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Modification of Spur Gear Stiffness Calculation Model

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Abstract

The time-varying mesh stiffness is the basis for the analysis of gear dynamics and is an important internal excitation of the gear. The correct calculation of its stiffness determines the importance of the subsequent dynamic analysis of the gear. In previous gear stiffness calculations, the gear is usually divided into two parts: the gear body and the gear teeth, but some energy is calculated more than once at the place where the tooth root circle and the gear body meet, so that the accuracy of the gear stiffness calculation is reduced. In this paper, the calculation of gear stiffness is corrected. By removing the over-calculated part from the gear tooth part, the gear stiffness was then calculated and the results before and after the correction were compared. It is found that the corrected gear stiffness calculation results are slightly larger than the pre-correction results. The method proposed in this paper has further improved the accuracy of the gear stiffness calculation results.

Keywords

Mesh Stiffness; Energy Method; Gear Body.

1. Introduction

Gear drive is one of the most important transmission devices in mechanical system. Time-varying mesh stiffness (TVMS) is an important internal excitation of gear system, which can directly affect the dynamic characteristics of gear transmission system, such as vibration and noise. Therefore, the calculation of gear stiffness is particularly important and has become the focus of research in the field of gear system dynamics in recent years.

For the calculation of gear stiffness, many studies have been done by many scholars. Weber [1] used the mechanics of materials method to calculate the deformation of spur gear in mesh state, and comprehensively considered the bending deformation, shear deformation and foundation deformation of gear teeth under load. Cornell [2] took into account the influence of stress on spur gear based on Weber, and proposed an improved analysis of gear tooth matrix. Yang et al. [3, 4] successively derived the contact stiffness calculation formula according to the Hertz contact theory, and proposed three methods to calculate the gear tooth mesh stiffness considering the Hertz contact deformation, radial compression deformation and bending deformation of the loaded gear teeth. The Hertz contact stiffness formula is widely used in the calculation of gear mesh stiffness. Tian [5] considered the influence of shear deformation on the basis of Yang's model based on the energy method, and recorded the mesh stiffness of gear teeth as the sum of four kinds of stiffness. Based on Muskhelishvili theory, Sainsot et al. [6] regarded the gear body with axle hole as a deformable elastic ring, and proposed a two-dimensional analytical formula to calculate the deformation of the elastic ring of the gear body under load. What's more, Chen et al. [7] regarded the deformation of loaded gear teeth as the sum of five kinds of deformation, which further improved the accuracy of gear mesh stiffness calculation. Then the mesh stiffness of the gear pair is deduced and the fixed end of the cantilever is

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expanded from the traditional base circle to the root circle [8]. In addition, they proposed an improved analytical calculation model of spur gear pair mesh stiffness, and analyzed the influence mechanism of gear structure coupling less than and tooth shape deviation on gear mesh stiffness [9]. However, the above gear stiffness calculation has two calculations for the energy of the gear body part. In this paper, this part of energy calculation will be corrected.

2. Gear Model

2.1 Gear Body Stiffness Calculation

In previous calculations of spur gear stiffness, the gear is usually divided into two parts for calculation: the gear body and the gear tooth part. The gear body is often considered as a deformable elastic ring, and the gear body stiffness is then solved for by an empirical formula [6]. The gear tooth stiffness is calculated by considering the gear tooth as a cantilever beam with a variable cross-section at the fixed end of the tooth root circle and solving its stiffness by the potential energy method, which is shown in Fig. 1.

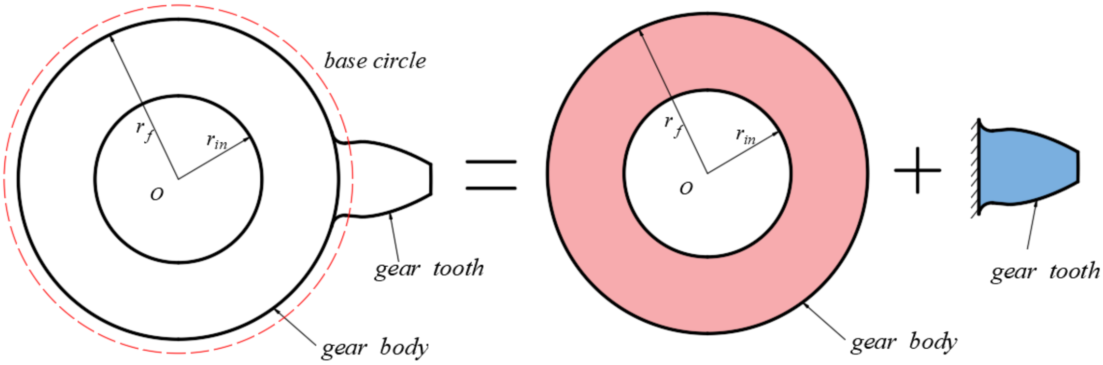

Fig. 1 Spur gear stiffness calculation diagram

However, in the calculation of the above gear stiffness, the gear body energy calculation has part of the energy calculation duplicated, and its schematic diagram is shown in Fig. 2.

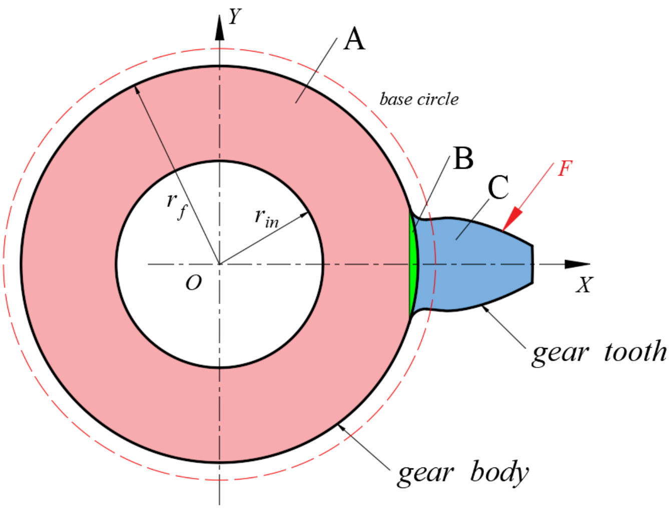

Fig. 2 Diagram of gear tooth stiffness calculation

From the above figure, the aforementioned calculation for the gear stiffness can be seen as:

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$$U_G = U_A + U_B + U_B + U_C \tag{1}$$

However, the gear stiffness should be calculated as:

$$U_G = U_A + U_B + U_C \tag{2}$$

That is, in the above calculation, the part of B in the figure is calculated more, so in order to improve the accuracy of gear stiffness calculation, it is necessary to make improvement here.

A schematic diagram of the spur gear stiffness calculation is shown in Fig. 2. The gear is divided into two parts: gear body and gear teeth. The former is obtained by an empirical formula, while the latter is treated as a variable section cantilever beam to obtain the stiffness. The formula for calculating the elastic deformation of the gear body is:

$$\delta_f = \frac{F}{Ew}\cos^2\alpha\left[L^*(h,\theta_f)\left(\frac{u}{S_f}\right)^2 + M^*(h,\theta_f)\frac{u}{S_f} + P^*(h,\theta_f)\left[1+Q^*(h,\theta_f)\tan^2\alpha\right]\right] \tag{3}$$

where u is the distance from the intersection of the line of force action and the centerline of the tooth to the root of the tooth; S_f is the length of the root arc of a single tooth; h is the ratio of the radius of the root circle to the radius of the shaft bore; and θ_f is the corresponding circular half-angle of the root arc. The specific calculation of other parameters can be found in the literature [6].

The gear body stiffness is calculated as:

$$k_f = \frac{F}{\delta_f} \tag{4}$$

2.2 Gear Tooth Stiffness Calculation

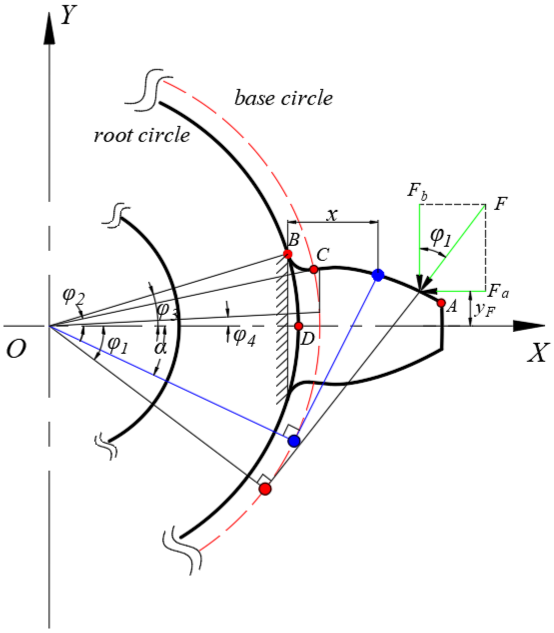

Fig. 3 Gear teeth force diagram

The potential energy method is used to calculate the gear tooth stiffness by considering the gear tooth as a cantilever beam constrained at the tooth root circle. The meshing force F is decomposed into forces Fa,Fb along the x,y directions as shown in Fig. 3.

The schematic diagram of the tooth part of the spur gear is shown in Fig. 3, point B is the intersection of the root circle and the transition curve, point C is the intersection of the transition curve and the involute, and point A is the intersection of the addendum circle and the involute. According to the Timoshenko beam theory, the tooth part takes the intersection point B of the root circle and the transition curve as a fixed boundary, and regards the tooth as a cantilever beam with variable cross-section to calculate its compressive energy U_a, shear energy U_s, and bending energy U_b. After cutting out the multi-computed part U_B, they can be expressed as:

$$U_a = \frac{F^2}{2k_a} = \int_{x_B}^{x_C} \frac{{F_a}^2}{2EA_{x1}} dx1 + \int_{x_C}^{x_F} \frac{{F_a}^2}{2EA_{x2}} dx2 - \int_{x_B}^{x_D} \frac{{F_a}^2}{2EA_{x3}} dx3 \tag{5}$$

$$U_s = \frac{F^2}{2k_s} = \int_{x_B}^{x_C} \frac{1.2{F_b}^2}{2GA_{x1}} dx1 + \int_{x_C}^{x_F} \frac{1.2{F_b}^2}{2GA_{x2}} dx2 - \int_{x_B}^{x_D} \frac{1.2{F_b}^2}{2GA_{x3}} dx3 \tag{6}$$

$$\begin{aligned} U_b = \frac{F^2}{2k_b} &= \int_{x_B}^{x_C} \frac{\left(F_b\left(x_F - x_1\right) - F_a y_F\right)^2}{2EI_{x1}} dx1 + \int_{x_C}^{x_F} \frac{\left(F_b\left(x_F - x_2\right) - F_a y_F\right)^2}{2EI_{x2}} dx2 \\ &- \int_{x_B}^{x_D} \frac{\left(F_b\left(x_F - x_3\right) - F_a y_F\right)^2}{2EI_{x3}} dx3 \end{aligned} \tag{7}$$

Where k_a, k_s and $k_b$are the bending, shear and axial compressive stiffness, respectively. $F_a = F \cdot sin\varphi_1$ is the horizontal component of the external load on the gear teeth, $F_b = F \cdot sin\varphi_1$ is the vertical component of the load on the gear teeth, E is the moment on the gear teeth, G is the elastic modulus, A_{x1}, A_{x2} and A_{x3} are the cross-sectional areas of the gear teeth at the transition curve, involute and at any x position in the region between point B and point D, respectively. I_{x1}, I_{x2} and I_{x3} are the moments of inertia of the gear teeth in the transition curve, involute and at any x position in the region between point B and point D, respectively.

$$I_{xi} = \begin{cases} \frac{2}{3}{y_1}^3 w, i = 1 \\ \frac{2}{3}{y_2}^3 w, i = 2 \\ \frac{2}{3}{y_3}^3 w, i = 3 \end{cases} \tag{8}$$

$$A_{xi} = \begin{cases} 2y_1 w, i = 1 \\ 2y_2 w, i = 2 \\ 2y_3 w, i = 3 \end{cases} \tag{9}$$

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Where $i = 1,2,3$ denote the root transition curve, the involute and the area of the region between point B and point D, respectively. y_1, y_2 and y_3 are the transition curve, the involute and the distance from the equation of the tooth root circle to the horizontal axis, respectively.

The references are to be numbered in the order in which they are cited in the text and are to be listed at To add the effect of teeth contact, the results derived by Yang and Sun [3] was taken into consideration to calculate the Hertzian contact stiffness as follows:

$$k_h = \frac{\pi EB}{4\left(1-\upsilon^2\right)} \tag{10}$$

where υ represent the Poisson's ratio.

According to the relationship between energy and stiffness, it can be deduced that the meshing stiffness of the gear pair when a single tooth meshes is:

$$\frac{1}{k_{ms}} = \frac{1}{k_{fi}} + \frac{1}{k_{ai}} + \frac{1}{k_{bi}} + \frac{1}{k_{si}} + \frac{1}{k_h} \tag{11}$$

where subscripts i=p,g represent the pinion and the gear, respectively, k_{ms} is the comprehensive meshing stiffness when single-tooth meshing.

Then, when two pairs of teeth are engaged in meshing at the same time, the total time-varying meshing stiffness of the gear can be expressed as:

$$\frac{1}{k_{ds}} = \frac{1}{k_{jtp}^i} + \frac{1}{k_{ap}^i} + \frac{1}{k_{bp}^i} + \frac{1}{k_{sp}^i} + \frac{1}{k_{ag}^i} + \frac{1}{k_{bg}^i} + \frac{1}{k_{sg}^i} + \frac{1}{k_{jtg}^i} + \frac{1}{k_h^i} \tag{12}$$

where superscript i represent i-th mesh tooth pair.

3. Stiffness Calculation and Comparison

The parameters of gear pairs used in this paper are shown in Table 1.

Table 1. Table of gear pair parameters

Parameter/property	Pinion	Gear
Number of teeth	30	25
Module	2	2
Pressure angle,°	20	20
Young's modulus(Pa)	207E9	207E9
Poisson's ratio	0.3	0.3
Teeth width(mm)	20	20

The above stiffness was calculated by matlab software, and the stiffness of single tooth before and after correction was calculated respectively. The comparison of the results can be seen in Fig. 4 and Fig. 5.

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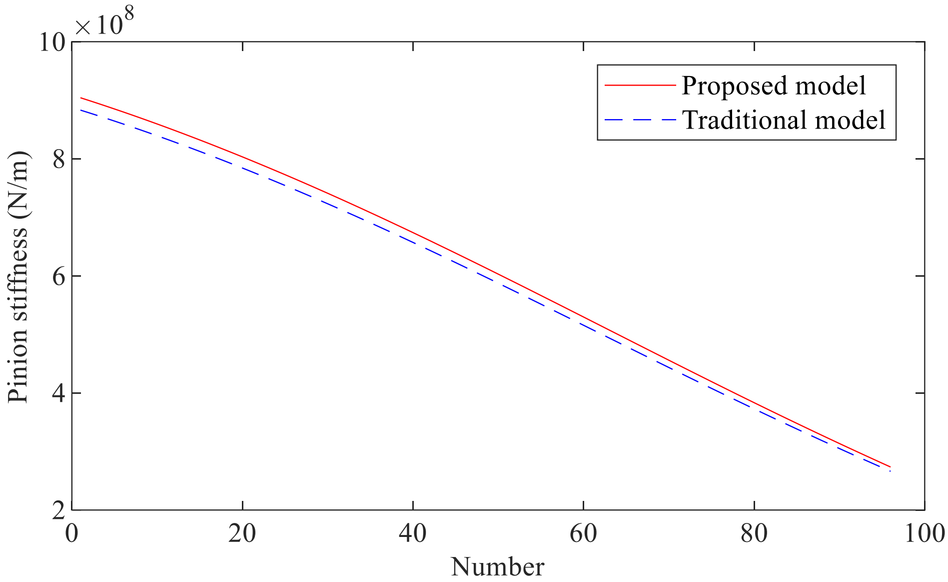

Fig. 4 Comparison of pinion stiffness

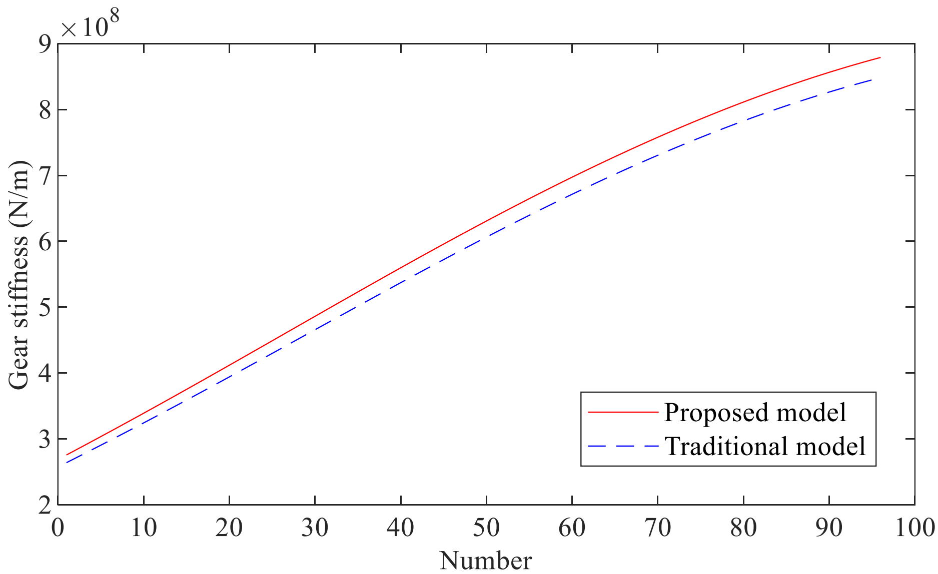

Fig. 5 Stiffness comparison of gear

As can be seen from Fig. 4 and Fig. 5, in the modified stiffness calculation, the stiffness of a single tooth is smaller than the previous stiffness calculation result, because part of the energy is less calculated, so the stiffness result will be slightly increased.

4. Conclusion

In this paper, some corrections are proposed in the gear stiffness calculation. In the previous gear stiffness calculation, an extra part of energy is calculated due to the empirical formula to solve the gear body stiffness and the potential energy method to solve the gear tooth stiffness. After the correction of the stiffness calculation formula proposed in this paper, the stiffness results are found to be slightly greater than the original calculation results. The correction method performed in this paper has improved the accuracy of gear stiffness calculation.

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