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Research and Development of Wave Transformation on Coral Reefs

Xueyi Zhao

School of Shanghai Maritime University, Shanghai 200000, China.

xueyizhaoshangqiu@163.com

Abstract

Wave propagation on coral reef topography with abrupt change of water depth is quite different from that on slow change of water depth. In this paper, the research progress of wave propagation on coral reef topography with abrupt change of water depth is reviewed, and the process of transforming general Boussinesq equation into conservation type governing equation is described in detail. The governing equation format is introduced: the mixed solution of finite difference method and finite volume method is adopted, the convection term in the equation is discretized by finite volume method, the interface flux is solved by HLL Riemannian solution, and the left and right variables of the control unit boundary are constructed by the fourth-order state interpolation method (muscl). Some existing problems and further research directions are pointed out.

Keywords

Boussinesq Equation; Coral Reef; Wave Breaking; Numerical Model.

1. Introduction

Coral reef is known as "tropical rain forest in the ocean", which has the highest biodiversity in marine ecosystem. Among the many coastal landforms in the world, coral reefs generally exist on the coastline of tropical and subtropical regions. For example, more than 70% of the coastline of Hawaii in the subtropical region is covered by coral reefs [1]. Secondly, China is one of the countries with many coral reef landforms, mainly distributed in Hainan Province, along the coast of Taiwan Province and the islands around the South China Sea. According to the relationship between coral reefs and shoreline, three types of reefs are divided: shore reef, barrier reef and Atoll. Shore reef is the most common type of coral reef, which is one of the classic landforms in offshore area. Shore reef, also known as "skirt reef" or "edge reef", is generally composed of reef front slope and reef flat, and the transition area between reef front slope and reef flat is called reef margin. Mature coral reefs generally have relatively horizontal reef flat, the width of which can reach several hundred meters, and most of its height is close to the average low tide level [2]. Similar to stepped topography, coral reefs have steep front slopes, usually on the order of 1:10. The reef flat is relatively flat and the water depth is shallow (usually less than 5 m). When the tide level is low, the waves break in front of the reef edge and the wave energy passing through the reef top is very small. At high tide level, the water depth at the top of the reef is generally 2-3m. When a storm surge passes, the water level may increase by about 0.5m. At this time, the smaller waves can pass through the reef top without breaking, while the larger waves will break. The wave height over the reef top is limited by the water depth. In recent years, with the increase of construction projects on coral reefs at home and abroad, it is more and more important to study the breaking law of waves on coral reefs. At the same time, it also provides theoretical support for reliable and effective engineering planning and design, construction and disaster prevention and reduction on coral reefs.

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1.1 Research status

In recent years, most of the researches on wave propagation and deformation on coral reefs focus on physical model tests, mainly on gentle slopes. The general physical model experiments are usually carried out in a long straight wave flume, which can generalize the coral reef topography into a model composed of a fixed slope, a horizontal platform reef and a slope behind the reef. Gourlay [3-4] conducted a series of model experiments for the first time to study the propagation and deformation of regular waves on coral reef terrain. Demirbilek [5] et al. Used the experimental model to study the water level change of coral reef flat and the wave climbing on the slope behind the reef under the combined action of wind wave and random wave. Yao [6] systematically studied the wave breaking law under different parameters such as reef flat water depth and slope gradient in front of the reef by using physical model, and gave the empirical formula of relevant breaking index. Roeber [7] used the experimental model to explore the wave propagation and deformation process on the coral reef, and compared the numerical simulation results based on Boussinesq equation with the experimental results. Mei Chen [8] found in the experiment that when the incident deep water wave height is high, the wave will break at the reef edge, the wave will decay rapidly, and then continue to spread to the reef flat, and reach the wave surface stable state. Liu Shuxue [9] and others studied the coral reef topography through cross-section physical model test. When the incident wave is regular wave and the wave height is small, the nonlinear effect of the wave is small, and the waveform is relatively stable; when the incident wave height is increased, the wave will break at the same position. Zheng Jinhai [10] and others think that the basic idea of Bragg resonance can be referred to, and the method of designing the crest elevation of the breakwater to be lower than the average low tide level can be used to achieve the purpose of wave dissipation. Because most of the coral reef flat is close to the average low tide level, it expands the idea of studying coral reef topography.

Numerical model test has the advantages of less cost, less manpower, short test period and easy to change experimental conditions. With the rapid development of computer technology, it has gradually become one of the mainstream methods in the study of wave in coastal waters. Most wave models are suitable for continental shelf coasts with gentle sloping water depths. However, because of the sharp change of water depth in the front slope of coral reef, the terrain is more complex than the flat and gentle slope coast. Nonlinearity and dispersion become important aspects of numerical model. With the expansion of Boussinesq equations, although the accuracy is different, it contains both dispersion and nonlinearity, and has the potential to simulate the propagation of tsunami waves. The numerical model based on nonlinear shallow water equation is usually used to simulate the near shore propagation of isolated waves. The nonlinear shallow water equation has high resolution and can deal with the wave breaking and climbing problems easily. However, the equation lacks dispersion, and there are some defects in simulating the wave propagation on the steep terrain of the shore reef. Compared with the nonlinear shallow water equation, Boussinesq equation has become an indispensable tool in coastal engineering because of its advantages of considering dispersion. Recent studies have shown that numerical models based on Boussinesq equation with shock capture capability have great advantages in simulating the process of breaking wave band [14-17]. That is, when certain judgment conditions are met, Boussinesq equation is degenerated to nonlinear shallow water equation to deal with wave breaking. Tonelli and petti[18] suggest that when the wave surface rises to a water depth ratio of 0.8, it is considered that the wave breaks.

2. Introduction of numerical model

2.1 Governing equation

The governing equations contain conservative forms of nonlinear shallow water equations to capture the shock related hydraulic processes. The Boussinesq equation is re expressed as a conservation equation. The TVD scheme (total variation reduction) is used to reconstruct the flux on both sides of the interface, and the Riemann solver is used to determine the interface flux. In addition, a conservative numerical scheme is used to simulate the wave propagation on the reef.

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$$\boldsymbol{\eta_t} + [(\boldsymbol{h}+\boldsymbol{\eta})\boldsymbol{U}]_x + [\left(\frac{z^2}{2}-\frac{h^2}{6}\right)\mathbf{h}\boldsymbol{U}_{xx} + (\mathrm{z}+\frac{h}{2})\mathbf{h}(\boldsymbol{hU})_{xx}]_x = \mathbf{0} \tag{1}$$

$$\boldsymbol{U_t} + \boldsymbol{UU_x} + \boldsymbol{g\eta_x} + \boldsymbol{z}\left[\frac{z}{2}\boldsymbol{U}_{txx} + (\boldsymbol{hU_t})_{xx}\right] + \boldsymbol{\tau} = \mathbf{0} \tag{2}$$

where the subscripts x and t denote partial derivatives with respect to space and time, g is gravitational acceleration, U is horizontal flow velocity, z is the reference depth, τ is the bottom shear stress in terms of the Manning coefficient.

The choice of evolution variables becomes critical when flow discontinuities develop. We re-derive the Boussinesq-type equations of Nwogu (1993) with the evolution variables H and HU for shockcapturing as in the conservative form of the nonlinear shallow-water equations. Since the bathymetry is stationary, $\boldsymbol{\eta_t}=\boldsymbol{H_t}$. There is Eqs (1) multiplied by u and Eqs (2) multiplied by H. through the chain derivation rule, a new momentum equation (3) is obtained. Eqs (1) and Eqs (3) form a new equation of conservation form.

$$(\boldsymbol{HU})_t + \left(\boldsymbol{HU}^2 + \frac{1}{2}\boldsymbol{gH}^2\right)_x - \boldsymbol{gHh_x} + \boldsymbol{U}\left[\left(\frac{z^2}{2}-\frac{h^2}{6}\right)\mathbf{h}\boldsymbol{U}_{xx} + \left(\mathrm{z}+\frac{h}{2}\right)\mathbf{h}(\boldsymbol{hU})_{xx}\right]_x$$

$$+\boldsymbol{H}(\boldsymbol{z}\left[\frac{z}{2}\boldsymbol{U}_{txx} + (\boldsymbol{hU_t})_{xx}\right] + \boldsymbol{\tau})=0 \tag{3}$$

The resulting set of governing Eqs. (1) and (3) contains the nonlinear shallow-water equations and second-order dispersion terms based on the flow velocity. The present approach can be applied to re-derive other Boussinesq-type equations to describe discontinuous flows.

$$\boldsymbol{Q_t} + \boldsymbol{F_x} = \boldsymbol{S} \tag{4}$$

where Q is the vector of conserved variables, F is the flux vector, and S is the source term. This gives rise to the identical flux term as in the nonlinear shallow-water equations that ensures proper handling of supercritical flows characterized by discontinuities. The individual vectors are

$$\mathbf{Q} = \begin{bmatrix}\boldsymbol{H}\\ \boldsymbol{P}\end{bmatrix};\ \mathbf{F} = \begin{bmatrix}\boldsymbol{HU}\\ \boldsymbol{HU}^2 + \mathbf{1/2}\boldsymbol{gH}^2\end{bmatrix};\ \mathrm{S}=\begin{bmatrix}\boldsymbol{\psi_c}\\ \boldsymbol{gHh_x} + \boldsymbol{U\psi_c} + \boldsymbol{\psi m} - \boldsymbol{H\tau}\end{bmatrix} \tag{5}$$

In which

$$\mathbf{P} = (\boldsymbol{HU}) + \boldsymbol{Hz}(\frac{z}{2}\boldsymbol{U}_{xx} + (\boldsymbol{hU})_{xx}) \tag{6}$$

$$\boldsymbol{\psi m} = \boldsymbol{H_t z}(\frac{z}{2}\boldsymbol{U}_{xx} + (\boldsymbol{hU})_{xx}) \tag{7}$$

$$\boldsymbol{\psi_c} = [\left(\frac{z^2}{2}-\frac{h^2}{6}\right)\mathbf{h}\boldsymbol{U}_{xx} + (\mathrm{z}+\frac{h}{2})\mathbf{h}(\boldsymbol{hU})_{xx}]_x \tag{8}$$

The term P contains all time-derivatives in the momentum Eq. (3) including part of the dispersion term obtained from a chain rule expansion. With $\boldsymbol{H_t}$ explicitly defined by the continuity Eq. (1), the dispersionterms, $\boldsymbol{\psi_c}$ and $\boldsymbol{\psi m}$, only contain spatial derivatives forinclusion in the source term.

2.2 Numerical method

The mixed scheme of finite volume and finite difference is used for spatial discretization. The flux term is calculated by the finite volume method, and the dispersion and left end term are calculated by the second order central difference method. The finite volume method is divided into two steps: first, the left and right variables of the Riemannian problem are constructed by using the appropriate scheme, and then the interface flux is obtained by solving the Riemannian problem. In this paper, the fourth-order muscl-tvd scheme is selected to construct the left and right variables of the interface. In this process, the choice of limiter will have a greater impact on the simulation results. Therefore, the fourth-order muscl scheme using minmod van leer hybrid limiter is presented in this paper, which makes the model more suitable for the steep coral reef terrain. The specific formula of this paper can be written as follows:

$$\emptyset^{\boldsymbol{L}}_{\boldsymbol{i+1/2}} = \emptyset_i + \frac{\mathbf{1}}{\mathbf{4}}[(\mathbf{1}-\boldsymbol{k_1})\boldsymbol{\chi}(\boldsymbol{r})\Delta^*\emptyset_{i+\frac{1}{2}} + (\mathbf{1}+\boldsymbol{k_1})\boldsymbol{\chi}(\mathbf{1}/\boldsymbol{r})\Delta^*\emptyset_{i+\frac{1}{2}}] \tag{9}$$

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$$\emptyset_{i-1/2}^{L} = \emptyset_i - \frac{1}{4}[(1+k_1)\chi(r)\Delta^*\emptyset_{i-\frac{1}{2}} + (1-k_1)\chi(1/r)\Delta^*\emptyset_{i+\frac{1}{2}}] \tag{10}$$

Where, $\emptyset_{i+1/2}^{L}$ is the reconstruction value on the left side of the element interface, and the value of $\Delta^*\emptyset$ is calculated by the following formula:

$$\Delta^*\emptyset_{i+\frac{1}{2}} = \Delta\emptyset_{i+1/2} - k_2\Delta^3\overline{\emptyset}_{i+\frac{1}{2}}/6 \tag{11}$$

$$\Delta\emptyset_{i+1/2} = \emptyset_{i+1} - \emptyset_i \tag{12}$$

$$\Delta^3\overline{\emptyset}_{i+\frac{1}{2}} = \Delta\overline{\emptyset}_{i+3/2} - 2\Delta\overline{\emptyset}_{i+\frac{1}{2}} + \Delta\overline{\emptyset}_{i-\frac{1}{2}} \tag{13}$$

$$\Delta\overline{\emptyset}_{i-1/2} = minmod(\Delta\emptyset_{i-\frac{1}{2}}, \Delta\emptyset_{i+\frac{1}{2}}, \Delta\emptyset_{i+\frac{3}{2}}) \tag{14}$$

$$\Delta\overline{\emptyset}_{i+1/2} = minmod(\Delta\emptyset_{i+\frac{1}{2}}, \Delta\emptyset_{i+\frac{3}{2}}, \Delta\emptyset_{i-\frac{1}{2}}) \tag{15}$$

$$\Delta\overline{\emptyset}_{i+3/2} = minmod(\Delta\emptyset_{i+\frac{3}{2}}, \Delta\emptyset_{i-\frac{1}{2}}, \Delta\emptyset_{i+\frac{1}{2}}) \tag{16}$$

Where minmod represents the minmod limiter function

$$\mathbf{minmod}(\mathrm{j},\mathrm{k},\mathrm{l}) = \mathrm{sign}(\mathrm{j})\max\{0, \min[|j|, 2\mathrm{sign}(\mathrm{j})\mathrm{k}, 2\mathrm{sign}(\mathrm{j})\mathrm{l}]\} \tag{17}$$

Where k_1, k_2 is the control parameter in compact structure, and $\chi(r)$ is the van leer limiting function

$$\chi(r) = \frac{r+|r|}{1+r} \tag{18}$$

Where

$$\mathbf{r} = \frac{\Delta^*\emptyset_{i+1/2}}{\Delta^*\emptyset_{i-1/2}} \tag{19}$$

The numerical flux is calculated by using HLL approximate Riemannian solution

$$\Theta(\psi^L, \psi^L) = \begin{cases} \Theta(\psi^L) & \text{if} \quad s_L \geq 0 \\ \Theta(\psi^R) & \text{if} \quad s_R \leq 0 \\ \Theta^*(\psi^L, \psi^L) & \text{if} \quad s_L < 0 < s_R \end{cases} \tag{20}$$

Where

$$\Theta^*(\psi^L, \psi^L) = \frac{s_R\Theta(\psi^L) - s_L\Theta(\psi^R) + s_Ls_R(\psi^R - \psi^L)}{s_R - s_L} \tag{21}$$

The characteristic wave velocity of Riemannian solution is

$$s_L = \min\left(U^L - \sqrt{gH^L}, U_s - \sqrt{\varphi_s}\right) \tag{22}$$

$$s_R = \min\left(U^R - \sqrt{gH^R}, U_s + \sqrt{\varphi_s}\right) \tag{23}$$

Where

$$U_s = \frac{1}{2}(U^L + U^R) + \sqrt{gH^L} - \sqrt{gH^R} \tag{24}$$

$$\sqrt{\varphi_s} = \frac{1}{2}\left(\sqrt{gH^L} + \sqrt{gH^R}\right) + \frac{1}{4}(U^L - U^R) \tag{25}$$

For the time step, the Runge Kutta method of the third-order strongly stable conservative (SSP) is used. Or the time format adopts the prediction correction method, the prediction step adopts the Adams bashforth scheme of the third-order compact scheme, and the correction step adopts the fourth-order Adams Moulton scheme. The adaptive time step is used to select the Δ t according to the stability condition of CFL.

3. Conclusion

Based on a large number of literatures at home and abroad, the development of main mathematical models of wave propagation and deformation on coral reefs is reviewed. It is not difficult to see that

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the research content of nearshore wave mathematical model is very rich and widely used. There are several conclusions.

a) Most of the physical model tests and numerical simulations conducted by scholars at home and abroad focus on the study of wave propagation and deformation on coral reefs with gentle slope. In fact, steep reefs account for a large proportion of coral reefs, so more and more scholars pay attention to the study of wave propagation and deformation on steep reefs.

b) There are three kinds of breakup models commonly used in Boussinesq wave model: 1. Eddy viscosity breakup model; 2. Mixed breakup model; 3. Water roll model.

c) In terms of numerical scheme, different limiters have different dissipative performance when using the mixed finite volume and finite difference scheme of shock capture for spatial discretization. Therefore, the choice of limiter may affect the accuracy of simulation results, which is also a direction to be studied in the future.

d) There are two indexes to evaluate the simulation effect of wave breaking: 1. Whether the position of wave breaking can be accurately captured; 2. Whether the energy loss caused by wave breaking can be accurately simulated. If you follow the “checklist” your paper will conform to the requirements of the publisher and facilitate a problem-free publication process.

References

[1] Battista, T.A., Costa, B.M., Anderson, S.M., 2007. Shallow-Water Benthic Habitats of theMain Eight Hawaiian Islands (DVD). NOAA Technical Memorandum NOS NCCOS61, Biogeography Branch. Silver Spring, MD.

[2] Z.Y. Sun, D.C. Huang: The progress of the engineering geology of coral reefs [J]. The progress of Geoscience 1999, 3(06):577-81.

[3] Gourlay M R. Wave transformation on a coral reef [J]. Coastal Engineering,1994 ,23(1):17-42.

[4] Gourlay M R. Wave set-up on coral reefs.2.Set-up on reefs with various profiles [J] Coastal Engineering, 1996, 28(1):17-55.

[5] Demirbilek Z, Nwogu O G, Ward D L. Laboratory Study of Wind Effect on Runup over Fringing Reefs. Report 1. Data Report[R]. Engineering Research and Development Center Champaign II Construction Engineering Research Lab. 2007.

[6] Yao Y, Huang Z, Monismith S G , et al. Characteristics of Monochromatic Waves Breaking over Fringing Reefs[J]. Journal of Coastal Research, 2013, 29(1):94-104.

[7] Roeber V,Cheung K F.Boussinesq-type model for energetic breaking waves in fringing reef environments [J].Coastal Engineering, 2012,70 (4):1-20.

[8] MEI Tao, GAO Feng. Flume experiment research on law of wave propagation on reef flat[J].Journal of Waterway andHarbor, 2013,34(1):13-18.

[9] LIU Shuxue, LIU Ning, LI Jinxuan, et al. Experimental researches on propagation characteristics on reefs terrain[J]. The Ocean Engineering,2015,33(2):42-49.

[10] ZHENG Jinhai, YU Haofeng, TAO Aifeng, et al. Research progress in wave Bragg resonance[J]. Advances in Science and Technology of Water Resources,2016,36(3):83-87.

[11] GELFENBAUM G, APOTSOS A, STEVENS A W, et al. Effects of fringing reefs on tsunami inundation: American Samoa[J]. Earth-Science Reviews, 2011,107(1-2): 12-22.

[12] LIANG Q, DU G, HALL J W, et al. Flood inundation modeling with an adaptive quadtree grid shallow water equation solver[J]. Journal of Hydraulic Engineering,2015,134(11): 1603-1610.

[13] ZHANG Zhicheng, WU Wei, LIU Hua, et al. A simplified method for calculation of wave height of trans-ocean tsunami in open ocean[J]. Journal of Hydrodynamics, 2017,32(4): 433-440.

[14] TONELLI M,PETTI M. Hybrid finite volume-finite difference scheme for 2DH improved Boussinesq equations[J]. Coastal Engineering,2009,56: 609-620.

[15] FANG K, ZOU Z, DONG P, et al. An efficient shock capturing algorithm to the extended Boussinesq wave equations[J]. Applied Ocean Research, 2013,43(5):11-20.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202107_7(7).0004

[16] KAZOLEA M, DELIS A I, SYNOLAKIS C E. Numerical treatment of wave breaking on unstructured finite volume approximations for extended Boussinesq-type equations [J]. Journal of Computational Physics, 2014, 271(9):281-305.

[17] SHI F, KIRBY J T, HARRIS J C, et al. A high-order adaptive time-stepping TVD solver for Boussinesq modeling of breaking waves and coastal inundation[J]. Ocean Modelling, 2012,43-44(2):36-51.

[18] Tonelli, M., Petti, M., 2010. Finite volume scheme for the solution of 2D extended Boussinesq equations in the surf zone. Ocean Engineering 37, 567–582.