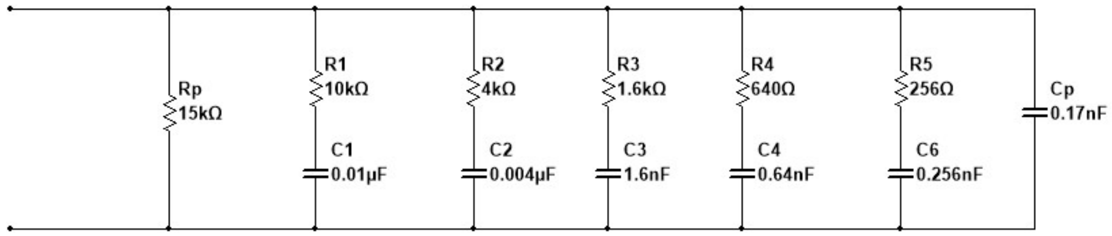

Figure 2. A 0.5-order fractional capacitance circuit

3. Modeling and Analysis of Fractional Chua's Chaotic Circuit

Professor L. O. Chua proposed the famous chaotic circuit[4] in 1983. It is the first circuit designed to produce chaotic phenomena, which is composed of two capacitors, one inductance, one resistance and one nonlinear resistance (as shown in Figure 3). From Kirchhoff's law, the state equation of the circuit can be deduced as:

$$C_1\left(\frac{dV_{C_1}}{dt}\right)=\frac{1}{R}\left(V_{C_2}-V_{C_1}\right)-g\left(V_{C_1}\right) \tag{8}$$

$$C_2\left(\frac{dV_{C_2}}{dt}\right)=\frac{1}{R}\left(V_{C_2}-V_{C_1}\right)+i_L \tag{9}$$

$$L\left(\frac{di_L}{dt}\right)=-V_{C_2} \tag{10}$$

i_L is the current flowing through L,V_{C_1}, V_{C_2} is the voltage of C_1, C_2, respectively,R is the resistance value of the resistance and $g\left(V_{C_1}\right)$ is the conductance of the nonlinear resistance, which can be written as:

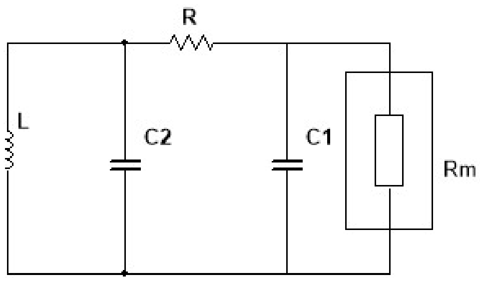

Figure 3. The chaotic circuit

$$g\left(V_{C_1}\right)=m_0\cdot V_{C_1}+\frac{1}{2}(m_1-m_0)\left[\left|V_{C_1}+E\right|+\left|V_{C_1}-E\right|\right] \tag{11}$$

In this paper, the fractional capacitance obtained by the integer order approximation method proposed by Valsa is used to replace the capacitance in Figure 3, and a fractional Chua's chaotic circuit (as shown in Figure 4) is built. Some phenomena different from the integer order are obtained by simulation with Multisim software.R=2080Ω,L=18mH,C_2=60nF,R_6=3.3kΩ, R_7=20kΩ, R_8=20kΩ, R_9=2.4kΩ, R_{10}=220Ω, R_{11}=220Ω.

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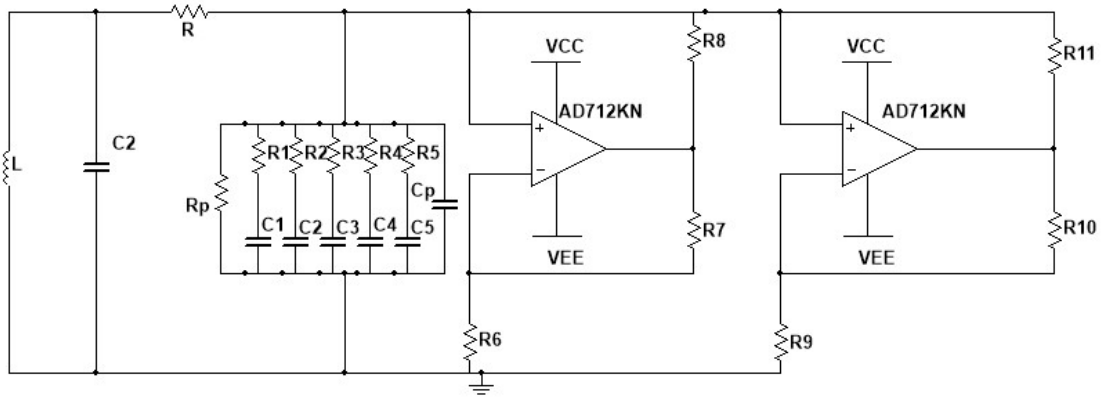

Figure 4. A fractional Chua's chaotic circuit

Chua's chaotic circuit is very sensitive to the change of resistance, so we constantly change the parameters R of fractional order Chua's chaotic system and integer order circuit system, and observe the changes of dynamic characteristics of the system.The following table summarizes the complex dynamic behavior of the two systems with parameters.

Table 1. Dynamic characteristics

R/Ω	α=0.5	α=1	R/Ω	α=0.5	α=1
2080	Double vortex chaotic attractor	Double vortex chaotic attractor	**2176**	Single vortex chaotic attractor	Double vortex chaotic attractor
2067	Stable focus	Double vortex chaotic attractor	**2336**	Single vortex chaotic attractor	Single vortex chaotic attractor
2058	Limit cycle	Stable focus	**2367**	Single vortex chaotic attractor	Limit cycle

4. Conclusion

Fractional capacitance is very important in the analysis and application of fractional circuits. Although the fractional capacitance obtained by integer order approximation is not accurate relative to chemical capacitance, it can meet the needs of research to a certain extent.Chua's chaotic circuit has strong sensitivity, and its specific experiment has certain technical difficulty. This is because chaotic motion is particularly sensitive to the error of electronic components. In this paper, a fractional Chua's chaotic circuit based on Fractal CPE capacitor model is established. Through Multisim software modeling and simulation, the different stability and dynamic properties of this system compared with normal Chua's chaotic circuit are studied. The dynamic behavior of the circuit still shows limit cycle, single vortex chaotic attractor and double vortex chaotic attractor, there will be greater changes in sensitivity.

References

[1] Valsa J , Vlach J . RC models of a constant phase element[J]. International Journal of Circuit Theory & Applications, 2013, 41(1):59-67.

[2] Carlson G, Halijak C.Approximation of Fractional Capacitors $(1/s)^{1/n}$ by a Regular Newton Process[J].IEEE Transactions on Circuit Theory,1964,11(2):210-213.

[3] A. Oustaloup, F. Levron, B. Mathieu, F. Nanot.Frequency-band complex noninteger differentiator: characterization and synthesis[J]. IEEE Transactions on Circuits and Systems I, 2000,47(1): 25-40.

[4] L. O. Chua and Sung Mo Kang, "Memristive devices and systems," in Proceedings of the IEEE, vol. 64, no. 2, pp. 209-223, Feb. 1976, doi: 10.1109/PROC.1976.10092.

[5] A. Adhikary, P. Sen, S. Sen, K. Biswas, Design and performance study of dynamic fractors in any of the four quadrants. Circuits Syst. Signal Process,2016,35(6): 1909-1932.