ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

TDE-based Model Reference Adaptive Second Order Sliding Mode Control for Overhead Crane

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Abstract

This paper proposes a model reference adaptive sliding mode control (MRASMC) scheme that uses time delay estimation (TDE) technique and adaptive modified super-twisting algorithm (AMSTA), then applies the scheme to overhead crane with uncertainties to ensure the payload can be transported to the desired position rapidly while simultaneously eliminating swing. Firstly, a desired reference model for improving transient performance was designed. More specifically, a new chattering-free reaching law is designed to achieve fast convergence of system state errors and improve system transient performance. To enhance the control performance, then, a time-delay estimation is employed to approximate the uncertainties and control law is designed based on modified adaptive super-twisting algorithm, which can reduce the chattering phenomenon and achieve finite-time convergence. The stability and convergence of the controlled system is analyzed using Lyapunov theory. Moreover, Simulation results are presented to illustrate the effectiveness of the proposed controller.

Keywords

Overhead Crane; Sliding Mode Control; Model Reference Adaptive; Time-delay Estimation; Modified Super-twisting Algorithm.

1. Introduction

In the past few decades, overhead cranes have been widely used in factories, ports, workshops and various industrial activities due to their strong load transport capacity. However, undesirable payload swings will inevitably occur due to the high-speed operation of overhead cranes in the complex industrial environment. At present, the effective payload swing suppression mainly depends on experienced manual operation, which not only leads to low transportation efficiency, but also causes safety accidents due to manual operation errors. Therefore, it has important theoretical and practical significance to study effective control strategies to achieve rapid positioning of the trolley and eliminate payload swing.

In the early stage, research on swing elimination was mainly open-loop control, which includes input shaping [1] and off-line trajectory planning [2]. The main idea of input shaping is to use the frequency characteristics and damping ratio of the system to plan the control signal reasonably, among which zero vibration input shaping is the most common type [3]. However, the control performance of this method depends heavily on the accuracy of model. The offline trajectory refers to planning the desired trajectory for trolley by analyzing the dynamic coupling relationship between the trolley and the payload. In the work of [4], a series of trolley reference trajectories were planned with the aid of nonlinear coupling analysis and the phase plane method to achieve good control performance. Due to the complexity of the work environment, combining with a feedback controller is usually needed to improve the robustness of the control system.

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In order to improve the robustness of the overhead crane system, the closed-loop control strategy receives much attention. The most representative closed-loop control methods mainly include energy control [5], partial feedback linearization [6]. Based on passivity or system energy method, which analyzing energy change rules of underactuated cranes without approximate linearization near the equilibrium point. For this reason, the strategy based on energy analysis naturally becomes the powerful control weapon for controlling underactuated cranes. Partial feedback linearization refers to the realization of system transformation by defining new control variables, and finally achieving local linearization. Regardless of open-loop control or closed-loop control, their control performance relies heavily on the accuracy of the model and generally requires complex matrix computation.

In particular, sliding mode control (SMC) has receive much attention, due to its strong robustness against both external disturbances and model parameter variations and simple structure, which suppress the matched disturbances or uncertainty from the input channel. One problem with SMC is that overhead crane system parameters are required when calculating its equivalent control law. The combination of SMC and Model Reference Adaptive Control (MRAC) is a very effective way to deal with the uncertainties of overhead cranes without prior knowledge of system parameters.[7] Another drawback with SMC mainly lie in chattering phenomenon. This is because the upper bound of the disturbance is generally unavailable, so switching gains are chosen large enough to cover a wide range of uncertainties. To restrain the chattering effect, replacing discontinuous sign function by saturation function [8] can avoid chattering. Although this produces a continuous control and eliminates chattering, it loses the robustness of the entire system against uncertainties. In addition, some scholars use boundary layer technology [9] and high-order control [10] to eliminate chattering. However, these approaches still have the limitation that they require information about the upper bound on the uncertain terms.

The key to solve this problem is to estimate the uncertainty and disturbance in the system. A variety of intelligent control methods have been proposed to find suitable estimation strategies for nonlinear systems, such as adaptive fuzzy control [11], adaptive control [12], neural network [13]. These intelligent algorithms are very effective theoretically, nonetheless, may make the implementation difficult or impossible due to computational complexity. In recent years, it is a very promising idea to combine sliding mode control with time delay estimation (TDE) method [14,15], providing a good estimation of uncertainty without requiring prior knowledge of uncertainty upper bound and the choice of switching gain can be reduced. However, the chattering problem has not been completely solved, which affecting the convergence time. Also, due to measurement noise and nonlinear characteristic in real experiment, TDE cannot accurately estimate the uncertainty and produce estimation errors inevitably in the system dynamics. Fortunately, the estimation error is bounded.To solve this problem, the high-order sliding mode control (HOSMC) technique [16] can reduce or eliminate chattering while simultaneously maintaining the robustness of the system. In order to improve the HOSMC algorithm, many algorithms have been presented that allows finite-time convergence and chatting suppression, such as twisting algorithm(TA),super-twisting algorithm(STA) and others [17]. Among them, the STA is the most effective algorithm because it is the only one that needs only sliding manifold measurements while the others require the measurements of its derivation [18], and thus becomes the most powerful algorithm based on STA and adaptive super-twisting algorithm(ASTA) control strategies.

Motivated by the work above, a model reference adaptive sliding mode control scheme based on TDE and AMSTA is designed, which solves the problem of locating the payload to the desired position while simultaneously eliminate the payload swing under uncertainty. Firstly, dynamic sliding mode control is used to design a reference model combined with a new chattering-free reaching law to improve transient performance of MRC.On this basis, the TDE provides an estimation of uncertainties and the AMSTA is used to eliminate the so-called TDE error, chattering phenomena and to ensure finite time convergence. Finally, the convergence and stability of the closed-loop system are strictly proven by Lyapunov theory. The simulation results verify the superiority of the controller. The main contributions of this work are summarized as follows:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

(1) To the authors' best knowledge, this work proposed for the first-time overhead crane control based on TDE and AMSTA.

(2) A new chattering-free reaching law combined with dynamic sliding mode control to improve the transient performance of the MRC.

(3) The method proposed in this paper requires no prior uncertainty upper bound, and the chattering phenomenon is well eliminated.

2. System model and problem statement

2.1 System model

According to Lagrangian Euler's law, 2D overhead crane model (as shown in fig.1) can be described as follows:

$$(M+m)\ddot{x} - m\sin\theta\,\ddot{l} - ml\cos\theta\,\ddot{\theta} - 2m\cos\theta\,\dot{l}\dot{\theta} + ml\sin\theta\,\dot{\theta}^2 = F_x \quad (1)$$

$$-m\sin\theta\,\ddot{x} + m\ddot{l} - ml\dot{\theta}^2 - mg\sin\theta = F_l \quad (2)$$

$$-ml\cos\theta\,\ddot{x} + ml^2\ddot{\theta} + 2ml\dot{l}\dot{\theta} + mgl\sin\theta = 0 \quad (3)$$

In the (1)(2)(3), $M, m, g \in R^+$ denote the trolley mass, payload mass, and gravitational constant respectively. $x(t), l(t), \theta(t) \in R$ represents the trolly position, rope length, the payload swing angle respectively. The force of the driving motors of trolly traveling and cargo lifting, which are characterized by $F_x(t), F_l(t)$ respectively.

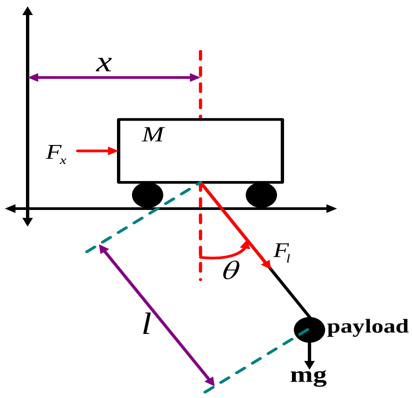

Fig. 1 Model of 2D overhead crane system

An overhead shown in Fig.1 is a nonlinear and highly coupled underactuated mechanical system, in which three output signals (x, l, θ) are driven by two actuators. The control input signal F_x, F_l directly control the actuated states (x, l) and indirectly drive the under-actuated state (θ). Based on the above reasons above, the overhead crane model (1)-(3) can be disassembled into the underactuated subsystem and the actuated subsystem after corresponding mathematical progressing. The underactuated dynamics can be determined by rewriting Eq. (3) as following:

$$\ddot{\theta} = \frac{-g\sin\theta}{l} - \frac{2\dot{l}\dot{\theta}}{l} + \frac{\ddot{x}\cos\theta}{l} \quad (4)$$

Substituting Eq. (4) into Eq. (1) together with Eq. (2), the actuated subsystem can be written in a state space from as follows:

$$\begin{bmatrix} \ddot{x} \\ \ddot{l} \end{bmatrix} = \begin{bmatrix} 0 \\ g\cos\theta + l\dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{1}{M} & \frac{\sin\theta}{M} \\ \frac{\sin\theta}{M} & \frac{M+m\sin^2\theta}{Mm} \end{bmatrix} \begin{bmatrix} F_x \\ F_l \end{bmatrix} \quad (5)$$

For simplicity and convenience, Eq. (5) can be rewritten as follows:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

$$\begin{cases} \dot{z_1} = z_2 \\ \dot{z_2} = f(z) + g(z)u \\ y = z_1 \end{cases} \tag{6}$$

where $z_1 = [z_{11} \quad z_{12}]^T = [x \quad l]^T$, $z_2 = [z_{21} \quad z_{22}]^T = [\dot{x} \quad \dot{l}]^T$ represents system states. $f(z), g(z)$ are known as nonlinear smooth function. $u = [F_x \quad F_l]^T$ denotes control input signal.

$$f(z) = \begin{bmatrix} 0 & g\cos\theta + l\dot{\theta} \end{bmatrix}^T, \quad g(z) = \begin{bmatrix} \frac{1}{M} & \frac{\sin\theta}{M} \\ \frac{\sin\theta}{M} & \frac{M + m\sin^2\theta}{Mm} \end{bmatrix}$$

2.2 Problem satatement

The control objective of this paper is to drive the output of the actual bridge crane to track the output of the reference model accurately, at the same time, to eliminate the swing angle of the payload in thepresence of a series of unknown friction and uncertain disturbances. This control objective can bemathematically quantified as follows:

$$z_1 \to z_d, \theta \to 0$$

Where z_d represents desired position.

3. Controller design

3.1 The reference model of overhead crane

During the process of MRAC controller design for overhead crane, the reference model of the closed-loop system can be designed as follows:

$$\ddot{z}_m = f_m(z_m) + g_m u_m \tag{7}$$

where $z_m = [x_m \quad l_m]^T, z_m \in R^{2\times1}$ represents the system state of overhead crane reference. $u_m = [F_{xm} \quad F_{lm}]^T, u_m \in R^{2\times1}$ represents the reference input signal. $f_m \in R^{2\times1}, g_m \in R^{2\times1}$ denotes the state matrix and output matrix, respectively.

In order to improve the transient performance of MRAC, a novel hyperbolic tangent reaching law is given by:

$$\dot{s} = -k_1 \tanh(as) - k_2 \sinh(b|s|^q) sign(s) \tag{8}$$

Where s is sliding surface, k_1, k_2, a, b denotes positive tuning parameter, q is positive odd number.

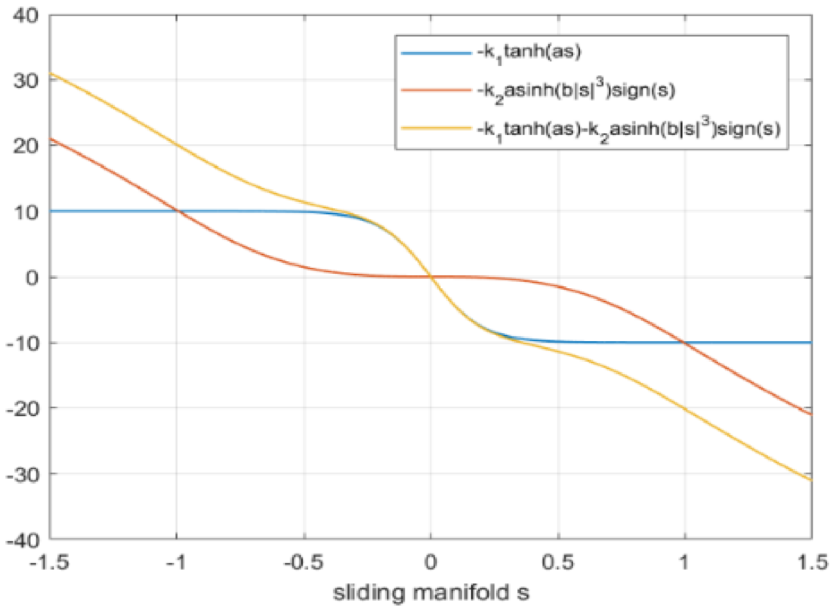

Fig. 2 The characteristics of the reaching law

ISSN: 2414-1895
DOI: 10.6919/ICJE.202108_7(8).0012

As depicted in Fig.2, it can be seen that the changing rate$\dot{s}$is mainly controlled by the second term $-k_2\, sinh(\, b|s|^q)sign(s)$ when sliding mode variable s is far away from zero. When s approach to zero, the changing rate$\dot{s}$depends on the first term $-k_1\, tanh(\, as)$, such that $\dot{s}$can infinitely close to zero due to pseudo linearization of $-k_1\, tanh(\, as)$ near the zero.

The tracking error between the overhead crane system (6) and the reference model (7) is defined as follows:

$$e = z_1 - z_d \tag{9}$$

where $e = [e_x \quad e_l]^T$,e_x,e_l represents the displacement error and rope error between the overhead system and the reference model respectively. A sliding mode surface including swing angle θ and the trolley displacement error can be chosen as follows:

$$s(t) = \hat{c}_1 e(t) + \hat{c}_2 \dot{e}(t) + \hat{c}_3 \int_0^t e(\xi)\, d\xi + \hat{c}_4 \theta(t) \tag{10}$$

where

$$\hat{c}_1 = diag\left(\hat{c}_{11},\ \hat{c}_{12}\right), \hat{c}_{1i} > 0, i = 1,2$$

$$\hat{c}_2 = diag\left(\hat{c}_{21},\ \hat{c}_{22}\right), \hat{c}_{2i} > 0, i = 1,2$$

$$\hat{c}_3 = diag\left(\hat{c}_{31},\ \hat{c}_{32}\right), \hat{c}_{3i} > 0, i = 1,2$$

$$\hat{c}_4 = \left[\hat{c}_{41} \quad 0\right]^T, \hat{c}_{4i} > 0, s = [s_x \quad s_l]^T, s \in R^{2\times1}$$

s_x,s_l denote the sliding mode surface of the trolley position with swing angle and rope length respectively

Differentiating Eq. (10) with respect to time and substituting Eq. (6)(10) into it, we have:

$$\dot{s} = \hat{c}_1 \dot{e} + \hat{c}_2 (f(z) + g(z)u - \ddot{z}_d) + \hat{c}_3 e + \hat{c}_4 \dot{\theta} \tag{11}$$

To keep the error variables on the sliding surface, the equivalent control $u_{eq}(t)$ can be calculated from $\dot{s} = 0$ as

$$u_{eq}(t) = (\hat{c}_2 g(z))^{-1}[-\hat{c}_1 \dot{e} - \hat{c}_2 f(z) + \hat{c}_2 \ddot{z}_d - \hat{c}_3 e - \hat{c}_3 \dot{\theta}] \tag{12}$$

Combining Eq. (12)(13), we can obtain the reference input of overhead reference model Eq. (6) as follows:

$$u_m(t) = u_{eq}(t) + u_{sw}(t) \tag{13}$$

Where $u_{sw}(t)$ is chosen as Eq. (22)

Substituting Eq. (13) into Eq. (7), the reference model acceleration can be derided as follows:

$$\begin{aligned} u_m(t) = \ddot{z}_d - (\hat{c}_2)^{-1}[\hat{c}_1 \dot{e} + \hat{c}_3 e + \hat{c}_4 \dot{\theta} + k_1 \tanh(as) + k_2 \sinh(b\,|\,s\,|^q) sign(s) \\ + s\left(\hat{c}_1 (e)^2 + \hat{c}_2 (\dot{e})^2 + \hat{c}_3 (\int e(\xi) d\xi)^2 + \hat{c}_4 (\theta)^2\right)] \end{aligned} \tag{14}$$

According to the underactuated subsystem Eq. (4) and the reference model Eq. (7), we can design the reference model of the payload swing angle as follows:

$$\ddot{\theta}_m = \frac{-g\, sin\theta_m}{l_m} - \frac{2\dot{l}_m \dot{\theta}_m}{l_m} + \frac{\ddot{x}\, cos\theta_m}{l_m} \tag{15}$$

3.2 Stability analysis

Theorem1: For the overhead crane nominal system (6), under the proposed reference law (13) and adaptive update law (19), the error (9) between the reference states and desired value will asymptotically converge to zero, which can be mathematically described as

$$\lim_{t\to\infty} e_m = 0 \Rightarrow \begin{cases} x_m \to x_d \\ l_m \to l_d \end{cases} \Rightarrow \theta_m \to 0$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

Proof: A Lyapunov function candidate can be chosen as:

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma_1}\tilde{c}_1^{\,2} + \frac{1}{2\gamma_2}\tilde{c}_2^{\,2} + \frac{1}{2\gamma_3}\tilde{c}_3^{\,2} + \frac{1}{2\gamma_4}\tilde{c}_4^{\,2} \tag{16}$$

where the variable $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{c}_4$ are the deviations between the adaptive gains in Eq. (11) and the initial gains, and are defined as

$$\tilde{c}_1 = \hat{c}_1 - c_1,\ \tilde{c}_2 = \hat{c}_2 - c_2,\ \tilde{c}_3 = \hat{c}_3 - c_3,\ \tilde{c}_4 = \hat{c}_4 - c_4 \tag{17}$$

Where c_1, c_2, c_3, c_4 are positive constant

Taking the time derivative of Eq. (18), we have:

$$\dot{\tilde{c}}_1 = \dot{\hat{c}}_1,\ \dot{\tilde{c}}_2 = \dot{\hat{c}}_2,\ \dot{\tilde{c}}_3 = \dot{\hat{c}}_3,\ \dot{\tilde{c}}_4 = \dot{\hat{c}}_4 \tag{18}$$

In order to make the derivative of the Lyapunov function to be a negative definite, i.e, the adaptive update law can be designed as:

$$\begin{cases} \dot{\hat{c}}_1 = \gamma_1 s^2 (e)^2 \\ \dot{\hat{c}}_2 = \gamma_2 s^2 (\dot{e})^2 \\ \dot{\hat{c}}_1 = \gamma_3 s^2 (\int e(\xi) d\xi)^2 \\ \dot{\hat{c}}_1 = \gamma_4 s^2 (\theta)^2 \end{cases} \tag{19}$$

Where $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are positive constant

The first derivative of Lyapunov function $\dot{V}$ is:

$$\dot{V} = s\dot{s} + \frac{1}{\gamma_1}\tilde{c}_1\dot{\tilde{c}}_1 + \frac{1}{\gamma_2}\tilde{c}_2\dot{\tilde{c}}_2 + \frac{1}{\gamma_3}\tilde{c}_3\dot{\tilde{c}}_3 + \frac{1}{\gamma_4}\tilde{c}_4\dot{\tilde{c}}_4 \tag{20}$$

Substituting Eq. (11)(14)(18) into Eq. (20), we have the following equality:

$$\dot{V} = s\left(\hat{c}_1\dot{e} + \hat{c}_2\left(f(z) + g(z)\left(u_{eq}(t) + u_{sw}(t)\right) - \ddot{z}_d\right) + \hat{c}_3 e + \hat{c}_4\dot{\theta}\right) + \frac{1}{\gamma_1}\left(\hat{c}_1 - c_1\right)\dot{\hat{c}}_1$$

$$+\frac{1}{\gamma_2}\left(\hat{c}_2 - c_2\right)\dot{\hat{c}}_2 + \frac{1}{\gamma_3}\left(\hat{c}_3 - c_3\right)\dot{\hat{c}}_3 + \frac{1}{\gamma_4}\left(\hat{c}_4 - c_4\right)\dot{\hat{c}}_4 \ = s\left(\hat{c}_2 g(z) u_{sw}(t)\right) + \frac{1}{\gamma_1}\left(\hat{c}_1 - c_1\right)\dot{\hat{c}}_1$$

$$+\frac{1}{\gamma_2}(\hat{c}_2 - c_2)\dot{\hat{c}}_2 + \frac{1}{\gamma_3}(\hat{c}_3 - c_3)\dot{\hat{c}}_3 + \frac{1}{\gamma_4}(\hat{c}_4 - c_4)\dot{\hat{c}}_4 \tag{21}$$

The switching control $u_{sw}(t)$ can be selected as:

$$u_{sw} = (\hat{c}_2 g(z))^{-1}\begin{pmatrix} -k_1 \tanh(as) - k_2 \sinh(b|s|^q)\,sign(s) \\ -s(\hat{c}_1(e)^2 + \hat{c}_2(\dot{e})^2 + \hat{c}_3(\int e(\xi)\,d\xi)^2 + \hat{c}_4\theta^2) \end{pmatrix} \tag{22}$$

Then, the Eq. (21) can be rewritten as:

$$\dot{V} = -k_1 s \tanh(as) - k_2 s a \sinh(b|s|^q)\,sign(s) + \hat{c}_1\left(\frac{1}{\gamma_1}\dot{\hat{c}}_1 - s^2(e)^2\right)$$

$$-\frac{1}{\gamma_1}c_1\dot{\hat{c}}_1 + \hat{c}_2\left(\frac{1}{\gamma_2}\dot{\hat{c}}_2 - s^2(\dot{e})^2\right) + \hat{c}_3\left(\frac{1}{\gamma_3}\dot{\hat{c}}_3 - s^2(\int e(\xi)\,d\xi)^2\right)$$

$$-\frac{1}{\gamma_3}c_3\dot{\hat{c}}_3 + \hat{c}_4\left(\frac{1}{\gamma_4}\dot{\hat{c}}_4 - s^2(\theta)^2\right) - \frac{1}{\gamma_4}c_4\dot{\hat{c}}_4 - \frac{1}{\gamma_2}c_2\dot{\hat{c}}_2 \tag{23}$$

Substituting Eq. (19) into Eq. (23), we obtain

$$\dot{V} = -k_1|s|\,tanh(as)sign(s) - k_2|s|a\,sinh(b|s|^q)$$
$$-s^2\left(c_1(e)^2 + c_2(\dot{e})^2 + c_3(\int e(\xi)\,d\xi)^2 + c_4(\theta)^2\right)$$
$$= -k_1|s|\,tanh(a|s|) - k_2|s|a\,sinh(b|s|^q)$$
$$-s^2(c_1(e)^2 + c_2(\dot{e})^2 + c_3(\int e(\xi)\,d\xi)^2 + c_4(\theta)^2) \quad \leq 0 \tag{24}$$

Remark: Where the term of $-k_1\,tanh(as) - k_2\,sinh(b|s|^q)sign(s)$ in (22) is the novel hyperbolic tangent reaching law proposed by Eq. (8) to eliminate chattering phenomenon and another term of $-s(\hat{c}_1(e)^2 + \hat{c}_2(\dot{e})^2 + \hat{c}_3(\int e(\xi)\,d\xi)^2 + \hat{c}_4\theta^2)$ is to design adaptive update law as (19)

3.3 TDE and AMSTA-based MRASMC design

A practice overhead crane system with uncertainties can be represented as follows:

$$\begin{cases} \dot{z_1} = z_2 \\ \dot{z_2} = f(z) + g(z)u + d \end{cases} \tag{25}$$

The error between the overhead crane and the reference model can be defined as follows:

$$e_r = z_1 - z_m,\ e_r = \theta - \theta_m \tag{26}$$

The sliding mode surface can be defined as follows:

$$s = \dot{e_r} + \lambda_1 e_r + \lambda_1 e_\theta \tag{27}$$

Where $\lambda_1 = diag(\lambda_{11}, \lambda_{12}), \lambda_2 = (\lambda_{21}, 0)^T$

Taking the derivative with respect to time of Eq. (27) and substituting Eq. (25) into it, we have:

$$\dot{s} = f(z) + g(z)u + d - \ddot{z_m} + \lambda_1\dot{e_r} + \lambda_1\dot{e_\theta} \tag{28}$$

Furthermore, the form of modified super-twisting algorithm is formulated as:

$$\begin{cases} \dot{s} = -k_1|s|^{0.5}\,sgn(s) - k_2 s + w \\ \dot{w} = -k_3 w - k_4\,sgn(s) \end{cases} \tag{29}$$

Where $|s|^{0.5} = diag(|s_1|^{0.5}, |s_2|^{0.5})$, $sgn(s) = diag(sgn(s_1), sgn(s_2))$, $k_i = diag(k_{ij})$, $i = 1,2,3,4$, $j = 1,2$. By resolving Eq. (29) using Eq. (28), the modified super-twisting algorithm is obtained as:

$$u = g^{-1}(z)[-f(z) + \ddot{z_m} - \lambda_1\ddot{e_r} - \lambda_2\dot{e_\theta} - k_1|s|^{0.5} - k_2 s + \int_0^t(-k_3 w - k_4\,sgn(s))dt - d] \tag{30}$$

Since d is uncertain, it will affect the control performance. Hence, the TDE scheme is utilized to estimate the uncertain term d, As the time delay L is chosen to be adequate small, it satisfies the following condition:

$$d(t) \cong d(t-L) \tag{31}$$

Therefore, the control law for overhead crane system can be rewritten as the following:

$$u = g^{-1}(z)[-f(z) + \ddot{z_m} - \lambda_1\ddot{e_r} - \lambda_2\dot{e_\theta} - k_1|s|^{0.5} - k_2 s + \int_0^t(-k_3 w - k_4\,sgn(s))dt - \hat{d}] \tag{32}$$

Where $\hat{d}$ is the estimation term of the uncertain term $d(t)$, and can be calculated as:

$$\hat{d} = \dot{z_2}(t-L) - f(z(t-L)) - g(z)u(t-L) \tag{33}$$

Theorem 2 considering the overhead crane system given in Eq. (25), the super-twisting control algorithm with time delay estimation Eq. (32) ensures the converge of the chosen sliding surface Eq. (27) to zero in a finite time. The finite reaching time to the sliding surface is estimated by

$$T = \frac{2}{\vartheta_2} ln(\frac{\vartheta_2}{\vartheta_1} V^{\frac{1}{2}}(s(0), w(0)) + 1) \tag{34}$$

Proof: Firstly, substituting the control Eq. (32) in the overhead crane system gives:

$$\begin{cases} \dot{s} = -k_1|s|^{0.5}\,sgn(s) - k_2 s + w \\ \dot{w} = -k_3 w - k_4\,sgn(s) + \dot{\varepsilon_d} \end{cases} \tag{35}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

where $\varepsilon_d = d - \overset{\Lambda}{d}$ is the estimation error, which can be assumed to be bounded. For simplicity, let us decompose the closed-loop dynamics above into two subsystems as follows:

$$\begin{cases} \dot{s_i} = -k_{1.i}|s|^{0.5}\,sgn(s_i) - k_{2.i}s_i + w_i \\ \dot{w_i} = -k_{3.i}w_i - k_{4.i}\,sgn(s_i) + \dot{\varepsilon_{di}} \end{cases} \tag{36}$$

Introducing a new vector $\eta = [\eta_1 \quad \eta_2]^T = [|s_i|^{0.5}\,sgn(s_i) \quad w_i]^T$,

According to the dynamic Eq. (36), let us take derivative of η, it follows that:

$$\begin{cases} \dot{\eta_1} = |s_i|^{-\frac{1}{2}}[-\frac{1}{2}k_{1.i} \quad \frac{1}{2}]\eta - \frac{1}{2}k_{2.i}\eta_1 \\ \dot{\eta_2} = |s_i|^{-\frac{1}{2}}[-k_{4.i}\eta_1 + \bar{\varepsilon_{di}}] - k_{3.i}\eta_2 \end{cases} \tag{37}$$

Where $\bar{\varepsilon_{di}} = |s_i|^{\frac{1}{2}}\dot{\varepsilon_{di}}$

Then, the Eq. (31) can be rewritten as follows:

$$\dot{\eta} = |s_i|^{-\frac{1}{2}}(A\eta + B\bar{\varepsilon_{di}}) - E\eta \tag{38}$$

Where $A = \begin{bmatrix} -\frac{1}{2}k_{1.i} & \frac{1}{2} \\ -k_{4.i} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E = \begin{bmatrix} -\frac{1}{2}k_{2.i} & 0 \\ 0 & k_{3.i} \end{bmatrix}$

For the stability analysis, the following Lyapunov function is chosen as:

$$V = \eta^T P \eta \tag{39}$$

Where P is symmetric and positive definite matric.

Differentiating the expression above yields:

$$\dot{V} = \frac{1}{|s|^{0.5}}[\eta^T(A^TP + PA)\eta + 2\eta^T PB\bar{\varepsilon_d}] - \eta^T R\eta \tag{40}$$

Where $R = E^TP + PE$

Otherwise, for better stability analysis, let us assume that the time derivative of TDE error is bound, i.e, $|\dot{\varepsilon}_{\mathrm{di}}| < \delta_i$. Then, the following inequality can be obtained:

$$2\eta^T PB\bar{\varepsilon_{di}} \le \varepsilon_{di}{}^2|\eta_1|^2 + \eta^T PBB^TP\eta \le \delta_i{}^2\eta^T C^T C\eta + \eta^T PBB^TP\eta \tag{41}$$

Where we utilized the Perfect square trinomial and $C = [1 \quad 0]$

Finally, using Eq. (40)(41), one has:

$$\dot{V} \le \frac{1}{|s|^{0.5}}[\eta^T(A^TP + PA + PBB^TP + \delta_i{}^2C^TC)\eta] - \eta^T R\eta \le -\frac{1}{|s|^{0.5}}\eta^T Q\eta - \eta^T R\eta \tag{42}$$

Where $A^TP + PA + PBB^TP + \delta_i{}^2C^TC = -Q$

By using Ragleigh's inequality, we can obtain:

$$\dot{V} \le -\frac{1}{|s|^{0.5}}\lambda_{min}(Q)||\eta||_2^2 - \lambda_{min}(R)||\eta||_2^2 \tag{43}$$

From Ragleigh's inequality

$$\lambda_{min}(P)||\eta||_2^2 \le V \le \lambda_{max}(P)||\eta||_2^2 \tag{44}$$

Through using Euclidean norm, the following inequality can be obtained as follows:

$$||\eta||_2^2 = |||s_i|^{0.5}\mathrm{sgn}(s_i)|| + ||w_i||^2 \ge ||s_i|| + ||w_i||^2 \ge ||s_i|| \tag{45}$$

Based on Eq. (44)(45), we can further obtain the following inequality:

$$||s_i|| \le ||\eta||_2^2 \le \frac{V}{\lambda_{min}(P)} \tag{46}$$

Then, it follows that:

$$-||s_i||^{-0.5} \le -(\frac{\lambda_{min}(P)}{V})^{\frac{1}{2}} \tag{47}$$

On the basis of Eq. (44)(47), Eq. (43) can be rewritten as follows:

$$\dot{V} \le -\frac{\lambda^{\frac{1}{2}}{}_{min}(P)\lambda_{min}(Q)}{\lambda_{max}(P)}V^{\frac{1}{2}} - \frac{\lambda_{min}(R)}{\lambda_{min}(P)}V = -\vartheta_1 V^{\frac{1}{2}} - \vartheta_2 V \tag{48}$$

Where $\vartheta_1 = \frac{\lambda^{\frac{1}{2}}{}_{min}(P)\lambda_{min}(Q)}{\lambda_{max}(P)}, \vartheta_2 = \frac{\lambda_{min}(R)}{\lambda_{min}(P)}$

The stability analysis shown above is sufficient if $k_{1.i}, k_{2.i}, k_{3.i}, k_{4.i}$ are constant. However, due to the complexity to determine $k_{1.i}, k_{2.i}, k_{3.i}, k_{4.i}$. Hence, the following over all stability is discussed in next step base on the $k_{1.i}, k_{2.i}, k_{3.i}, k_{4.i}$ are adaptive parameters. Then, the Eq.(30) can be rewritten as:

$$\begin{cases} \dot{s_i} = -\hat{k}_{1.i}|s|^{0.5}\, sgn(s_i) - \hat{k}_{2.i}s_i + w_i \\ \dot{w_i} = -\hat{k}_{3.i}w_i - \hat{k}_{4.i}\, sgn(s_i) + \dot{\varepsilon}_{di} \end{cases} \tag{49}$$

Theorem 3 Considering the system Eq. (33). Suppose that the time derivative of TDE error $\dot{\varepsilon}_{di}$ satisfies $|\dot{\varepsilon}_{di}| < \delta_i$ with unknown constant δ_i, then the sliding surface s=0 will be reached in finite time with adaptive gains:

$$\begin{cases} \begin{cases} \dot{\hat{k}}_{1.i} = \gamma_1\sqrt{\frac{\eta_1}{2}}, & if\ s \ne 0 \\ 0, & if\ s = 0 \end{cases} \\ \hat{k}_{4.i} = \hat{k}_{1.i}\,\theta \end{cases} \tag{50}$$

$$\begin{cases} \begin{cases} \dot{\hat{k}}_{2.i} = \gamma_2\sqrt{\frac{\eta_2}{2}}, & if\ s \ne 0 \\ 0, & if\ s = 0 \end{cases} \\ \hat{k}_{3.i} = \frac{1}{2}k_{2.i} \end{cases} \tag{51}$$

Where $\gamma_1, \eta_1, \gamma_2, \eta_2, \theta$ are arbitrary positive constants.

Proof: For analyzing the stability using the adaptive parameters the following Lyapunov is taken into account:

$$V_1 = V + \frac{1}{2\eta_1}\tilde{k}_{1.i}^{\,2} + \frac{1}{2\eta_2}\tilde{k}_{2.i}^{\,2} + \frac{1}{2\eta_3}\tilde{k}_{3.i}^{\,2} + \frac{1}{2\eta_4}\tilde{k}_{4.i}^{\,2} \tag{52}$$

Where

$$\tilde{k}_{1.i} = k_{1.i} - \hat{k}_{1.i} < 0, \tilde{k}_{2.i} = k_{2.i} - \hat{k}_{2.i} < 0, \tilde{k}_{3.i} = k_{3.i} - \hat{k}_{3.i} < 0, \tilde{k}_{4.i} = k_{4.i} - \hat{k}_{4.i} < 0$$

Where

$$V = \eta^T P \eta \tag{53}$$

$$P = \begin{bmatrix} \lambda + 4\theta & -2\theta \\ -2\theta & 1 \end{bmatrix} \tag{54}$$

P is symmetry and positive definite for any $\lambda > 0$

Taking the derivative of Eq. (52) yields:

$$\dot{V_1} = \dot{\eta}^T P \eta + \dot{\eta} P \eta^T - \frac{1}{\eta_1}\left(k_{1.i} - \hat{k}_{1.i}\right)\dot{\hat{k}}_{1.i} - \frac{1}{\eta_2}\left(k_{2.i} - \hat{k}_{2.i}\right)\dot{\hat{k}}_{2.i}$$
$$-\frac{1}{\eta_3}(k_{3.i} - \hat{k}_{3.i})\dot{\hat{k}}_{3.i} - \frac{1}{\eta_4}(k_{4.i} - \hat{k}_{4.i})\dot{\hat{k}}_{4.i} \tag{55}$$

The first two term of Eq. (55) are computed taking account Eq. (38). Then the symmetric matric Q, R can be deduced as:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

$$Q=\begin{bmatrix}\hat{k}_{1.i}(\lambda+4\theta^2)-4\theta\hat{k}_{4.i}-4\theta^2-\delta^2 & * \\ -\hat{k}_{1.i}\theta+\hat{k}_{4.i}-\frac{1}{2}(\lambda+4\theta^2)+2\theta & 2\theta\end{bmatrix} \tag{56}$$

By choosing $\hat{k}_{4.i}=\hat{k}_{1.i}\theta$, the matrix Q will be positive definite with minimum eigenvalue $\lambda_{min}\ (Q)=\varepsilon>0$, if

$$\hat{k}_{4.i}>\frac{[2\theta-\frac{1}{2}(\lambda+4\theta^2)]^2}{2\theta-1}+4\theta^2-\delta^2,\theta>\frac{1}{2} \tag{57}$$

$$R=\begin{bmatrix}\hat{k}_{2.i}(\lambda+4\theta^2) & * \\ -\theta(\hat{k}_{2.i}+2\hat{k}_{3.i}) & 2\hat{k}_{3.i}\end{bmatrix} \tag{58}$$

By choosing $\hat{k}_{2.i}=2\hat{k}_{3.i}$, the matrix R will be positive definite with minimum eigenvalue $\lambda_{min}\ (R)=\mu\geq 0$

Now, in view of the above analysis and taking into account Eq. (37), the derivative of Lyapunov Eq. (34) can be obtained as follows:

$$\begin{aligned}V_1 &\leq -\frac{\varepsilon\lambda^{\frac{1}{2}}{}_{min}\ (P)}{\lambda_{max}\ (P)}V^{\frac{1}{2}}-\frac{\mu}{\lambda_{max}\ (P)}V-\frac{1}{\eta_1}\left(k_{1.i}-\hat{k}_{1.i}\right)\dot{\hat{k}}_{1.i}-\frac{1}{\eta_2}\left(k_{2.i}-\hat{k}_{2.i}\right)\dot{\hat{k}}_{2.i} \\ &\quad -\frac{1}{\eta_3}\left(k_{3.i}-\hat{k}_{3.i}\right)\dot{\hat{k}}_{3.i}-\frac{1}{\eta_4}\left(k_{4.i}-\hat{k}_{4.i}\right)\dot{\hat{k}}_{4.i} \\ &\leq -\frac{\varepsilon\lambda^{\frac{1}{2}}{}_{min}\ (P)}{\lambda_{max}\ (P)}V^{\frac{1}{2}}-\frac{\gamma_1}{\sqrt{2\eta_1}}\left|k_{1.i}-\hat{k}_{1.i}\right|-\frac{\gamma_2}{\sqrt{2\eta_3}}\left|k_{2.i}-\hat{k}_{2.i}\right|-\frac{\gamma_3}{\sqrt{2\eta_3}}\left|k_{3.i}-\hat{k}_{3.i}\right| \\ &\quad -\frac{\gamma_4}{\sqrt{2\eta_4}}\left|k_{4.i}-\hat{k}_{4.i}\right|-\frac{1}{\eta_1}\left(k_{1.i}-\hat{k}_{1.i}\right)\dot{\hat{k}}_{1.i}-\frac{1}{\eta_2}\left(k_{2.i}-\hat{k}_{2.i}\right)\dot{\hat{k}}_{2.i} \\ &\quad -\frac{1}{\eta_3}\left(k_{3.i}-\hat{k}_{3.i}\right)\dot{\hat{k}}_{3.i}-\frac{1}{\eta_4}\left(k_{4.i}-\hat{k}_{4.i}\right)\dot{\hat{k}}_{4.i}+\frac{\gamma_1}{\sqrt{2\eta_1}}\left|k_{1.i}-\hat{k}_{1.i}\right|+\frac{\gamma_2}{\sqrt{2\eta_3}}\left|k_{2.i}-\hat{k}_{2.i}\right| \\ &\quad +\frac{\gamma_3}{\sqrt{2\eta_3}}|k_{3.i}-\hat{k}_{3.i}|+\frac{\gamma_4}{\sqrt{2\eta_4}}|k_{4.i}-\hat{k}_{4.i}|\end{aligned} \tag{59}$$

According to the Jensen's inequality, we can get:

$$\begin{aligned}V_1 &\leq -\chi\sqrt{V+\frac{1}{2\eta_1}(k_{1.i}-\hat{k}_{1.i})^2+\frac{1}{2\eta_2}(k_{2.i}-\hat{k}_{2.i})^2+\frac{1}{2\eta_3}(k_{3.i}-\hat{k}_{3.i})^2+\frac{1}{2\eta_4}(k_{4.i}-\hat{k}_{4.i})^2} \\ &\quad +\left(k_{1.i}-\hat{k}_{1.i}\right)\left(\frac{\gamma_1}{\sqrt{2\eta_1}}-\frac{1}{\eta_1}\dot{\hat{k}}_{1.i}\right)+\left(k_{2.i}-\hat{k}_{2.i}\right)\left(\frac{\gamma_2}{\sqrt{2\eta_3}}-\frac{1}{\eta_2}\dot{\hat{k}}_{2.i}\right) \\ &\quad +\left(k_{3.i}-\hat{k}_{3.i}\right)\left(\frac{\gamma_3}{\sqrt{2\eta_3}}-\frac{1}{\eta_3}\dot{\hat{k}}_{3.i}\right)+(k_{4.i}-\hat{k}_{4.i})(\frac{\gamma_4}{\sqrt{2\eta_4}}-\frac{1}{\eta_4}\dot{\hat{k}}_{4.i})\end{aligned} \tag{60}$$

Where $\chi=min(\frac{\varepsilon\lambda^{\frac{1}{2}}{}_{min}\ (P)}{\lambda_{max}\ (P)},\gamma_1,\gamma_2)$

Let us definite:

$$\begin{aligned}\varphi &=\left(k_{1.i}-\hat{k}_{1.i}\right)\left(\frac{\gamma_1}{\sqrt{2\eta_1}}-\frac{1}{\eta_1}\dot{\hat{k}}_{1.i}\right)+\left(k_{2.i}-\hat{k}_{2.i}\right)\left(\frac{\gamma_2}{\sqrt{2\eta_3}}-\frac{1}{\eta_2}\dot{\hat{k}}_{2.i}\right) \\ &\quad +(k_{3.i}-\hat{k}_{3.i})(\frac{\gamma_3}{\sqrt{2\eta_3}}-\frac{1}{\eta_3}\dot{\hat{k}}_{3.i})+(k_{4.i}-\hat{k}_{4.i})(\frac{\gamma_4}{\sqrt{2\eta_4}}-\frac{1}{\eta_4}\dot{\hat{k}}_{4.i})\end{aligned} \tag{61}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

In order to achieve finite time, converge, we have to assume $\varphi = 0$. Then, the adaptation of gains $\dot{\hat{k}}_{1.i}, \dot{\hat{k}}_{2.i}, \dot{\hat{k}}_{3.i}, \dot{\hat{k}}_{4.i}$ can be obtained as follows:

$$\begin{cases} \dot{\hat{k}}_{1.i} = \gamma_1 \sqrt{\frac{\eta_1}{2}} \\ \dot{\hat{k}}_{4.i} = \gamma_4 \sqrt{\frac{\eta_4}{2}} \end{cases} \tag{62}$$

$$\begin{cases} \dot{\hat{k}}_{2.i} = \gamma_2 \sqrt{\frac{\eta_2}{2}} \\ \dot{\hat{k}}_{3.i} = \gamma_3 \sqrt{\frac{\eta_3}{2}} \end{cases} \tag{63}$$

By selecting $\theta = \frac{\gamma_4}{\gamma_1}\sqrt{\frac{\eta_4}{\eta_1}}$, $\frac{\gamma_2}{\gamma_3} = 2\sqrt{\frac{\eta_3}{\eta_2}}$, Eq. (57) and Eq. (62), Eq. (60) and Eq. (63) coincide respectively. It is worth noting that for the finite tine convergence $\hat{k}_{1.i}, \hat{k}_{2.i}$ must satisfy inequality Eq.(58) and Eq.(60) ,it means that $\hat{k}_{1.i}$ is supposed to increase in according with Eq.(60) until Eq.(58) is met that guarantee the positive definiteness of the matrix Q, after that the finite convergence is guaranteed according to Eq.(62). Also, as soon as the sliding variable S and its derivative converges to zero, it does not make sense to increase by making. Therefore, we obtain the gain-adaptation law, theorem is proven.

4. Simulation results

Relevant simulation experiments conducted using MATLAB/Simulink to verify the control performance of the proposed method.

In order to verify the controller performance proposed in this paper, as well as the robustness of the controller to the overhead crane parameter and the control requirement, The simulation is carried out in following three cases.

4.1 Comparison with existing methods

Consider the overhead crane system described in(25)with the nominal parameter $M = 26.7kg$, $m = 2kg$, $g = 9.81m/s^2$, $x_0 = 0m, l_0 = 0.1m$ stand for initial value of the trolley position and rope length respectively, while the desired value is set to $x_d = 1m$, $l_d = 0.8m$. The parameters of the reference model in Eq.(43) are set as $k_{11} = k_{12} = 30$, $k_{21} = k_{22} = 0.01$, $a_1 = a_2 = 0.05$, $b_1 = b_2 = 1$, $q_1 = 3$, $q_2 = 3$; The parameters of the controller in Eq.(30) are set as $k_{31} = k_{32} = 10$, $k_{41} = k_{42} = 1$, $k_{51} = k_{52} = 0.05$; External disturbance $d = [\cos(9t), 0.8\cos(8t)]^T$.

The proposed method is compared with adaptive dynamic sliding mode control (ASMC) in [19], and a sliding mode control (SMC) law with exponential approach law is defined as follows:

$$u_{smc} = -G^{-1}(z)[F(z) - \ddot{z}_d + \beta_s \dot{e}_r + \rho_s \dot{e}_\theta + \ddot{e}_\theta + k_s S + \eta_s sign(S)]$$

Where $\beta_s = diag(\beta_{s1}, \beta_{s2}), \rho_s = [\rho_{s1}, 0]^T$ are the matrices of the parameters to be designed and $\beta_{si} \in R^+$; $i = 1,2$. $\rho_{si} \in R^+$; $i = 1$. $k_s = [k_{s1}, k_{s2}]^T$ represent control gains and $k_{si} \in R^+; i = 1,2$, $\eta_s = [\eta_{s1}, \eta_{s2}]^T$ are switching gains and $\eta_{si} \in R^+$; $i = 1,2$. The parameter of SMC controller is set as:

$$\beta_{s1} = 0.55, \beta_{s2} = 0.7, \rho_{s1} = 2.1, k_{s1} = 1, k_{s2} = 1, \eta_{s1} = 0.5, \eta_{s1} = 0.5$$

Simulation results of the three controllers are shown in Fig. 3

As can be seen from Fig. 3(a) and (b) that the trolley moves to the desired position $x_d = 1m$ at approximately 7.7 sec without any overshoot while the swing angle θ converges to the minimal neighborhood of zero. In Fig3. (b), although the range of the swing angle get increasing owing to the

ISSN: 2414-1895
DOI: 10.6919/ICJE.202108_7(8).0012

trolley speed increasing compared with adaptive dynamic SMC method, using the proposed method makes the convergence of speed of the swing angle θ faster. Fig. 3(d) and (e) show the control force u is rather smooth, and the chattering phenomenon gets constrained effectively by using the proposed control law.

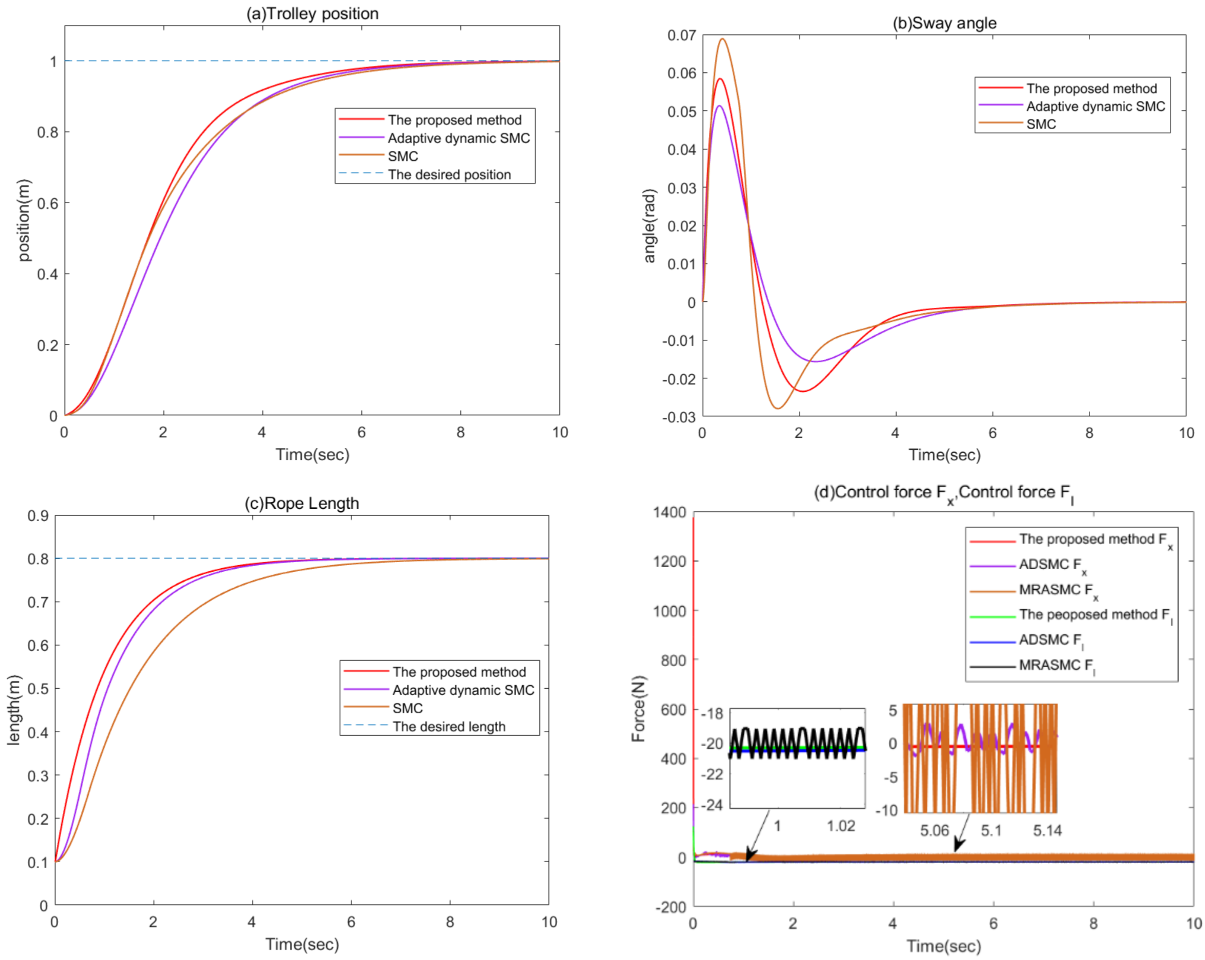

Fig. 3 Simulation results of different methods

4.2 Robustness verification against parameter uncertainties

In order to verify the robustness of proposed method, the overhead crane system parameter values changing as following two arbitrary sets of parameters. All the parameter values in this simulation study are kept as same as that in last simulation case except that the payload mass and trolley mass. The simulation results are depicted in Fig.4.

Case 1: Parameter uncertainties. The model parameters are changed to $M = 5kg$, $m = 2kg$, respectively, while the nominal values remain $M = 2kg$, $m = 0.8kg$

Case 2: Two different transportation task. The initial and desired positions for crane system are changed to $x_0 = 0.01m$, $l_0 = 0.2m$, $x_d = 1.2m$, $l_d = 1m$ (and $x_d = 1.5m$, $l_d = 1.2m$)

Limited by the paper length, the figures for the control inputs are not provided in Figs.4,5, since they are very similar with those in the first set of simulations.

The simulation results are shown in Fig.4. It can be seen from Fig.(a), (b)and(c) that, even under various parameter uncertainties, the proposed method still acquires great control performance, namely, the trolley accurately reaches the targeted position with no angle swings. Furthermore, from the results in Fig 5. (a), (b)and(c), it is concluded that the simulation carried out intentionally in Case 2

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

further verifies the proposed method's adaptability to different control objective. In short, as shown in simulation experimental results, the proposed controller has great robustness against model parameter uncertainties and different control objectives.

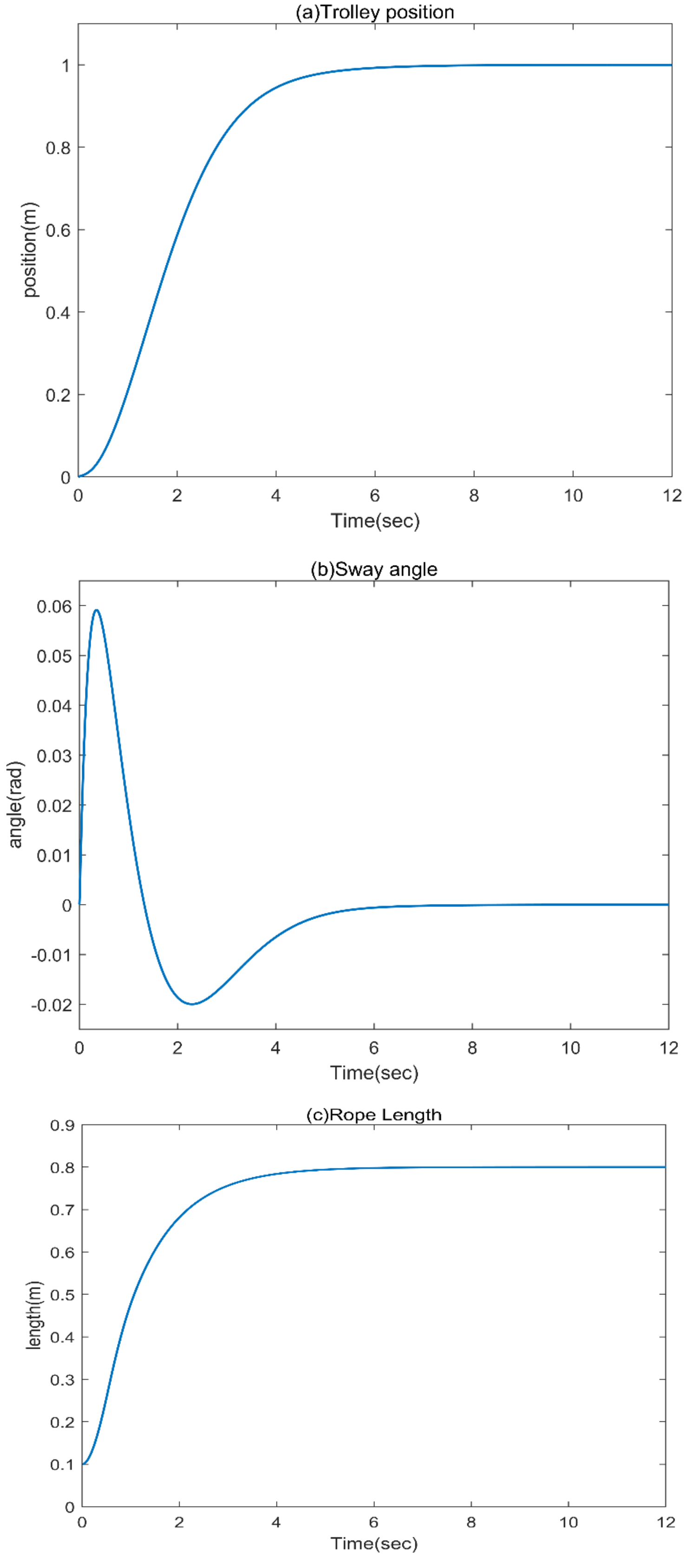

Fig.4 Simulation results: test of parameter uncertainty

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

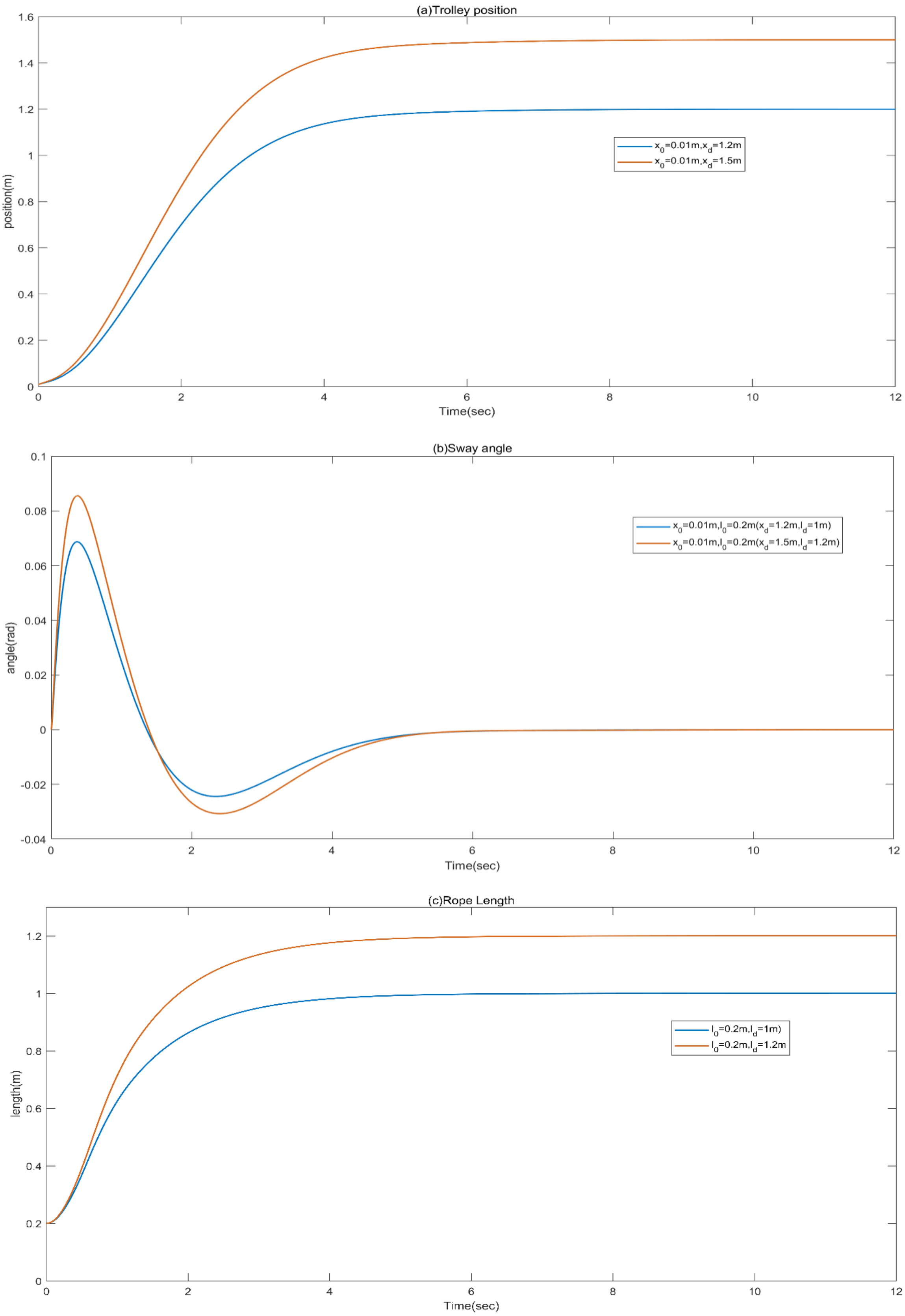

Fig. 5 Simulation results: test of a different transportation task

5. Conclusion

This paper proposed an integral sliding mode controller for 2-D under-actuated cranes with time varying reference model, which achieves precise trolley positioning, with improved transient

ISSN: 2414-1895 DOI: 10.6919/ICJE.202108_7(8).0012

performance on swing suppression. The proposed controller and the corresponding stability analysis are derived directly from the system's original nonlinear model instead of a linearized model around the equilibrium point. Simulation results are provided to validate the effectiveness of the proposed controller. Consider input saturation in future work.

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