

Research on the Sinking Factor of Saxon Bowl

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Abstract

A bowl with a hole in the bottom will sink when placed in water. The Saxons used this device in time. This paper studies the parameters that influence the sinking time. Through the timing function, it is guessed that the bowl will drop at a constant speed from immersion to the immersion process, and the factors affecting the sinking time are the diameter and height of the bowl. Then through the establishment of a physical model, combined with fluid mechanics and Bernoulli equation, using the bowl mass, liquid density, notch area and other factors to deduce the bowl drop as a uniform process. Therefore, while ignoring the surface tension, the influence of bowls with different pore diameters and heights on the falling time is studied, and the relationship between the falling time and the height of the bowl and the size of the pore size is fitted by MATLAB.

Keywords

Saxon Bowl; Fluid Mechanics; Sinking Time; MATLAB.

1. Background analysis

The Saxons usually refer to the Germanic people who immigrated and ruled England from the time of the Norman conquest in the fifth century AD. In that era, timing tools were relatively scarce. The Saxons invented the Saxon bowl, a timing device. They opened a hole in the bottom of the bowl, placed the bowl in the water to sink naturally, and timed it by observing the sinking phenomenon of the bowl in the water. In fact, in ancient times, there were many ways to achieve the "timekeeping" function. For example, the ancient Tibetans in my country invented the "standard watch" and the "measuring" time measuring device." Li Qiang et al. studied the characteristics of the ancient timekeeping tool "horse, engraving on the top", and Chen Ningxin et al. studied another ancient timekeeping tool. The timer "water clock." In the West, the "hourglass" is also a timing device, first invented by Alexander in the third century.

Because the Saxon Bowl timer has the characteristics of convenient production and relatively accurate timing, the study of related physics problems in the sinking process of the Saxon Bowl has become one of the competition topics of the 2020 International Young Physicists Championship (1YPT). This paper analyzes the sinking time of the Saxon Bowl timing device theoretically and experimentally, and theoretically establishes a physical model, and derives the relationship between the sinking time of the Saxon Bowl and related physical parameters. Experimentally, the related physical parameters are studied.

2. Theoretical analysis

2.1 During the preliminary experiment, two obvious phenomena were discovered

In the middle process, the difference between the inside and outside of the container maintains a stable value, and the difference is related to the quality of the container.

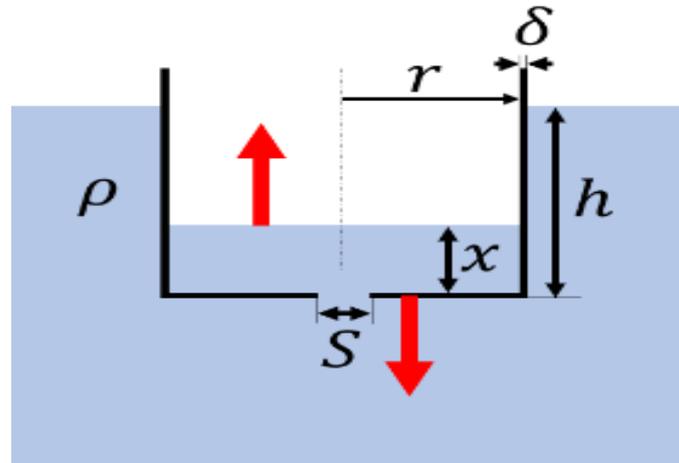


Figure 1. Saxon bowl model

2.2 In the middle process, the whole container descends at a constant speed

If the above two phenomena are approximately true, then a period of time can be drawn on the container wall. When the liquid level is at this height, the Saxon bowl has a good linear relationship, and this period can be regarded as the time interval.

A cylindrical container is selected for theoretical analysis, and the internal cross section of the container is S_0 , the cross section of the wall is δ , the area of the small hole is S , the water pressure on the bottom of the container is P , and the mass of the container is m . It is hoped that through the previous experiments, the relationship between the speed of water entering and the difference between the internal and external liquid level, the quality of the container, the area of the small hole and a fixed proportional coefficient can be found, so as to extend the experimental conclusions to a more practical Saxon bowl.

Before doing theoretical calculations, make a few basic simplifying assumptions:

- (1) $\delta \ll S_0$ takes into account two aspects. One is that the inner wall of the bowl used in the experiment is very thin, and δ is a small amount relative to S_0 ; the other is that ignoring the high-order small amount of δ can simplify the calculation difficulty.
- (2) The assumption of $S \ll S_0$ is also based on making the flow velocity of only a small piece near the small hole and its surroundings be v , and the flow velocity of most of the water in the container is v' , only a very small part of the water. The flow rate is somewhere in the middle of the two.
- (3) Slowly changing hypothesis. This hypothesis is based on the second assumption above and the second phenomenon in the pre-experimental phenomenon. Generally speaking, except for the stage when the bowl has just entered the water and is about to sink, the process of entering the water in the middle can be approximately regarded as quasi-static. A hypothesis is of great significance for simplifying our physical model.

3. Analysis

Assuming that the sink is large enough, there is no change in the liquid level during the sinking process. For the force analysis of the container, there are:

$$mg + P_0\delta + P_0(S - S_0) - (P_0 + \rho gh)(S_0 + \delta - S) = \frac{md^2h}{dt^2}$$

Analysis of the force of the water in the container:

$$P(S_0 - S) + (P_0 + \rho gh)S - P_0S_0 - \rho S_0 xg = \frac{dp}{dt} = \frac{dm}{dt} v + \frac{dv}{dt} m$$

where $\frac{dv}{dt} = \rho x S_0 \left(\frac{d^2x}{dt^2} - \frac{d^2h}{dt^2} \right)$, $v \frac{dm}{dt} = \rho \left(\frac{dx}{dt} \right)^2 S_0$

Bernoulli equation, because the container reference frame is a non-inertial frame, under the premise of slowly changing, the acceleration of the container can be regarded as constant within a certain time in the micro-element. At this time, g in Bernoulli's equation can be corrected to $g - \frac{d^2h}{dt^2}$

Then you can get: $P_0 + \rho x \left(g - \frac{d^2h}{dt^2} \right) + \frac{1}{2} \rho \left(\frac{dx}{dt} \right)^2 = P + \frac{1}{2} \rho \left(\frac{dx}{dt} \right)^2$

Solutions have to $P = P_0 + \rho x \left(g - \frac{d^2h}{dt^2} \right)$

Simultaneously obtain a system of second-order differential equations:

$$S \left[hg - x \left(g - \frac{d^2h}{dt^2} \right) \right] = S_0 \left[\left(\frac{dx}{dt} \right)^2 + x \frac{d^2x}{dt^2} \right]$$

$$[m + \rho x(S_0 - S)] \left(g - \frac{d^2h}{dt^2} \right) - \rho gh(S_0 + \delta - S) = 0$$

According to the experimental phenomenon, it is observed that the liquid level difference between the inside and outside of the intermediate timing interval is a certain value, namely

$$h - x = c(c = \text{constant})$$

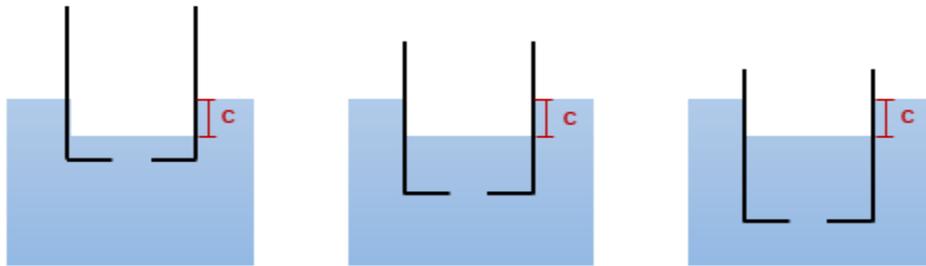


Figure 2. Saxon bowl model

Then: $\frac{dx}{dt} = \frac{dh}{dt}$, $\frac{d^2x}{dt^2} = \frac{d^2h}{dt^2}$

$$gS(h - x) = v^2 S_0 + (S_0 - S)(xv dv/dx)$$

$$\frac{dv^2}{v^2 - gSc/S_0} = 2 \frac{dx}{x} \frac{S_0}{S - S_0}$$

$$v_0 = 0, x_0 \ll 1$$

Points earned: $v^2 = \frac{gSc}{S_0} \left(1 - \frac{x_0}{x} \right)^{\frac{2S}{S_0 - S}} \approx gSc/S_0$

The bowl moves approximately at a constant speed in the process of movement, and the study of the falling process of the Saxon bowl only studies the process of uniform speed.

4. Experimental phase

4.1 Experiment on the relationship between aperture and falling time

Experiment with a bowl with an inner diameter of 40.00mm and different apertures, and record the falling time.

Table 1. The relationship between falling time and pore size

The inside diameter /mm	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00
Aperture /mm	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00
Fall time /mm	8.26	5.71	4.86	3.01	2.08	1.86	1.79	1.21	1.08

Through theoretical analysis and experimental data recording, the theoretical value is basically consistent with the experimental value, as shown in Figure 3:

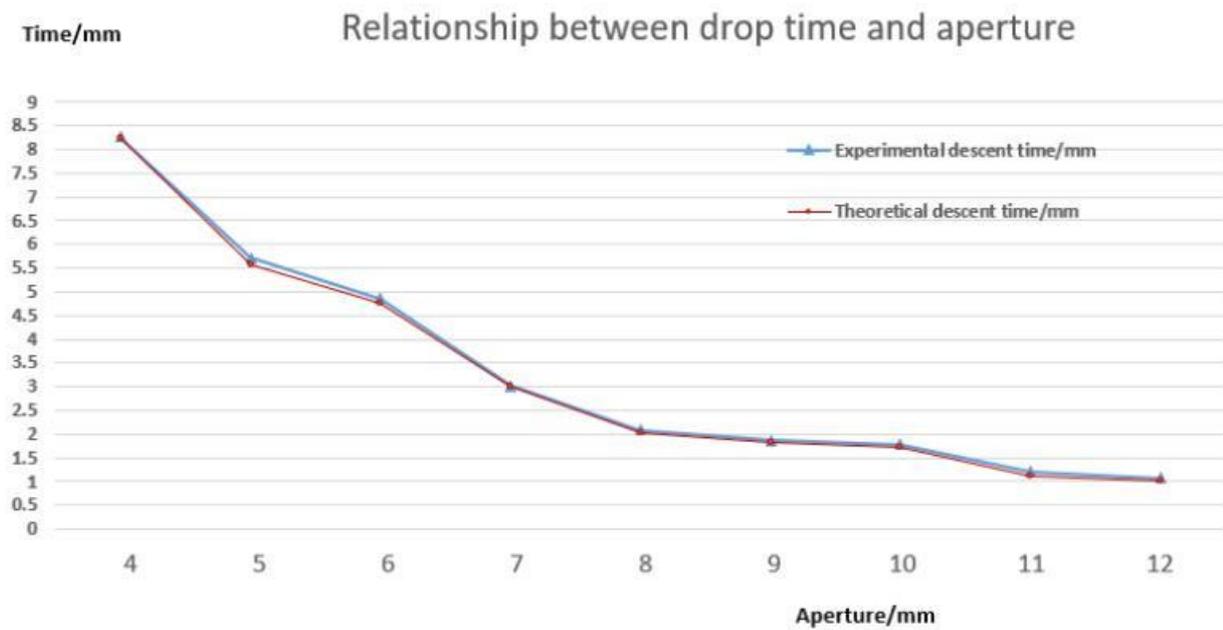


Figure 3. The relationship between fall time and aperture

Use MATLAB to fit the relationship between aperture and time, as well as the reciprocal square relationship between time and aperture, as shown in Figure 4, 5.

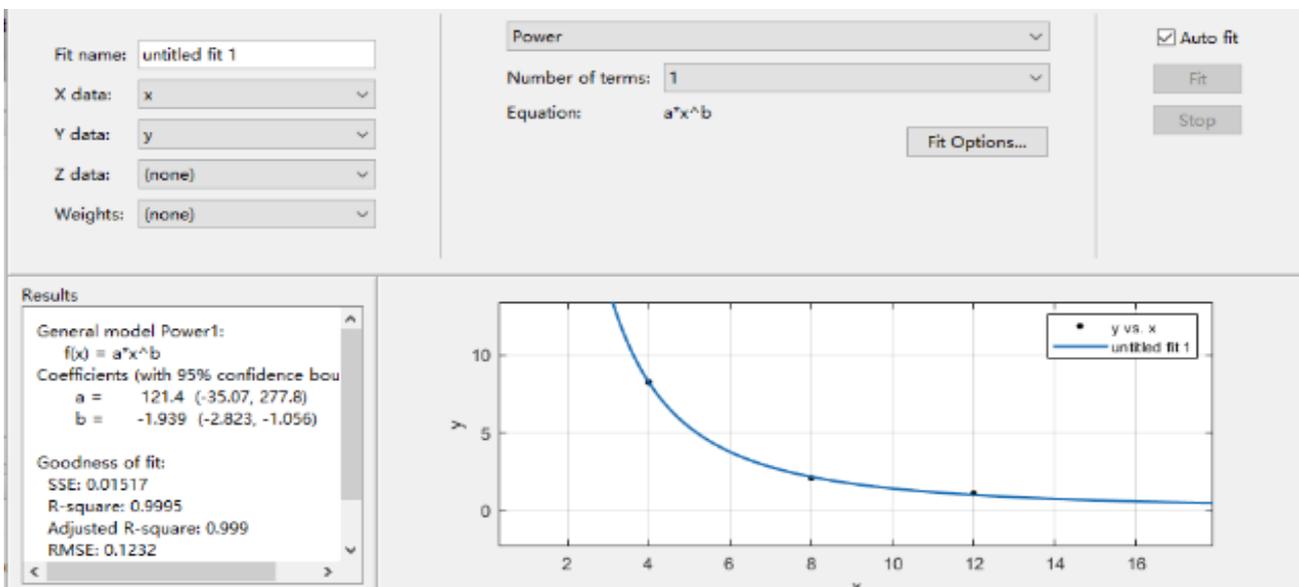


Figure 4. MATLAB fits the relationship between aperture and time

$$y = 121.4 \cdot x^{-1.9}$$

$$R^2 = 0.9995$$

4.2 The relationship between height and falling time

Through experiment 1, it is found that the bowl with 8.00mm aperture is the easiest to record the descending process. Therefore, the experiment of different heights with 8.00mm aperture is selected. In the experiment, coins are hung to ensure that the Saxon bowl has the same quality for easy control variable.

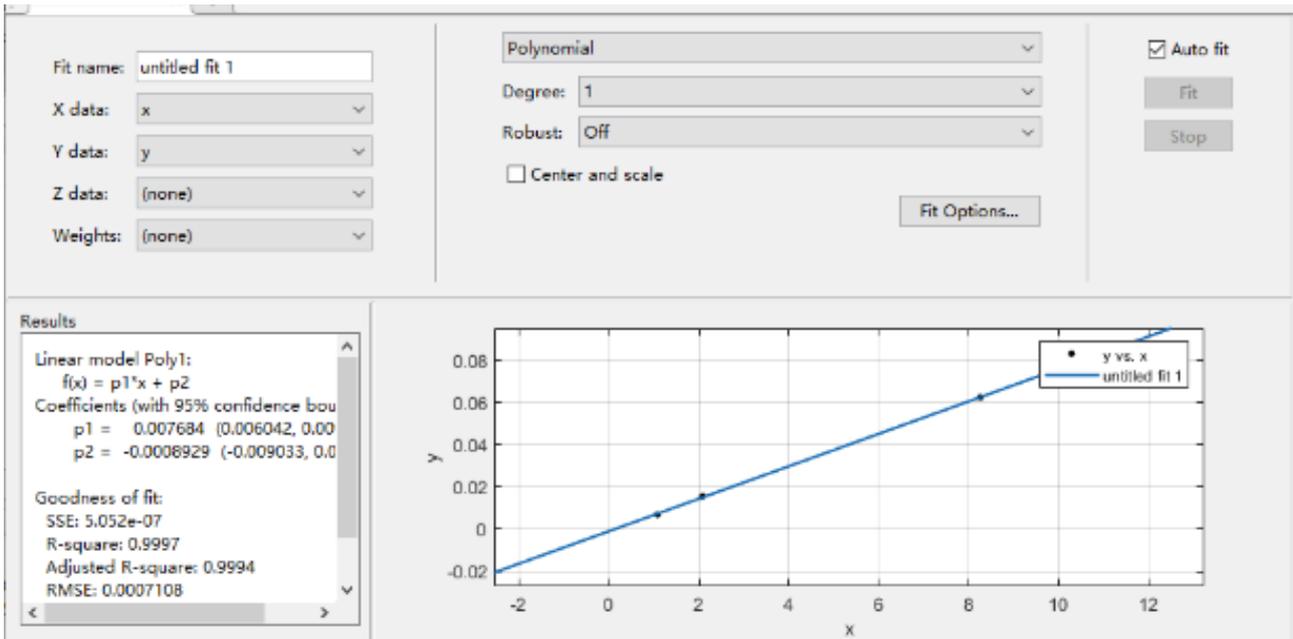


Figure 5. Time is linear with the reciprocal of the square of the aperture

$$t = \frac{\pi}{S}$$

Table 2. The relationship between falling time and height

Aperture /mm	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
Height /mm	40.00	42.50	45.00	47.50	50.00	52.50	55.00	57.50	60.00
Fall time /mm	5.23	7.65	10.34	13.08	15.35	18.09	19.45	21.09	23.09

Through theoretical analysis and experimental data recording, the theoretical value is basically consistent with the experimental value, as shown in Figure 6.

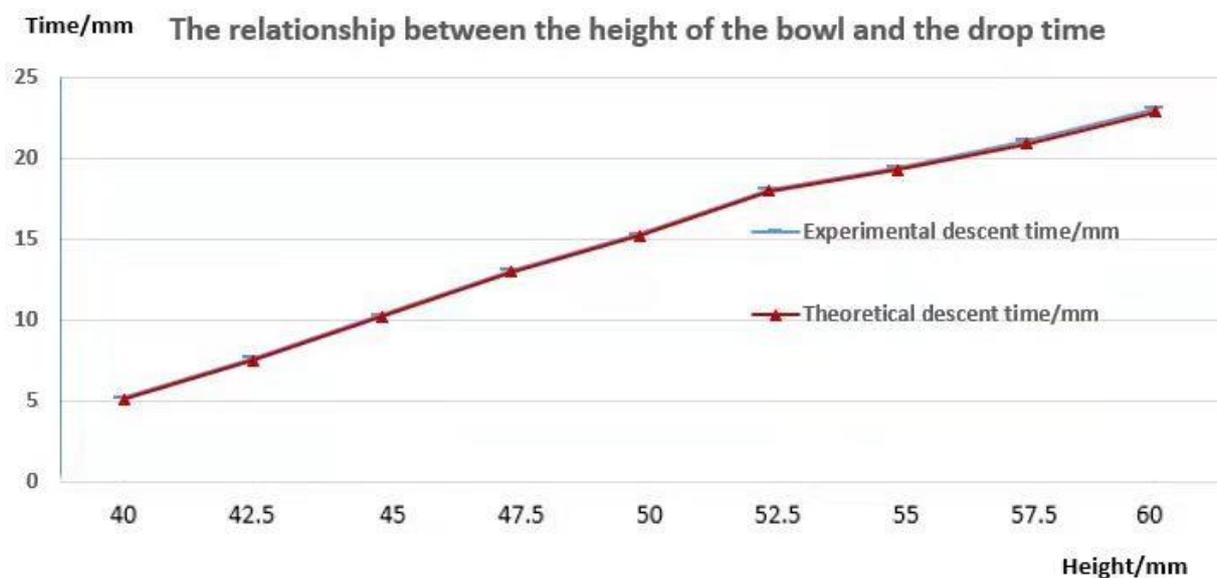


Figure 6. The relationship between falling time and bowl height

MATLAB fits the relationship between the height of the bowl and the falling time, as shown in Figure 7:

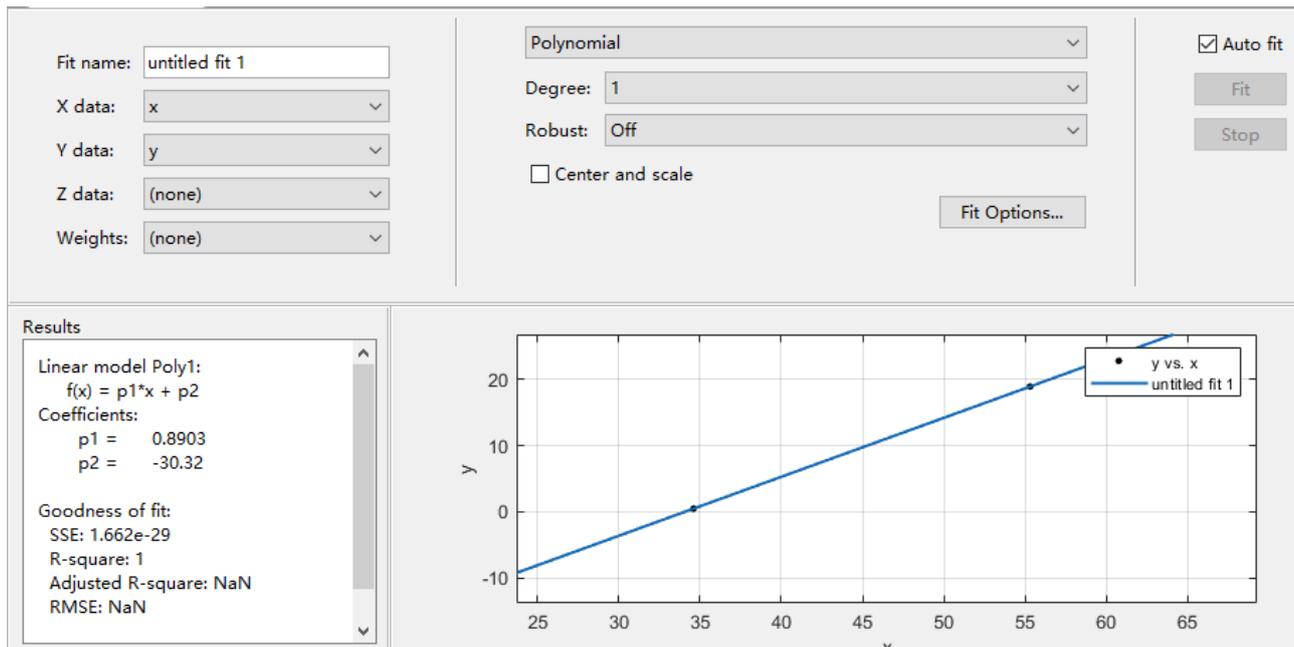


Figure 7. The relationship between the height of the bowl and the falling time

$$y = 0.89x - 30$$

5. Conclusion

The Saxon bowl timing device has been researched theoretically and experimentally, a relatively simple physical model has been established, and the relationship between the sinking time of the Saxon bowl and the hole diameter and bowl height has been analyzed. Theoretical analysis shows that: the larger the Saxon aperture, the faster the decline; the higher the altitude, the slower the decline. On the basis of theoretical analysis, experiments are carried out, and the aperture and height are both factors that affect the fall time using MATLAB software, and they have a definite numerical relationship. Therefore, when choosing the Saxon bowl, choose the appropriate specifications.

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