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Numerical Simulation of Wave-current Interaction on Submerged Dike Terrain

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Abstract

The source wave generation method is a common method to determine the incident boundary conditions when establishing a wave propagation numerical model. This paper improves the mass source wave making method, rewrites the wave mass source term function into the current mass source term function to form a stable background water flow, and uses Boussinesq equation as the governing equation, using MIKE21BW software to establish a numerical simulation model suitable for wave-current interaction. Numerical simulation of wave propagation is carried out in the topography of the submerged embankment. Comparing the calculated results with analytical solutions or physical model experiment results shows that the model can effectively simulate wave propagation and deformation under different background current environments.

Keywords

Mass Source Wave Making Method; Boussinesq Type Equation; Wave-current Interaction; MIKE21BW Model.

1. Introduction

The interaction of waves and currents plays an important role in the changes of the nearshore hydrodynamic environment and the evolution of beaches, and is one of the frontier hot research issues in the coastal engineering community. With the rapid development of modern computer technology, numerical models have become a common method for studying wave propagation and deformation.

One of the difficulties in simulating wave-current interaction in a numerical flume is wave-making. The wave making methods in the numerical flume are mainly the pusher wave making method and the source wave making method. Among them, the source wave making method is divided into mass source wave making method and momentum source wave making method. Brorsen and Larsen [1] proposed a mass source wave-making method based on the theory of potential flow. The wave source can be placed at any position of the wave numerical model, and the purpose of wave simulation can be achieved by controlling the flow in the wave source area. Lin et al. [2] developed and improved the mass source wave making method, and concluded that the wave source set below the water surface is less disturbed by the water surface fluctuation. Therefore, the mass source wave making method is less affected by the reflected wave and can be applied to Numerical simulation of multiple wave types.Wei et al. [3] introduced the mass source wave making method into Boussinesq-type equations, and used one-dimensional and two-dimensional numerical models to simulate single-frequency waves and random waves. Compared with other methods, the mass source wave-making method has the advantage of being less affected by reflected waves, so this method is also suitable for wave

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numerical simulation of complex near-shore terrain [4]. Therefore, this paper will use the mass source wave-making method to carry out the numerical simulation of wave-current interaction.

Boussinesq equation is a powerful tool to describe wave propagation and deformation in coastal water. Boussinesq [5] derived a one-dimensional nonlinear wave equation containing a third-order derivative term suitable for uniform depth water. Peregrine [6] used the perturbation expansion method to derive a two-dimensional Boussinesq-type equation with slowly varying water depth, but it has only weak dispersion and weak nonlinearity. Literature [7-10] and others have improved the Boussinesq-type equation to varying degrees. In terms of wave numerical simulation, MIKE 21 BW model software is widely used today. It can better simulate many physical phenomena in the wave propagation process in port and coastal engineering. The model contains two parts: one-dimensional model and two-dimensional model, both of which are numerically solved based on Boussinesq-type equations. Both one-dimensional and two-dimensional models use flow rate as a variable to solve Boussinesq equations that improve linear dispersion characteristics.Xu Junfeng et al. [11] used the MIKE 21 BW model to analyze the influence of the channel on wave propagation, and Wang Weiyuan et al. [12] used MIKE 21 software to simulate the wave number propagation in the Hangzhou Bay.

The purpose of this paper is to use Boussinesq-type equation as the governing equation and use MIKE 21 BW software to improve the quality source wave making method. Set the wave source as a constant flow water source to realize the numerical simulation of wave propagation under different background currents, compare the wave deformation of wave in downstream and upstream environments, and analyze and discuss the influence of water current on wave propagation and deformation.

2. Mathematical model

2.1 Basic equation

The MIKE 21 BW two-dimensional model adopts the Boussinesq-type equation improved by literature [13] without considering the crushing effect as the governing equation. The governing equation is:

Continuity equation:

$$n\frac{\partial \eta}{\partial t}+\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0 \tag{1}$$

Momentum equation in the x direction:

$$n\frac{\partial P}{\partial t}+\frac{\partial}{\partial x}\left(\frac{P^2}{d}\right)+\frac{\partial}{\partial y}\left(\frac{PQ}{d}\right)+F_x n^2 gd\frac{\partial \eta}{\partial x}+n^2 P\left[\alpha+\beta\frac{\sqrt{P^2+Q^2}}{d}\right]+\frac{gP\sqrt{P^2+Q^2}}{d^2C^2}+n\psi_1=0 \tag{2}$$

Where:

$$\begin{aligned}\psi_1=&-\left(B+\frac{1}{3}\right)h^2\left(P_{xxt}+Q_{xyt}\right)-nBgh^3(\eta_{xxx}+\eta_{xyy})\\&-hh_x\left\{\frac{1}{3}P_{xt}+\frac{1}{6}Q_{yt}+nBgh\left(2\eta_{xx}+\eta_{yy}\right)\right\}-hh_y\left(\frac{1}{6}Q_{xt}+nBgh\eta_{xy}\right)\end{aligned} \tag{3}$$

Momentum equation in the y direction:

$$n\frac{\partial Q}{\partial t}+\frac{\partial}{\partial y}\left(\frac{Q^2}{d}\right)+\frac{\partial}{\partial x}\left(\frac{PQ}{d}\right)+F_y n^2 gd\frac{\partial \eta}{\partial y}+n^2 Q\left[\alpha+\beta\frac{\sqrt{P^2+Q^2}}{d}\right]+\frac{gQ\sqrt{P^2+Q^2}}{d^2C^2}+n\psi_2=0 \tag{4}$$

Where:

$$\begin{aligned}\psi_2=&-\left(B+\frac{1}{3}\right)h^2\left(Q_{yyt}+P_{xyt}\right)-nBgh^3(\eta_{yyy}+\eta_{xxy})\\&-hh_y\left\{\frac{1}{3}Q_{yt}+\frac{1}{6}P_{xt}+nBgh\left(2\eta_{yy}+\eta_{xx}\right)\right\}-hh_x\left(\frac{1}{6}P_{yt}+nBgh\eta_{xy}\right)\end{aligned} \tag{5}$$

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Where P, Q is the velocity (flux) in the x and y directions integrated along the depth in the Cartesian coordinate system; η is the surface height; t is the time;h is the static water depth; g is the gravitational acceleration; n is the porosity; B is the Boussinesq dispersion factor (when B =1/15, it can better reflect the dispersion characteristics [14]); C is the Chezy coefficient; $d=h+\eta$ is the total water depth; α is the resistance coefficient of porous media laminar flow; β is the resistance coefficient of turbulent flow in porous media; The dispersion term ψ_1 and ψ_2 sum adopts the expression of literature [15]; Fx is the horizontal stress in the x direction, and Fy is the horizontal stress in the y direction.

2.2 Mass source wave making method

The mass source wave making method is a wave making method that is only suitable for numerical models. The principle is to add the wave-making source term function to the continuity equation, and realize the generation of various wave patterns through different source term functions. The wave source is set inside the model water area and is divided into point source, line source and finite volume source types. MIKE 21 BW model is suitable for the line source type. Figure 1 is a schematic diagram of the numerical water flume of the mass source wave making method. If you want to simulate the numerical simulation of dual-frequency waves, you need to set up two wave sources before and after the wave numerical flume. Of course, if you want to carry out the numerical simulation of single-frequency waves, you need to set up one wave source. The length of the water tank is set to 10 times the wavelength. The left and right boundaries of the water flume are both set as the solid wall boundary, and the top boundary is set as the pressure inlet.

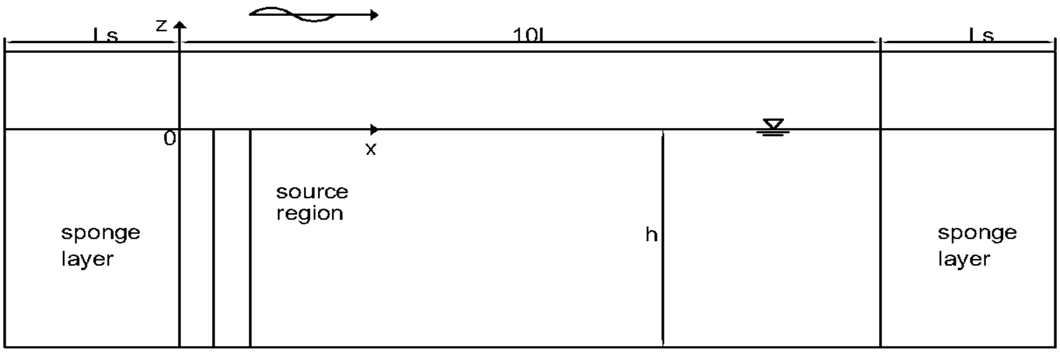

Figure 1. Schematic diagram of wave numerical flume

The wave source expression of the single frequency wave mass source wave making method is [16]:

$$S_m(t)=\frac{\rho c H}{A_\Omega}\cos(\omega t) \tag{6}$$

In the formula, $S_m(t)$ is the quality source term function that changes with time; c is the wave speed; A_Ω is the total area of the grid covered by the wave-making area; H is the wave height; $\rho$is the density; ω is the circular frequency.

The quality source of the MIKE 21 BW model adopts the line source type, and the position of the line source has a certain influence on the wave-making effect, which is discussed in literature [16].

In this paper, the quality source term function of waves is rewritten as the quality source term function of water flow, so as to achieve the purpose of numerical simulation of wave propagation under the background of steady current. The function expression of the flow quality source term is:

$$S_m(t)=\frac{2\rho U_c(t)}{A_\Omega} \tag{7}$$

In the formula, $U_c(t)$ is the flow rate function per unit width, which is related to the flow velocity and the change of water depth at uniform water depth.

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Set the current source term at both ends of the wave numerical flume to simulate a stable background current; then set the wave source term between the two current source terms to establish a numerical model of wave propagation under the background current.

2.3 Wave elimination method

In the wave elimination region, the variable P,Q in the continuity equation and the variable η in the momentum equation are multiplied by the damping coefficient $\upsilon(x)$ in the numerical solution, so as to achieve the wave elimination effect [17].

3. Numerical calculation and verification

3.1 Wave and current interaction on the topography of the submerged embankment

The deformation of waves on the submerged dike is a more complicated problem. After the waves cross the submerged dike, due to the strong nonlinear action, different orders of harmonics are generated. The energy of the wave will be transferred from the main frequency to the high frequency. Therefore, the results of submerged dike terrain experiment are often used to verify the calculation results of the nonlinear wave numerical model.

Literature [18] conducted a physical model experiment of wave propagation on the submerged embankment terrain. Figure 2 is a schematic diagram of the experiment in Literature [18]. This paper performs a numerical simulation of the experimental terrain. The length of the numerical flume is 38.5m, the water depth before and after the submerged dike is 0.5m, the elevation of the top of the submerged dike is 0.25m, the front of the dike is composed of two slopes of 1:20 and 1:60, and the back of the dike is composed of two slopes. It is composed of 1:40 and 1:10 slopes, and wave-eliminating layers are set at both ends of the flume.

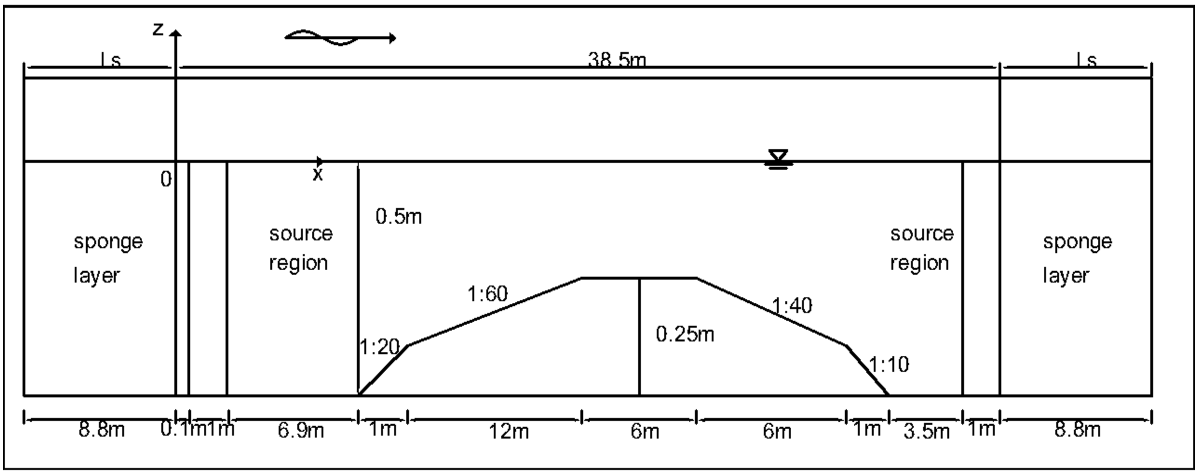

Figure 2. Schematic diagram of topographic wave-current interaction model of submerged embankment

The theoretical value of the velocity distribution on the topography of the submerged dike is:

$$U_c(x)=\frac{u_0 h_0}{h(x)} \tag{8}$$

In the formula, u_0 is the flow velocity at the uniform water depth in front of the submerged dike; h_0 is the water depth in front of the submerged dike; and $h(x)$ is the water depth change function.

3.2 Interaction between single frequency wave and water current

Given the incident wave T=1.1s, a=0.02m, and L=1.78m; set the water flow rate function per unit width in the uniform depth water area in front of the submerged dike as, respectively, assuming that the wave current is in the same direction and the wave current is in the opposite direction. Figure 3

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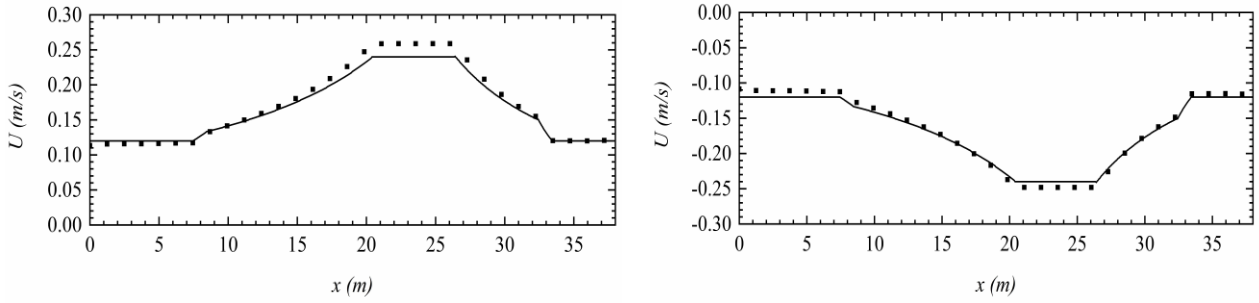
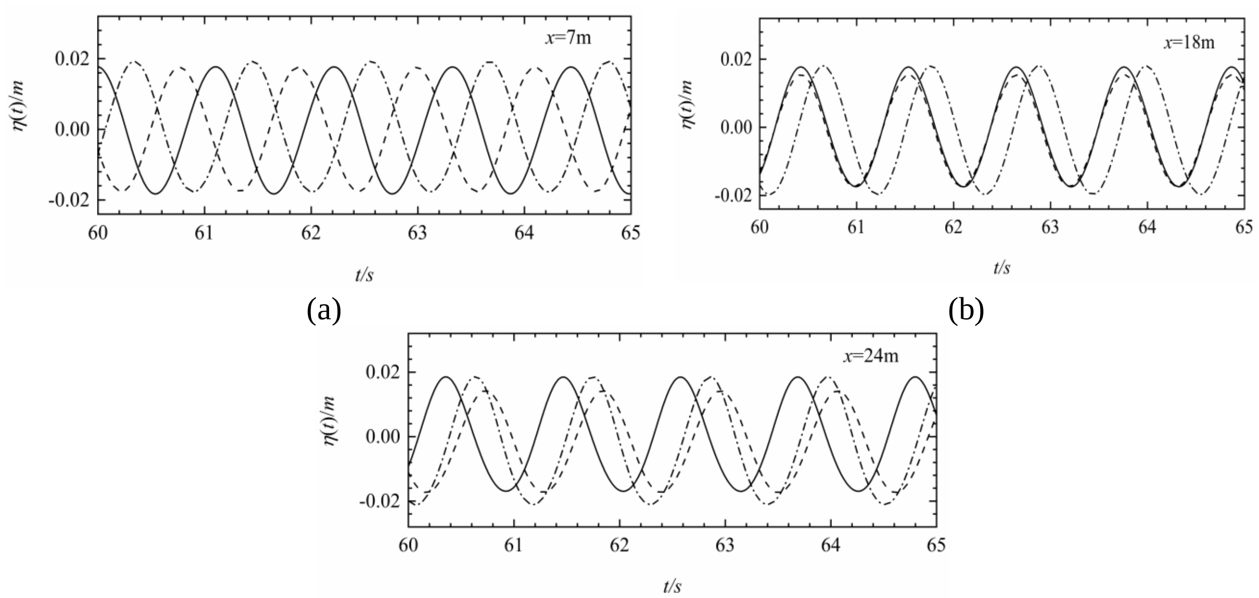
(a) and (b) are the comparison of the velocity distribution value and the theoretical value of the downstream and upstream flow velocity on the topography of the submerged embankment, respectively. It can be seen from the figure that the flow velocity distribution value calculated in this paper is consistent with the theoretical value, which shows that it is feasible to simulate the water flow by setting the flow quality source term in the model.



(a) (b)

Figure 3. The numerical solution of MIKE 21 BW model is compared with the theoretical solution in the velocity distribution of submerged embankment, (a) is downstream, (b) is upstream (▬ :theoretical solution, ■: numerical solution)



(a) (b)

(c)

Figure 4. Comparison of time varying process lines of wave surface at different positions under different background water flow (solid line: no flow, dotted line: downstream, dot dash line: upstream)

Figure 4 shows the wave surface changes with time at three points (x=7m, 18m, 24m) in front of the embankment, in the middle of the embankment, and at the top of the embankment (x=7m, 18m, 24m) under different background water flows. It can be clearly seen from the figure that the topography affects the wave height and wavelength. The wave at the top of the embankment is most affected by the current, and the fluctuation process at different positions is very uniform and stable. It is known from the changing process of waves on the submerged dike that during the climbing process of waves, as the water depth decreases, coupled with the nonlinear effect, the energy of the waves has been transmitted from the main frequency to the high frequencies, so the higher-order harmonics in the waves Will continue to increase. In the upstream environment, the waves become steeper and the peaks and troughs become sharper and higher; in the downstream environment, the waves become

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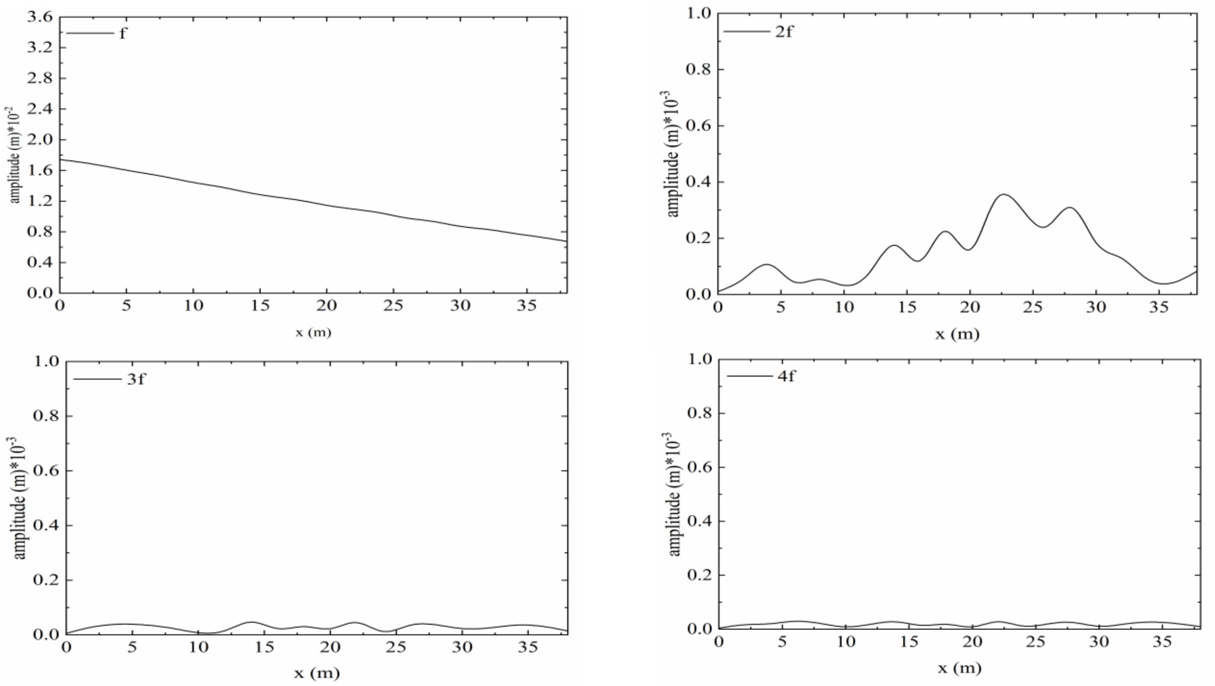
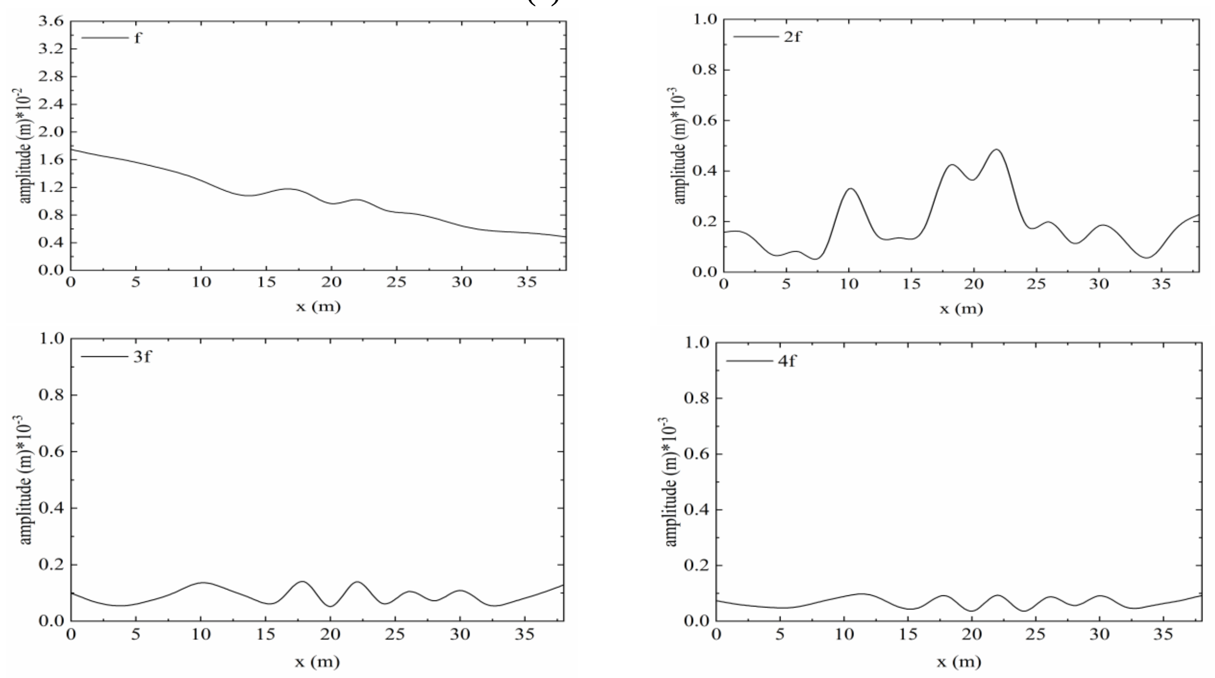
flatter and the peaks and troughs become wider and shallower. Figure 9 shows the Fourier analysis of waves at different locations to better see the changing characteristics of waves when they pass through the submerged dike in a current environment.

Figure 5 shows the amplitude comparison of harmonics generated in different water flow environments. It can be seen that the wave propagates to the submerged embankment to produce harmonics, and the amplitude of the harmonics is also affected by the current. The amplitude of the harmonics is reduced in a downstream environment, and the amplitude of the harmonics is increased in an upstream environment. In a no-current environment, the amplitude of the dominant frequency, that is, the first-order amplitude, is significantly reduced after climbing over the submerged dike, while the amplitude of higher-order harmonics is significantly increased. In a countercurrent environment, this trend is more obvious; in a downstream environment, the opposite is true. The main frequencies of the generated harmonics are shown in Table 1.



(a) no-current



(b) upstream

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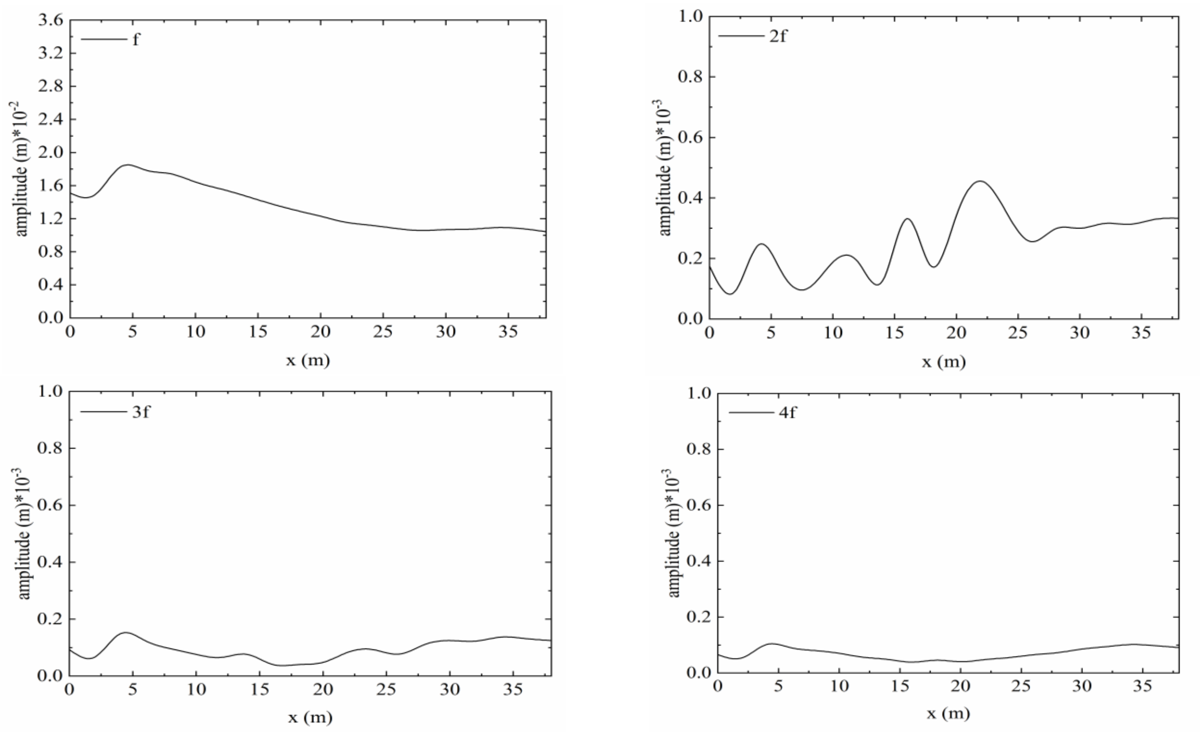

(c) downstream

Figure 5. Comparison of the amplitude of harmonics generated by the MIKE 21 BW model under different water flow backgrounds

Table 1. Harmonic main frequency (unit: Hz)

f	2f	3f	4f
0.90	1.80	2.70	3.60

4. Conclusion

In this paper, the quality source wave-making method is improved, the wave quality source term is improved to the current source term, and a constant current source point is set in the wave numerical model to realize the numerical simulation of waves in a current environment. Using Boussinesq-type equations as the governing equations, the MIKE 21 BW software was used to simulate the wave-current interaction in the topography of the submerged embankment, and to compare and analyze the wave deformation under the action of currents with different directions and velocities. Numerical simulation results show that the amplitude of waves decreases under the action of downstream flow, and the wavelength increases; under the action of countercurrent, the amplitude increases and the wavelength decreases, and the influence of water flow on waves increases with the increase of velocity. The numerical simulation results are reasonable, which proves that the improved mass source wave-making method can effectively simulate the wave propagation deformation under the action of water current.

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