Optimization of ADRC Based on Fin Stabilizer

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Abstract
In this paper, we introduce the mathematical model of wave disturbance and fin stabilizer system, and optimize the expansion state observer (ESO) of active disturbance rejection control (ADRC) controller for the phenomenon that the input signal of the ship fin stabilizer model has a sudden change of derivative that affects the control effect of the system. Simulation of the system by Matlab shows that the optimized fin stabilizer active disturbance rejection controller can improve its state observability of the signal, improve the control effect and reduce the ship transverse rocking.

Keywords
ADRC; Fin Stabilizer; ESO; Matlab.

1. Introduction
In the actual control engineering, the control strategy of "error elimination based on error" is mostly used, the most representative control method is "PID" control, Han Jingqing scholars on the core idea of PID, the use of modern microprocessor digital technology to improve the efficiency of error signal processing developed a new controller with better quality and higher efficiency controller——active disturbance rejection controller[1].

The most prominent feature of the ADRC is that all uncertainties acting on the controlled object are attributed to the "unknown disturbance" and the object input and output data estimate and compensate for it. The expansive state observer observes in real time the unknown disturbances and the real time effects generated by the system model, and converges quickly through the nonlinear state error feedback control law (NLSEF) "small error with large gain, large error with small gain". Among them, the expansive state observer has a large impact on the tracking of the signal to the error feedback, and for the input signal of the ship's fin stabilizer system has the phenomenon of abrupt change in the derivative, the expansive state observer is easy to cause high-frequency chattering due to its non-smooth characteristics, which is not conducive to the practical application of engineering.

This paper firstly introduces the mathematical model of the ship, wave disturbance, and then describes the structure of the ADRC controller and the fin stabilizer system. The dynamic performance of the self-turbulence controller is improved by using the continuous smooth function $qin()$ instead of the $fal()$ function to solve the problem of sudden change of derivatives[2]. The overall analysis is carried out by Matlab to verify the optimization effect.

2. Wave Model and Fin Stabilizer System Model
2.1 Wave Model
In the field of wave research, waves consist of a ternary irregular wave. To facilitate the study, we only study waves of a fixed direction of the ship, whose crest and trough lines are perpendicular to the forward direction and parallel to each other, called "long-peaked waves". In this paper, the long-
peaked random waves are equated to an infinite number of micro-amplitude cosines of different wave amplitudes and wavelengths. We use the following wave amplitude model expression:

$$\zeta(t) = \sum_{i=1}^{\infty} \zeta_{ai} \cos(k_i \xi - \omega_i t + \epsilon_i)$$  \hspace{1cm} (1)

where $\zeta_{ai}$, $k_i$, $\omega_i$ are respectively the amplitude, wave number, and angular frequency of the $i$ times harmonics; $\epsilon_i$ is the random initial phase with uniform distribution between; $\xi$ is the coordinates of a point at sea level; $\zeta(t)$ is the coordinates of the fluctuating water surface at a fixed point and the stationary water surface at the time.

The peak wave equation at $\xi = 0, \eta = 0$ point at sea level:

$$\zeta(t) = \sum_{i=1}^{\infty} \zeta_{ai} \cos(\omega_i t + \epsilon_i)$$  \hspace{1cm} (2)

Nowadays, the descriptive formulas of wave energy spectrum are mainly Newman wave energy spectrum, Pearson Moscovici wave energy spectrum (PM spectrum), ITTC single- and two-parameter wave energy spectrum, and JONSWAP wave energy spectrum[3]. In this paper, the ITTC single-parameter wave energy spectrum recommended by the International Ship Model Test Cell Conference is selected as the simulation simulation spectrum. Its equation as described in:

$$S_{\zeta}(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-\frac{3.11}{h_{1/3}^2 \omega^2}\right)$$  \hspace{1cm} (3)

where $g$ is the gravitational acceleration; $h_{1/3}$ is the significant wave height, ; $S_{\zeta}(\omega)$ is the wave energy spectrum density.

The significant wave height are selected as 2m, 4m, 6m and 7m, and equation (3) is modeled using Matlab. Its wave energy spectrum simulation curve is shown in Fig.1.

From the Fig.1, we can see that the wave spectrum rate is concentrated in $0.25 \leq \omega \leq 1.6$.

The function for the wave inclination angle at a fixed point, as described in:

$$\hat{\alpha}(t) = \sum_{j=1}^{N} \sqrt{2} \int_{\omega_{j-1}}^{\omega_{j}} S_{\zeta}(\omega) d\omega \cos(\hat{\alpha}_j t + \epsilon_j)$$  \hspace{1cm} (4)
where \( S_{\alpha}(\omega) \) is the wave inclination angle spectrum density.

The relationship between the wave inclination angle spectrum and the wave height spectrum, as described in:

\[
S_{\alpha}(\omega) = K_p^2 K_T^2 \frac{\omega^3}{g^2} S_s(\omega)
\]

where \( K_p, K_T \) are the correction coefficient considering the effect of ship's width and ship's draft.

During the navigation of the ship at ocean, the waves actually act on the ship at a certain encounter frequency, and the relationship between the encounter frequency and the natural frequency, as described in:

\[
\omega_e = \omega + \frac{\omega^2}{g} V \cos \mu
\]

where \( V \) is the speed of the ship when sailing; \( \mu \) is the wave angle.

According to the above equation, the encounter frequency wave inclination angle formula, as described in:

\[
\hat{\alpha}_e(t) = \sum_{i=1}^{N} \sqrt{2 \int_{\omega_{\min}^i}^{\omega_{\max}^i} S_{\varepsilon}(\omega) d\omega} \cos(\hat{\omega}_n t + \varepsilon_i) \sin \mu
\]

According to the equal energy discrete sampling method, the sampling frequency is determined to obtain the simulation curve of wave inclination angle when the wave angle is 45°, and the wind speed is 15 m/s as shown in Fig.2.

**Figure 2. The simulation of wave inclination angle**

### 2.2 Ship motion model

The mathematical model of ship's linear transverse rocking motion has higher accuracy, and the ship sails mostly with small angle transverse rocking, and its nonlinear characteristics are not outstanding, so when analyzing the ship's transverse rocking, the linear mathematical model can be used instead of the nonlinear mathematical model[4]. Ship transverse rocking linear, as described in:
\[(I_s + \Delta I_s)\ddot{\phi} + 2N\mu \dot{\phi} + Dh\phi = -(\square I_s \ddot{\alpha}_e + 2N\mu \dot{\alpha}_e + Dh\alpha_e)\]  
\hspace{5cm} (8)

where \(I_s\) is the ship's own moment of inertia; \(\Delta I_s\) is the additional moment of inertia; \(\phi\) is the transverse sway angle; \(N\mu\) is the ship's transverse sway damping moment coefficient; \(h\) is the transverse stability center height; \(D\) is the ship's displacement; \(\alpha_e\) is the effective wave inclination angle.

In the right term of the above equation, it is experimentally proven that the value of \(Dh\alpha_e\) is much greater than the sum of the values of \(\square I_s \ddot{\alpha}_e\) and \(2N\mu \dot{\alpha}_e\). For the sake of analysis, the above equation can be simplified as:

\[(I_s + \Delta I_s)\ddot{\phi} + 2N\mu \dot{\phi} + Dh\phi = -Dh\alpha_e\]  
\hspace{5cm} (9)

Laplace transform of equation (9) yields:

\[G_\phi(s) = \frac{\phi(s)}{\alpha_e} = \frac{1}{T^2_s s^2 + 2T_s s + 1}\]  
\hspace{5cm} (10)

where \(T_\phi = \sqrt{\frac{I_s + \Delta I_s}{Dh}}\) is the inherent period of the ship's transverse rocking motion.

### 2.3 Model of the fin stabilizer system

In order to reduce the wave disturbance to the ship's stability, the fins are used to generate a stabilizing moment that is opposite to the wave disturbance moment. The fin stabilizer system measures the actual ship's transverse velocity through the angular velocity sensor, and then the fin stabilizer controller processes the obtained transverse velocity information and transmits it to the servo system to drive the fin movement to generate the offset torque to achieve the effect of reducing the shaking.

#### 2.3.1 Angular rate sensor

The ship converts the actual angular rate signal of the ship's transverse rocking while sailing into a voltage signal through the angular rate sensor measurement element with the following transfer function:

\[G_o(s) = \frac{400s}{s^2 + 80s + 4000}\]  
\hspace{5cm} (11)

#### 2.3.2 Amplifier

The role of the amplifier is to amplify the output signal of the upper level, the amplification depends on the specific situation, and drive the next level components, the specific transfer function is as follows:

\[G_Q(s) = K_Q\]  
\hspace{5cm} (12)

#### 2.3.3 Speed sensitivity regulator and wave level sensitivity regulator

The speed sensitivity regulator can reduce the sensitivity of the control system to the sea state and ensure the normal operation of the fins in high sea state by adding a wave level sensitivity regulator to the fin system[5].

#### 2.3.4 Servo system

The servo system uses an electro-hydraulic servo system to obtain a larger turning fin torque with the following transfer function.

\[G_e(s) = \frac{550}{s^2 + 15s + 225}\]  
\hspace{5cm} (13)
2.3.5 Fin
The fin angle is converted into a wave inclination angle due to the drive of the servo system. This link can be equated to a proportional link $K_a$ in the simulation.

3. ADRC Controller
The core of the ADRC technique actively extracts disturbance information from the input/output signal of the controlled object before the disturbance obviously affects the final output of the system, and then eliminates it with the control signal as soon as possible. Specifically, TD is used to obtain the differential signal and realize the configuration of the over process, NLESO is used to estimate the state and disturbance information of the system in real time, and NLSEF is used to realize the state feedback of the nonlinear state and disturbance, so that the controlled object full of disturbance and uncertainty is reduced to the standard integral series type, and the active suppression and abatement of disturbance is realized[6].

3.1 ADRC Basic Structure
3.1.1 Tracking Differentiator

$$\begin{cases}
fh = fh(an(x_1 - v, x_2, 2h)) \\
x_1 = x_1 + hx_2 \\
x_2 = x_2 + hfh \\
y = x_1
\end{cases} \quad (14)$$

3.1.2 Nonlinear State Feedback Law

$$u_0 = fh(an(e_1, ce_2, r, h_1)) \quad (15)$$

3.1.3 Expansion State Observer

$$\begin{cases}
\dot{e}_1 = z_4 - y \\
\dot{z}_4 = z_2 - \beta_{01}e_1 \\
\dot{z}_2 = z_3 - \beta_{02}fal(e_1, \frac{1}{2}, \delta) + bu \\
\dot{z}_3 = -\beta_{03}fal(e_1, \frac{1}{4}, \delta)
\end{cases} \quad (16)$$

where $fal(x,a,\delta) = \begin{cases} x \delta^{(1-a)}, |x| \leq \delta \\ sign(x)|x|^\alpha, |x| > \delta \end{cases}$

3.2 ESO Optimization
The ADRC controller is a data-driven algorithm, and the accuracy of the data directly affects its control effect. Among them, the ESO is the core part of the ADRC controller for solving the core problem of perturbation observation[7]. ESO in order to effectively suppress the role of the unknown function, we take the following nonlinear feedback form $g(e,a)$ based on the nonlinear feedback (feedback of the pairwise system) effect:

$$g(e,a) = |e|^\alpha \text{sign}(e) \quad (17)$$

To avoid high-frequency chattering, we transform the function into a continuous power function $fal(x,a,\delta)$ with linear segments near the origin. Further, to solve the problem that the derivative affects the system performance when the $fal(x,a,\delta)$ function is not derivable and $\delta$ takes small values, we adopt the $qin(e,a,\delta)$ function:
\[
\begin{align*}
\begin{cases}
\text{sign}(e)|e|^{a}, |e| > \delta \\
(a-1)\delta^{a-3}e^3 - (a-1)\delta^{a-2}e^2 \text{sign}(e) + \delta^{a-1}e, |e| \leq \delta
\end{cases}
\end{align*}
\]

(18)

Figure 3. \(g, \text{fal}, \text{qin}\) Performance at zero point

The performance of the \(g, \text{fal}, \text{qin}\) function in is shown in the Figure 3. It can be seen from the above graph that the \(\text{qin}\) function is roughly similar to the \(\text{fal}\) function in the field of \(\delta\), but \(\text{qin}\) is continuous and smooth in the field of \(\delta\). The effect of sudden changes in the derivatives of the waves with large transient variations is a problem to be considered in the dilated state observer, and the application of \(\text{qin}\) functions instead of \(\text{fal}\) functions can improve this problem. A new ESO is used for the wave problem as follows:

\[
\begin{align*}
\varepsilon_1 &= z_1 - y \\
\dot{z}_1 &= z_2 - \beta_0z_1 \\
\dot{z}_2 &= z_3 - \beta_0qin(\varepsilon_1, \frac{1}{2}, \delta) + bu \\
\dot{z}_3 &= -\beta_0qin(\varepsilon_1, \frac{1}{4}, \delta)
\end{align*}
\]

(19)

4. Simulation Analysis

Apply \(\text{qin}(e, a, \delta), \text{fal}(x, a, \delta)\) respectively to the ESO and compare their performance.

Figure 4. Performance of \(\text{qin}(e, a, \delta)\) and \(\text{fal}(x, a, \delta)\) function tracking on input
It can be seen from the Figure.4 that the ESO using the \textit{fal} function is not ideal for the observation of the input, on the other hand, the ESO using the \textit{qin} function is better for the observation of the input and can quickly observe its change when the input changes drastically. Further, the performance of the control effect of the optimized ESO is analyzed. The simulation of the fin stabilizer control system is carried out by MATLAB software, which mainly investigates the performance of the ADRC controller for the optimized eESO under some ocean wave conditions. The ocean wave state: the significant wave height is 3m, wind speed is 15kn, and wave angle is 45°; the parameters of the \textit{fal} function based ESO: $\beta_{01} = 100, \beta_{02} = 300, \beta_{03} = 1000, \delta = 0.05$ and the parameters of the \textit{qin} function based ESO: $\beta_{01} = 800, \beta_{02} = 50, \beta_{03} = 50, \delta = 0.05$.

From the above Figure.5, it can be seen that the performance of the controller optimized by the ESO is better than the original controller, with the average wave tilt angle of 0.17 value from zero for the former and 0.32 value from zero for the latter, and a significant performance improvement can be found from the average value.

5. Conclusion

In this paper, we design a ADRC controller for a ship fin stabilizer model by using a \textit{qin} function to optimize the ESO. The continuous smoothness of the qin function can be improved for the case where the parameter $\delta$ of the \textit{fal} function is too low to affect the observation performance due to the sudden change of the derivative of the input signal. Simulations show that the \textit{qin} function-based ESO can improve the tracking performance of the fin stabilizer system and thus improve the dynamic quality of the controller.

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References


