LÉVy Noise Induced Coherence Resonance in a Single Neuron Model

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Abstract

This paper mainly studies the coherence resonance(CR) in the single neuron model induced by Lévy noise.The characteristic function of Lévy noise has two more parameters than white Gaussian noise,which allows it describe much more complicated features.The effect of various parameters on the CR is exhibited by means of numerical simulation. The influences of various parameters on the SR will be exhibited by means of numerical simulation. The Rulkov model could exhibit different discharge mode such as quasi periodic discharge mode and cluster discharge mode under different parameters. In this paper, appropriate parameters are employed to prepare resting mode. Different parameters are adopted to compare the coherence resonance. Mean first passage time (MFPT)as one of the most commonly used measurement of coherence resonance is also adopted here.

Keywords

LÉVy Noise, Neuron Model, Coherence Resonance.

1. Introduction

The two-dimensional Rulkov model was introduced to study the regularization of array of chaotic oscillators[1]. For the past the decades, Rulkov model had received a lot of research interests owing to its simple form.Wang, QY et al. [2] studied the pattern formation and firing synchronization in two types of 2D square lattice map neural networks. Later on,X Sun et al. [3] investigate the effect of correlated white Gaussian noise on the spatiotemporal pattern formation of the studied system.

It is well-known that noise can play a constructive role in nonlinear dynamical systems. There are many works having been done about white Gaussian noise induced stochastic resonance and coherence resonance[4-8]. Coherence resonance is a phenomenon in which the signal of the excitable system enhanced by an appropriate amount of noise[20]. While a white Gaussian noise can only describe some disturb without jumps[9], the reality systems are always driven by various disturbance combined with discontinuously unpredictable jumps[10-13]. Since A Weron .et.al proposed α-stable Lévy noise[14], it has been widely applied to varies fields such as physics, natural science and social science[15-18]. Compared with Gaussian white noise, Lévy noise has two more parameters, charactterstic exponent a and skewness parameter b, which enable it to be used to characterize multiplied stochastic models with jump terms. Wu et al. introduced the effect of electromagnetic induction on the Hodgkin-Huxley model to detect the stochastic resonance in HH model[21]. However, no relevant research on the influence of Lévy noise on Rulkov model by now.

In this paper, we mainly study the coherence resonance phenomenon of the Rulkov model induced by Lévy noise. The bifurcation of Rulkov model is analyzed.The fact that Rulkov model would show quasi periodic discharge mode and cluster discharge mode under different parameters is also obtained. By changing the parameters of Lévy noise,we calculated the MFPT of the evolution from resting state

 Volume 8 Issue 1, 2022
DOI: 10.6919/ICJE.202201_8(1).0001

to excited state and discuss the coherence resonance induced by Lévy noise by means of numerical simulation. By changing the parameters of Lévy noise, the MFPT of the evolution process from resting state to excited state is calculated.The coherence resonance induced by Lévy noise is discussed by means of numerical simulation. The effects of the characteristic exponent and the skewness parameter are analyzed. In addition, we find the system has a different scale coefficient with the skewness parameter between the parameter is greater thane zero and less than zero.As for the characteristic exponent a, the result exhibits that within a certain range, the larger a is, more difficult it is for coherence resonance to occur.

2. Single Rulkov Model:

Consider the following noise free Rulkov model:

$$\begin{cases} x_{n+1} = \dfrac{\alpha}{1+x_n^2} + y_n = F(x_n, y_n) \\ y_{n+1} = y_n - \eta(x_n - \beta) = G(x_n, y_n) \end{cases}$$

With n is the discrete time scale,α,η=0.001, β are the system parameters, x_n represents the fast variable membrane voltage, y_n denotes the recovery(slow) variable.

It can be easily seen from the equation that the model has a unique fixed point $A(x^*, y^*)$:

$$\begin{cases} x^* = \beta \\ y^* = \beta - \dfrac{\alpha}{1+\beta^2} \end{cases}$$

And the Jacobian matrix of the equation at fixed point A is as follows:

$$J = \begin{pmatrix} \dfrac{\partial F}{\partial x} & \dfrac{\partial F}{\partial y} \\ \dfrac{\partial G}{\partial x} & \dfrac{\partial G}{\partial y} \end{pmatrix}_{|A(x^*, y^*)} = \begin{pmatrix} -\dfrac{2\alpha\beta}{(1+\beta^2)^2} & 1 \\ -\eta & 1 \end{pmatrix}$$

By solving the characteristic equation of the Jacobian matrix we can obtained:

$$\lambda^2 + [\frac{2\alpha\beta}{(1+\beta^2)^2} - 1]\lambda + \eta - \frac{2\alpha\beta}{(1+\beta^2)^2} = 0$$

$$\Delta = \left[\frac{2\alpha\beta}{(1+\beta^2)^2}\right]^2 - 4\left[\eta - \frac{2\alpha\beta}{(1+\beta^2)^2}\right]$$

When $\Delta > 0$,the system can present complex discharge modes, such as cluster discharge, peak discharge and quasi periodic discharge and so on.While $\Delta = 0$, system will be at rest mode or under

ISSN: 2414-1895 DOI: 10.6919/ICJE.202201_8(1).0001

limit cycle. In this article we adopt $\alpha = 1.8375$ and $\beta = -1$ so that the system would be at rest mode without any disturbance.

3. LÉVy Noise and MFPT

In the past decades, Gaussian white noise has been widely favored by scholars for its classicality and stablity. However, the noise in reality surroundings is not as stable as we expected, and the proposal of the Lévy noise fills the gap in this aspect. In this section, the probability distribution function of the Lévy noise and MFPT as a measurement of coherence resonance phenomenon are introduced.

3.1 LÉVy Noise

Compare with Gaussian white noise, the Lévynoise has more complex mathematical form, which allows it cover varied circumstances. By selecting appropriate parameters, the distribution function of noise can be degenerated to some classical distributions. For example, $a = 2.0, b = 0.0$ and $a = 1.0, b = 0.0$ are Gaussian distribution and Cauchy distribution respectively.

$$\phi(k) = \begin{cases} \exp[-D^a |k|^a (1 - ib\,\mathrm{sgn}(k)\tan\frac{\pi a}{2})], & a \neq 1 \\ \exp[-D|k|(1 + ib\,\mathrm{sgn}(k)\frac{2}{\pi}\ln|k|)], & a = 1 \end{cases}$$

The characteristic exponent a and skewness parameter b describe the stability and asymmetry of Lévy noise. Here, we consider the McCulloch method:

$$\xi_n(a,b,D) = L(n\Delta t) - L[(n-1)\Delta t] = D\Delta t^{1/a}\xi_n(a,b,1),$$

And we simply $\xi_n(a,b,1)$ as ξ_n:

$$\xi_n = \begin{cases} M\dfrac{\sin(aU+N)}{[\cos(U)]^{\frac{1}{a}}}\left[\dfrac{\cos[(1-a)U-N]}{W}\right]^{\frac{1-a}{a}}, & a \neq 1 \\ \dfrac{2}{\pi}\left[\left(\dfrac{2}{\pi}+bU\right)\tan(U) - b\ln\left(\dfrac{\pi W\cos(U)/2}{\frac{2}{\pi}+bU}\right)\right], & a = 1 \end{cases}$$

$$M_{a,b} = \left\{\cos\left[\arctan\left(b\tan\frac{a\pi}{2}\right)\right]\right\}^{\frac{1}{a}}$$

$$N_{a,b} = \arctan\left(b\tan\frac{a\pi}{2}\right)$$

Here U is a random variable with uniform distribution on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.W is a random variable with standard exponential distribution.U and W are independent to each other.

3.2 Mean First-Passege Time(MFPT)

The exact expression of the MFPT for a system to reach the final state x_2 ,from the initial state x_1, is given by [7,19].

$$T(x_1 \to x_2) = \int_{x_1}^{x_2} \frac{dx}{B(x)P_{st}(x)} \int_{-\infty}^{x} P_{st}(y)dy$$

Where the $B(x)$ and $P_{st}(x)$ is obtained by Fokker-Plank equation.In particular, we choose x_1=0, and x_2 as the value of the peak so that the time of the first pulse is represented by $T(x_1 \to x_2)$.

4. Numerical Simulation Results

In this paper, we consider the Rulkov model driven by periodic applied current and Lévy noise as follows:

$$\begin{cases} x_{n+1} = \dfrac{\alpha}{1+x_n^2} + y_n + D\Delta t^{1/a}\xi_n \\ y_{n+1} = y_n - \eta(x_n - \beta) \end{cases}$$

Where $D\Delta t^{1/a}$ represent the Lévy noise signal, D is the density of the Lévy noise, Δt is the time step. Here, we adopt the α =1.8375, η =0.001, β =-1 .Under such parameters, the model will reach a stable resting state after oscillation.

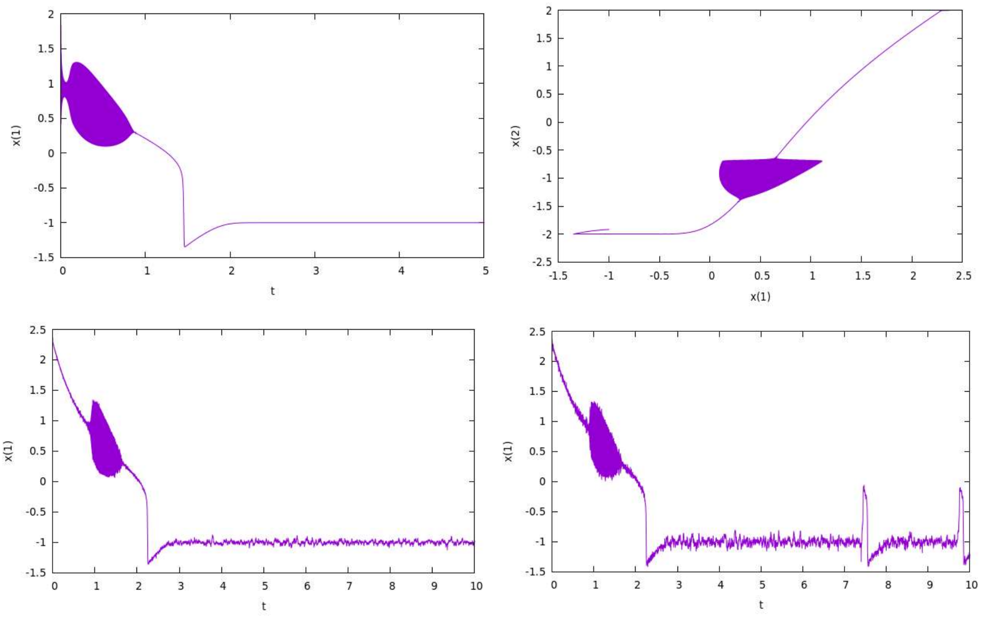

Figure 1. (a) $x_n - t$ curve of Rulkov model.(b) the phase diagram of noise-free Rulkov model. $x_n - t$ curve under $D = 0.005$ (c) and $D = 0.010$ with $a = 1.5$ and $b = 0.0$.

In figure 1(a) and 1(b), we take the initial conditions, $t_0 = 0$, $x_0 = y_0 = 2$ and the time step $\Delta t = 0.001$ in numerical calculations, the evolution process and phase gram of the system without Lévy noise are plotted. Then we gradually increase the intensity of Lévy noise(figure1(c)and figure1(d)). As the noise increase, the result exhibits that the system will appear excite signal.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202201_8(1).0001

It is obvious that the Lévy noise induced coherence resonance in our Rulkov model. We will calculate the MFPT function of system under different characteristic exponent a and skewness parameter b in the following subsections.

4.1 Characteristic Exponent A

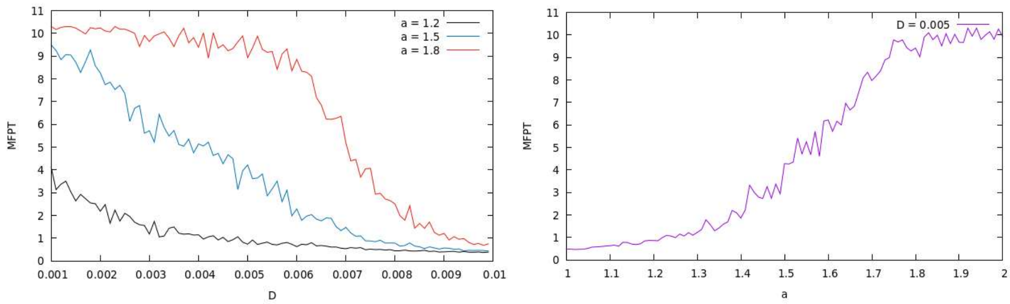

Figure 2. (a)MFPT-D curves under different characteristic exponent a with skewness parameter $b = 0.0$.(b)MFPT-a curve with $b = 0.0$ and $D = 0.005$.

In figure 2(a),the MFPT of Rulkov model is presented under different characteristic exponent a with the skewness parameter given as $b = 0.0$ .The MFPT exhibits a downward trend as the increase of noise intensity which implies that the Rulkov model under the Lévy noise formed coherence resonance phenomenon. In figure 2(b),we plot the MFPT as a function of characteristic exponent under the certain noise intensity $D = 0.005$ .Due to the Lévy noise contains huge jumps, the curve shows the characteristics of vibration. Anyhow, we can still recognize the positive correlation between characteristic exponent and MFPT, which means the larger a is,the more difficult to observe coherence resonance. The figure shows when $a < 1$, the MFPT of system is nearly equals to zero, which means the first spike would occur as soon as the noise signal applied. With the increase of a, the MFPT would show a significant raise and finally arrive at the end of simulation.

4.2 Skewness Parameter B

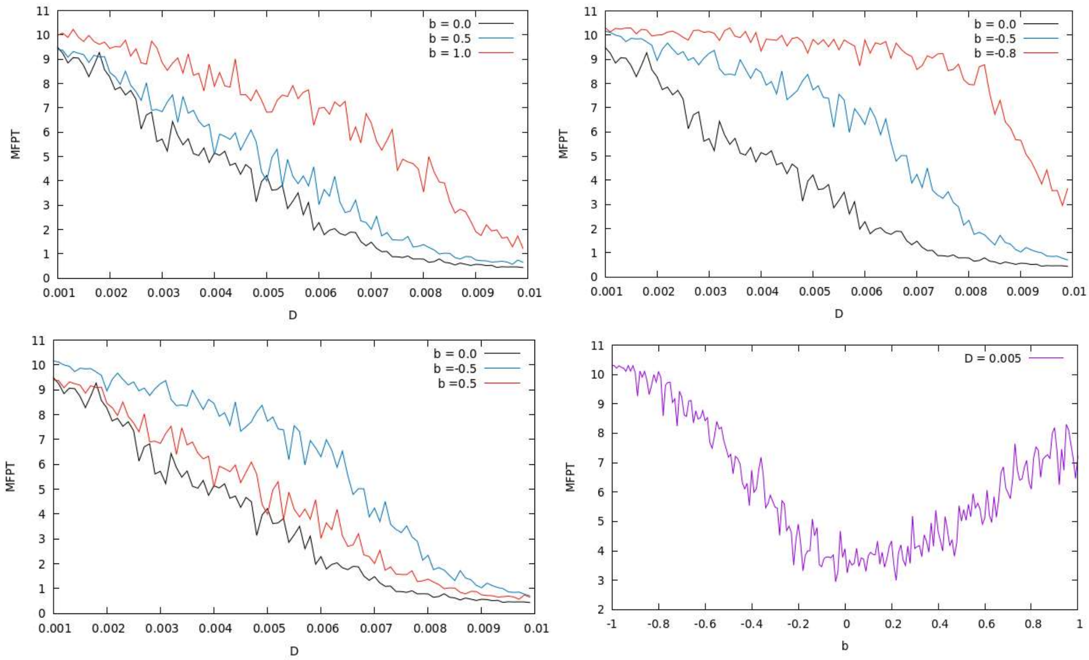

Figure 3. (a)-(c)MFPT-Dcurves under different skewness parameter b with characteristic exponent $a = 1.5$.(d)MFPT-b curve with $a = 1.5$ and $D = 0.005$

The effect of the skewness parameter b on the coherence resonance phenomenon of Rulkov model is considered in figure(3). We compared different values of skewness parameter under the certain characteristic parameter $a = 1.5$. The figure 3(a)shows the comparison when $b > 0$,while the figure 3(b) shows the comparison when $b < 0$. In figure 3(c), we compare the case where b has the same absolute value, because most literature think that it is the absolute value of b rather than the absolute value of b counts. However, the result of our numerical simulation exhibits a different outcoming. From the figure 3(d), we can easily figure that the tendency of $b < 0$ is greater than that of $b > 0$. We speculate that this is due to the fact that the resting membrane voltage of Rulkov model is less than zero, which implies that the coherence resonance phenomenon is more obvious when the skewness parameter of the Lévy noise is coupled with the parameters of the system.

5. Conclusion

The Rulkov model has attracted a lot of interests because of its complex discharge properties. At the same time, the Lévy noise is also widely concerned because of its unique characteristics. However, there is no relevant research on the phenomenon when applying the Lévy noise on the Rulkov model.

In this paper, the Jacobian matrix of the Rulkov model is analyzed, and the appropriate parameters are selected to make the system without noise at rest mode. Then a moderate noise is applied to make coherent resonance occur. Through the results of numerical simulation, we find that the MFPT of system positively correlated with characteristic exponent a,which implies that the smaller a is, the easier coherence resonance occurs.In addition, we also researched the effect of skewness parameter b by changing the value. Interestingly, we find that, unlike other literature, although the MFPT and the skewness parameter of the system are positively correlated, the proportional coefficient is not the same in the case of $b > 0$ and $b < 0$. Generally speaking, when $b > 0$, it is easier to observe coherence resonance.

In future work, we have some further ideas. Maybe we can try to apply a periodic signal on the single Rulkov model and study the dynamic mechanism of Lévy noise induced stochastic resonance in Rulkov model. Besides, the coherence resonance induced by Lévy noise on a neural network of Rulkov model is also attractive.

6. Conflict of Interests

The authors declares that they have no conflict of interest.

7. Ethical approval

All procedures preformed in studies involving human participants were in accordance with the ethical standards of the institutional and /or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article do not contain studies with human participants or animals by any of the authors.

Acknowledge

The author is grateful to multiple reviewers for their time and effort spent in reviewing the relation area.

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