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Kozai-Lidov Mechanism in the Simulations of Hierarchical Three-Body Systems

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Abstract

Simulations of hierarchical three-body systems - a wide variety of astrophysical systems astronomers observed - are done in this work to study Kozai-Lidov Mechanism, a dynamical phenomenon affecting the orbit of a binary system perturbed by a distant third body under certain conditions. The orbit's argument of pericenter oscillate about a constant value, which contributes to a periodic exchange between its eccentricity and inclination. An algorithm is developed to simulate the relative motion of rigid bodies in hierarchical three-body systems through the Law of Universal Gravitation and Newtonian Mechanics. Two arbitrary hierarchical three-body systems are constructed and simulated to obtain the orbital inclinations, eccentricities, mutual inclination, and Kozai-Lidov constant's periodic change over time. Results from the simulations agree and correspond to the original Hamiltonian analysis of Kozai-Lidov mechanism which is discussed in the work to, which confirm Kozai-Lidov Mechanism's successful application on typical three-body gravitational interaction cases.

Keywords

Hierarchical Three-Body System; Kozai-Lidov Mechanism; Simulation, Hamilton Mechanism; Planetary Orbits.

1. Introduction

The three body problem is one of the longest standing problem in physics. Due to the complexity of its chaotic quasi-stable nature, no general analytical solution exists expect in special cases (Poincare 1892). There are mainly two states that a three body system can evolve in: a hierarchical state in which the orbit of a binary system perturbed by a distant third body; or a state where three bodies in approximate energy equipartition rapidly exchanging energy and angular momentum (e.g. Anosova & Orlov 1985; Valtonen & Mikkola 1991). The Kozai–Lidov mechanism focuses on hierarchical three body systems, in which a body called "perturber" far away from the other two orbits their center of mass, compose the outer binary, as the two bodies in the center comprise the inner binary, causing the eccentricity and the inclination to exchange periodically [1]. The effect was first described in 1961 by the Soviet space scientist Mikhail Lidov while analyzing the orbits of artificial and natural satellites of planets [2]. The work is presented at the Conference on General and Applied Problems of Theoretical Astronomy held in Moscow on November 20–25, 1961. In 1962, a Japanese astronomer Yoshihide Kozai published this same result, in application to the orbits of the asteroids

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perturbed by Jupiter [3]. The number of citations of the 1961 and 1962 papers by Lidov and Kozai has risen sharply in the 21st century. As of 2017, the Kozai mechanism is among the most studied astrophysical phenomena. The Kozai-Lidov mechanism has wide applications in interpreting numerous astrophysical systems. For example, it is of great importance in exoplanet configurations and obliquities [4-9]. In addition, it also has key position in the triple evolution that close stellar binaries with two compact objects are likely produced through [6,10-21]. Furthermore, it can also be applied to the growth of black holes at the centers of dense star clusters, and the formation of short-period binary black holes [22-25], and tidal disruption events [26-30].

2. Hamiltonian analysis of kozai-lidov mechanism

In this part, after looking at the introductory information of how Kozai-Lidov mechanism is originated and can applied, we look at it more theoretically in this part from a mathematical and analytical approach to study the mechanism.

In a typical three-body system studied under Kozai-Lidov mechanism, the eccentricity of the secondary has a certain relation with the inclination of the secondaries. To analyze the periodical oscillations of inclination and eccentricity over time, the calculation must involve in large number of variables, making it hard to complete with Newton's mechanism. In this derivation, the Hamiltonian Mechanism is applied to facilitate the derivation.

Let the system composed of the primary and the secondary *inner binary* and let the perturber and the center of mass of the inner binary comprise the *outer binary*.

$$H = H_{in} + H_{out} + H_{perturber} \tag{1}$$

The third term is a coupling, the term showing the interaction. For the coupling term, we apply power series of the fraction of the semi-major axis.

To put further in this equation [31],

$$H = \frac{Gm_1m_2}{2a_1} + \frac{Gm_3(m_1+m_2)}{2a_2} + \frac{G}{a_2}\sum_{j=2}^{\infty} \alpha^j M_j (\frac{r_1}{a_1})^j (\frac{a_2}{r_2})^{j+1} P_j(\cos\phi) \tag{2}$$

Where

m_1 = mass of the primary

m_2 = mass of the secondary

m_3 = mass of the perturber

a_1 = ratio between the semi-major axes of the inner binary system and the outer binary system

r_1 = the vector from the primary to the secondary

r_2 = the vector from the center of mass of the inner binary to the perturber

ϕ = the angle between r_1 and r_2

P_j = Legendre polynomials which allows us to work with Hamiltonian in a spherical coordinate system

Here 'c.m.' denotes the center of mass of the inner binary, containing objects of masses m1 and m2. The separation vector r1 points from m1 to m2; r2 points from 'c.m.' to m3. The angle between the vectors r1 and r2 is Φ.

Given the Hamiltonian of the whole system, we are able to get a constant that doesn't change over time. And back to the Kozai-Lidov mechanism itself, we are able to find the angular momentum of a certain planet which allows us to find the constant which connects the inclination and the eccentricity together.

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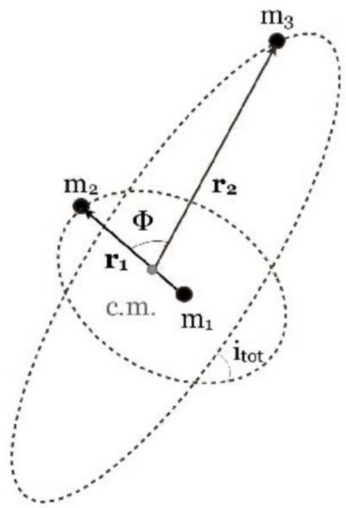

Fig. 1.[4] Coordinate system (not to scale) used to describe the hierarchical triple system.

Given the mass of the planets and their semi-major axis of their orbits, it's not hard to find the conjugate momentum of two systems, which later leads us to discover the angular momentum and the constant we need.

$$L_1 = \frac{m_1 m_2}{m_1+m_2}\sqrt{G(m_1+m_2)a_1} \tag{3}$$

for the conjugate momentum of the inner orbit[4], and

$$L_2 = \frac{m_3(m_1+m_2)}{m_1+m_2+m_3}\sqrt{k^2(m_1+m_2+m_3)a_2} \tag{4}$$

for the conjugate momentum of the outer orbit. Knowing the eccentricity of the orbits, we are able to obtain the angular momentum for each system. We obtain the magnitude of these angular momenta from the conjugate momentum.

$$G_1 = L_1\sqrt{1-{e_1}^2} \tag{5}$$

$$G_1 = L_2\sqrt{1-{e_2}^2} \tag{6}$$

Since the inclination of both orbits are known, we obtain the component of G_1 that is parallel to G_2.

$$G_1 \cos(\mathrm{i1}\ +\ \mathrm{i2}) = L_z \tag{7}$$

$$L_z = \sqrt{(1-{e_1}^2)}\cos i \tag{8}$$

We find the relationship between the inclination of the outer orbit and the eccentricity of the inner orbit.

And also, according to the previous study[32](Merritt 2009), the period which the oscillation happens is

$$T_{\_\{Kozai\}} = \frac{2\pi\sqrt{GM}}{Gm_2}\frac{a_2^3}{a^{\frac{3}{2}}}(1-e_2^2)^{\frac{3}{2}} = \frac{T_2^2}{T}(1-e_2^2)^{\frac{3}{2}} \tag{9}$$

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where $T_{\_\{Kozai\}}$ respectively represents the oscillation period of the either secondary or the perturber. In the following section we verify this result by doing simulation.

3. Simulations of hierarchical three-body systems

3.1 Simulation Objective

The simulation of a hierarchical three-body system is done by using MatplotLib in the Python Interpreter. To model a situation where Kozai-Lidov Mechanism can reasonably apply, this work constructs a planetary system with one star, one planet orbiting the star, and another planet orbiting the system at relatively farther radius (the distant third body perturber).

In the simulation, the planets and stars are treated as point mass with no radius. Also, the inner planet is treated as a test particle with negligible mass compared to the outer perturber. The model systems are constructed so that the star's own motion due to gravitational pull from the two planets is also negligible (ensuring the planetary masses are negligible to the stellar mass). The center of mass of the star is set as the origin of a three dimensional Cartesian coordinate system.

The simulation of the kinematics of the system is completely based on Newtonian mechanics, using the Kepler's Laws and the Law of Universal Gravitation, not considering relativistic effects.

To observe larger Kozai-Lidov effects in relatively short amount of time due to computation power constraints, we invent two special cases of three body systems to use in our simulations. In these two cases, the initial mutual inclination between the inner and outer orbits is set to very large value. In addition, the initial velocity vector is given a small direction tweak by applying coefficients to the components of the vector.

3.2 Simulation Strategy

Firstly, mass and initial orbit radius of the two planets are assigned. We call the inner planet Planet P and the outer planet Planet Q. Initial position vector and initial velocity vectors are then expressed according to the Kepler's law, by

$$\mathrm{r_p} \equiv \text{ Planet P's position vector from the star}$$

$$\mathrm{r_q} \equiv \text{ Planet Q's position vector from the star}$$

$$\mathrm{v_p} = \sqrt{\frac{\mathrm{GM}}{\mathrm{r_p}}} \tag{10}$$

$$\mathrm{v_q} = \sqrt{\frac{\mathrm{GM}}{\mathrm{r_q}}} \tag{11}$$

Where $\mathrm{r_{planet}}$ is the arbitrarily assigned special value for any Planet chosen, G is the universal gravitational constant, and M is the mass of the star.

The three components of the initial velocity vectors of the two planets are then expressed by assigning x-component, y-component, and calculating z-component.

$$\mathrm{v_{px}} = \mathrm{v_p} \cdot \mathrm{k_{px}} \tag{12}$$

$$\mathrm{v_{py}} = \mathrm{v_p} \cdot \mathrm{k_{py}} \tag{13}$$

$$\mathrm{v_{pz}} = \sqrt{\mathrm{v_p} - \mathrm{v_{px}}^2 - \mathrm{v_{py}}^2} \tag{14}$$

$$\mathrm{v_{qx}} = \mathrm{v_q} \cdot \mathrm{k_{qx}} \tag{15}$$

$$\mathrm{v_{qy}} = \mathrm{v_q} \cdot \mathrm{k_{qy}} \tag{16}$$

$$\mathrm{v_{qz}} = \sqrt{\mathrm{v_q} - \mathrm{v_{qx}}^2 - \mathrm{v_{qy}}^2} \tag{17}$$

Where $\mathrm{k_{*x}}$ and $\mathrm{k_{*y}}$ are coefficients are manually assigned special value for any Planet chosen.

After the initial values are assigned, we can obtain the following variables:

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$$m_p \equiv \text{ Planet P's mass}$$
$$m_q \equiv \text{ Planet Q's mass}$$
$$r_p \equiv \text{ Planet P's position vector from the star}$$
$$r_q \equiv \text{ Planet Q's position vector from the star}$$
$$r_{pq} \equiv \text{ Planet Q's position vector from Planet P}$$
$$v_p \equiv \text{ Planet P's velocity}$$
$$v_q \equiv \text{ Planet Q's velocity}$$
$$x_p \equiv \text{ x component of Planet P's position vector from the star}$$
$$y_p \equiv \text{ y component of Planet P's position vector from the star}$$
$$z_p \equiv \text{ z component of Planet P's position vector from the star}$$
$$x_q \equiv \text{ x component of Planet Q's position vector from the star}$$
$$y_q \equiv \text{ y component of Planet Q's position vector from the star}$$
$$z_q \equiv \text{ z component of Planet Q's position vector from the star}$$
$$v_{px} \equiv \text{ x component of Planet P's velocity}$$
$$v_{py} \equiv \text{ y component of Planet P's velocity}$$
$$v_{pz} \equiv \text{ z component of Planet P's velocity}$$
$$v_{qx} \equiv \text{ x component of Planet Q's velocity}$$
$$v_{qy} \equiv \text{ y component of Planet Q's velocity}$$
$$v_{qz} \equiv \text{ z component of Planet Q's velocity}$$

From these variables, we can further express the acceleration vectors of Planet P and Planet Q, and their respective x, y, and z components.

The accelerations of Planet P due to the star and Planet Q are:

$$a_{p-s} \equiv \text{ acceleration of Planet P due to the star } = \frac{GM}{{r_p}^2} \tag{18}$$

$$a_{p-q} \equiv \text{ acceleration of Planet P due to Planet Q } = \frac{Gm_q}{{r_{pq}}^2} \tag{19}$$

Combined, we can get

$$a_{px} \equiv \text{ x component of Planet P's acceleration} = -\frac{a_{p-s}x_p}{r_p} - \frac{a_{p-q}(x_p - x_q)}{r_{pq}} \tag{20}$$

$$a_{py} \equiv \text{ y component of Planet P's acceleration} = -\frac{a_{p-s}y_p}{r_p} - \frac{a_{p-q}(y_p - y_q)}{r_{pq}} \tag{21}$$

$$a_{pz} \equiv \text{ z component of Planet P's acceleration} = -\frac{a_{p-s}z_p}{r_p} - \frac{a_{p-q}(z_p - z_q)}{r_{pq}} \tag{22}$$

The accelerations of Planet Q due to the star and Planet P are:

$$a_{q-s} \equiv \text{ acceleration of Planet P due to the star } = \frac{GM}{{r_q}^2} \tag{23}$$

$$a_{q-p} \equiv \text{ acceleration of Planet P due to Planet Q } = \frac{Gm_p}{{r_{pq}}^2} \tag{24}$$

Combined, we can get

$$a_{qx} \equiv \text{ x component of Planet Q's acceleration} = -\frac{a_{q-s}x_q}{r_q} - \frac{a_{q-p}(x_q - x_p)}{r_{pq}} \tag{25}$$

$$a_{qy} \equiv \text{ y component of Planet Q's acceleration} = -\frac{a_{q-s}y_q}{r_q} - \frac{a_{q-p}(y_q - y_p)}{r_{pq}} \tag{26}$$

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$$a_{qz} \equiv \text{ z component of Planet Q's acceleration} = -\frac{a_{q-s}z_q}{r_q} - \frac{a_{q-p}(z_q - z_p)}{r_{pq}} \tag{27}$$

From these variables, we can also express the specific angular momentum and the total mechanical energy of the two planets:

$$h_p \equiv \text{ Planet P's specific angular momentum } = \begin{vmatrix} i & j & k \\ x_p & y_p & z_p \\ v_{px} & v_{py} & v_{pz} \end{vmatrix}$$

$$= \sqrt{(y_p v_{pz} - v_{py} z_p)^2 + (z_p v_{px} - v_{pz} x_p)^2 + (x_p v_{py} - v_{px} y_p)^2} \tag{28}$$

$$h_q \equiv \text{ Planet Q's specific angular momentum} = \begin{vmatrix} i & j & k \\ x_q & y_q & z_q \\ v_{qx} & v_{qy} & v_{qz} \end{vmatrix}$$

$$= \sqrt{(y_q v_{qz} - v_{qy} z_q)^2 + (z_q v_{qx} - v_{qz} x_q)^2 + (x_q v_{qy} - v_{qx} y_q)^2} \tag{29}$$

$$E_p \equiv \text{ Planet P's total mechanical energy} = \frac{1}{2} m_p {v_p}^2 - \frac{GMm_p}{r_p} \tag{30}$$

$$E_q \equiv \text{ Planet Q's total mechanical energy} = \frac{1}{2} m_q {v_q}^2 - \frac{GMm_q}{r_q} \tag{31}$$

Where i, j, and k are unit vectors in x, y, and z directions, respectively.

From these newly expressed variables, we can eventually express the inclination and eccentricity of the two planets:

$$i_p \equiv \text{ Planet P's instantaneous orbit inclination to X} - \text{Y Plane}$$

$$= \tan^{-1}\left(\frac{z_p}{\sqrt{{x_p}^2 + {y_p}^2}}\right) \cdot \frac{180°}{\pi} \tag{32}$$

$$i_q \equiv \text{ Planet Q's instantaneous orbit inclination to X} - \text{Y Plane}$$

$$= \tan^{-1}\left(\frac{z_q}{\sqrt{{x_q}^2 + {y_q}^2}}\right) \cdot \frac{180°}{\pi} \tag{33}$$

$$e_p \equiv \text{ Planet P's instantaneous orbit eccentricty} = \sqrt{1 + \frac{2E_p}{m_p} + \left(\frac{h_p}{GM}\right)^2} \tag{34}$$

$$e_q \equiv \text{ Planet Q's instantaneous orbit eccentricty} = \sqrt{1 + \frac{2E_q}{m_q} + \left(\frac{h_q}{GM}\right)^2} \tag{35}$$

To achieve our purpose, we must include a graph that shows how the instantaneous eccentricity of the inner planet and the inclination between the two orbiting disks of the planets are related to a periodically constant value.

$$i \equiv \text{ Inclination angle between the two planetary orbits } = |i_p - i_q| \tag{36}$$

$$e \equiv \text{ Eccentricity of the inner Planet P } = e_p \tag{37}$$

$$L_z = \sqrt{1 - e^2} \cdot \cos(i) \tag{38}$$

Where L_z is the periodic constant value that relates i and e by equation (8) in the part 2.

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3.3 Programming Method

Next, we move on to the simulation process. Firstly, we define a time indicator to express an instantaneous moment of the simulation.

$$\mathrm{t} \equiv \text{ Time Indicator}$$

For certain time interval $[0, t_{limit}]$, we set each day as one iteration step. In each iteration step, we update the variables according to the equations defined above in the order of:

Accelerations of the two planets

Velocities of the two planets

Positions of the two planets

Specific Angular Momentum of the two planets

Total Mechanical Energy of the two planets

Instantaneous Inclinations of the orbit of the two planets

Instantaneous Eccentricities of the orbit of the two planets

$\mathrm{L_z}$ of the Kozai-Lidov system

Time Indicator

At the end of each iteration step, we append the Time Indicator, t, Instantaneous Inclination of the orbit of Planet P and Planet Q, ip and iq, and Instantaneous Eccentricity of the orbit of Planet P and Planet Q, ep and eq, each to a separate Python List for the two planets. $\mathrm{L_z}$ is also appended to a Python List. After the entire iteration loop ends, we input ip, iq, ep, eq, $\mathrm{L_z}$ and t Python Lists to MatplotLib methods, which plot the graphs of inclination versus time and eccentricity versus time of the two planets and $\mathrm{L_z}$ versus time.

Since $\mathrm{L_z}$'s periodic nature, we also need to determine the average value of $\mathrm{L_z}$ in the simulation process, by:

$$\mathrm{L_{z\_average}} = \frac{\sum_{\mathrm{t}=0}^{t_{limit}} \mathrm{L_z}}{t_{limit}} \tag{39}$$

3.4 Models used in Simulation

Two models of hierarchical three-body systems are invented to use in the simulation program.

First Model: System X includes a star with the mass of the Sun, an inner planet with the mass of Earth, initially orbiting at distance of semi-major axis of Earth from the Sun, and an outer planet with the mass of Jupiter, initially orbiting at distance of semi-major axis of Jupiter from the Sun.

With all the previous variables defined, we assign the following initial values to start System X at t = 0:

$$\mathrm{m_p} = 5.97 \times 10^{24}\mathrm{kg} \quad \mathrm{r_p} = 1.000\ \mathrm{AU} \quad \mathrm{k_{px}} = 0 \quad \mathrm{k_{py}} = 0.99$$
$$\mathrm{m_q} = 1.90 \times 10^{27}\mathrm{kg} \quad \mathrm{r_q} = 5.202\ \mathrm{AU} \quad \mathrm{k_{qx}} = 0 \quad \mathrm{k_{qy}} = 0.98$$
$$\mathrm{x_p} = 0.98475\mathrm{r_p} \quad \mathrm{y_p} = 0.01000\mathrm{r_p} \quad \mathrm{z_p} = 0.17368\mathrm{r_p}$$
$$\mathrm{x_q} = 0.98475\mathrm{r_q} \quad \mathrm{y_q} = 0.01000\mathrm{r_q} \quad \mathrm{z_q} = -0.17368\mathrm{r_q}$$
$$\mathrm{i_p} = 10° \quad \mathrm{i_q} = -10° \quad \mathrm{i_{tot}} = 20°$$

Second Model: System Y includes a star with the mass of the Sun, an inner planet with the mass of Earth, initially orbiting at distance of semi-major axis of Earth from the Sun, and an outer planet with the mass of Saturn, initially orbiting at distance of semi-major axis of Saturn from the Sun.

And we assign the following initial values to start System Y at t=0:

$$\mathrm{m_p} = 5.97 \times 10^{24}\mathrm{kg} \quad \mathrm{r_p} = 1.000\ \mathrm{AU} \quad \mathrm{k_{px}} = 0 \quad \mathrm{k_{py}} = 0.99$$
$$\mathrm{m_q} = 5.68 \times 10^{26}\mathrm{kg} \quad \mathrm{r_q} = 9.582\ \mathrm{AU} \quad \mathrm{k_{qx}} = 0 \quad \mathrm{k_{qy}} = 0.98$$

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$$x_p = 0.98475r_p \quad y_p = 0.01000r_p \quad z_p = 0.17368r_p$$
$$x_q = 0.98475r_q \quad y_q = 0.01000r_q \quad z_q = -0.17368r_q$$
$$i_p = 10° \quad i_q = -10° \quad i_{tot} = 20°$$

Notice that we set the initial mutual inclination very large in both models so that Kozai-Lidov effects could be observed in relatively short amount of time. And the mass of the inner planet (secondary) is much less than the outer perturber.

Simulations of the System X and Y are done in time interval of 50 years and 80 years respectively, with one year equivalent to 365 days.

4. Results

We obtain four graphs from System X and System Y respectively. In the graphs, blue line represents planet P's property and red line represents that of planet Q. Fig. 2. graph shows the inclination between the planetary orbit of planet P and the X-Y plane versus time and the inclination between the planetary orbit of planet Q and the X-Y plane versus time. Fig. 3. shows the eccentricity of the planetary orbits of planet P and planet Q versus time, respectively. Fig. 4. shows the mutual inclination between the two planetary orbits, defined in equation (36). Fig. 5. shows the Kozai-Lidov constant, which is defined in equation (28), versus time. The blue dashed line shows the average Lz over the trial time length.

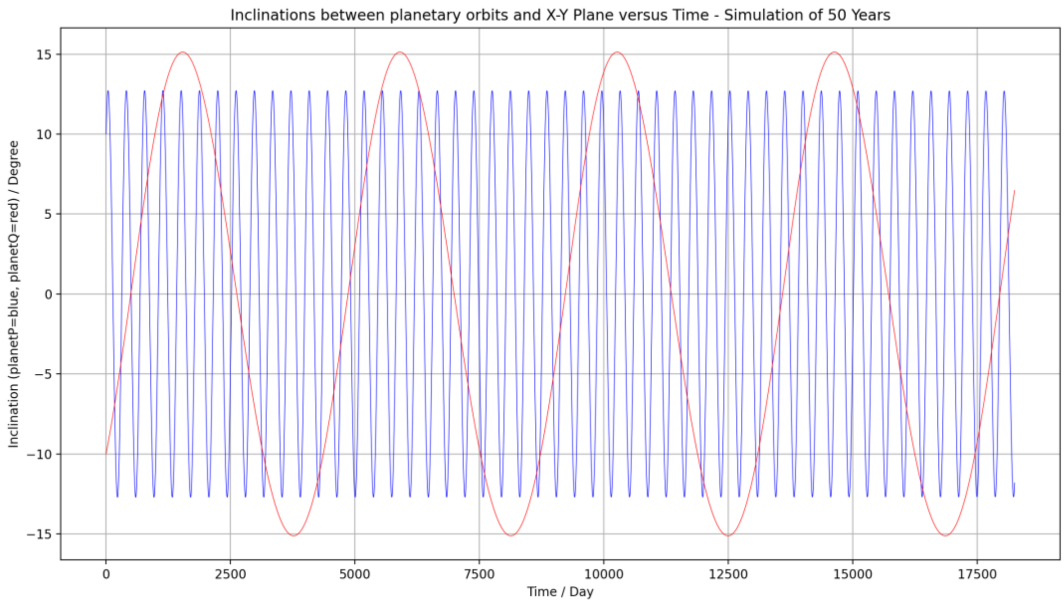

Fig 2. Results of simulation on system X in a 50 year time interval.

System X is a three-body system with arbitrarily chosen primary of mass equals 1M☉, secondary of mass 1MEarth, and perturber of mass 1MJupiter. The inner orbit has a=1AU and the outer orbit has a=5.202AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. This graph shows the inclination between the planetary orbit of planet P and the X-Y plane versus time and the inclination between the planetary orbit of planet Q and the X-Y plane versus time.

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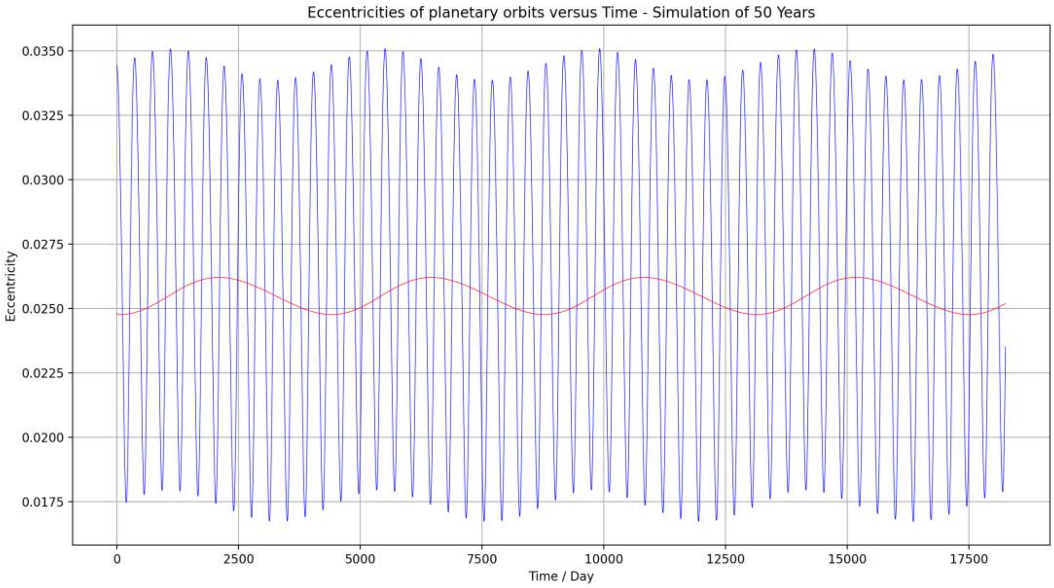

Fig 3. Results of simulation on system X in a 50 year time interval.

System X is a three-body system with arbitrarily chosen primary of mass equals 1M☉, secondary of mass 1MEarth, and perturber of mass 1MJupiter. The inner orbit has a=1AU and the outer orbit has a=5.202AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. This graph shows the eccentricity of the planetary orbits of planet P and planet Q versus time, respectively.

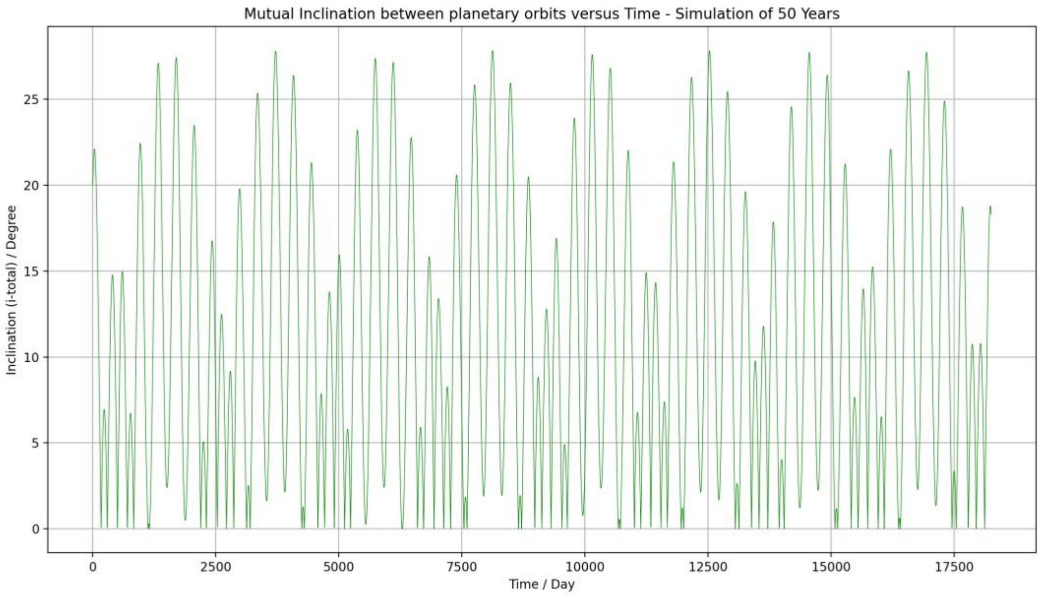

Fig 4. Results of simulation on system X in a 50 year time interval.

System X is a three-body system with arbitrarily chosen primary of mass equals 1M☉, secondary of mass 1MEarth, and perturber of mass 1MJupiter. The inner orbit has a=1AU and the outer orbit has a=5.202AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. This graph shows the mutual inclination between the two planetary orbits, defined in equation (36).

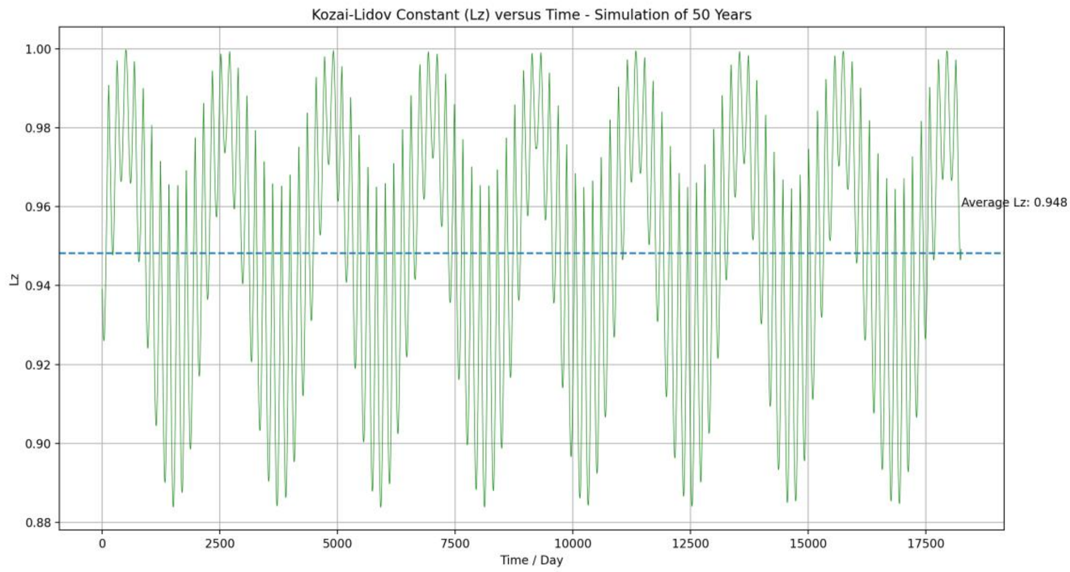

Fig 5. Results of simulation on system X in a 50 year time interval.

System X is a three-body system with arbitrarily chosen primary of mass equals 1M⊙, secondary of mass 1MEarth, and perturber of mass 1MJupiter. The inner orbit has a=1AU and the outer orbit has a=5.202AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. This graph shows the Kozai-Lidov constant, which is defined in equation (28), versus time. The blue dashed line shows the average Lz over the trial time length.

We obtain four graphs from the System Y simulation too.

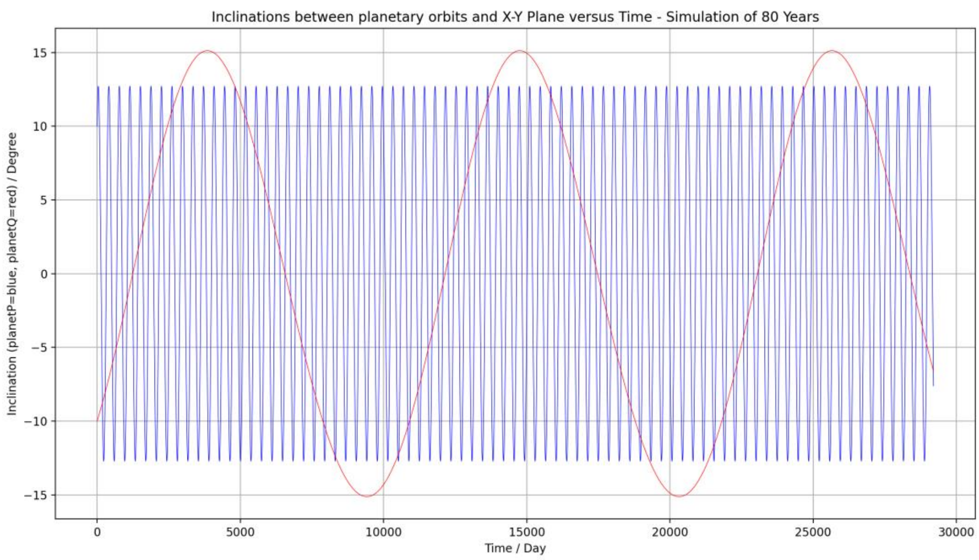

Fig 6. Results of simulation on system Y in a 80 year trial.

System Y is a three-body system with arbitrarily chosen primary of mass equals 1M⊙, secondary of mass 1MEarth, and perturber of mass 1MSaturn. The inner orbit has a=1AU and the outer orbit has a=9.582AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. The graph shows the inclination between the planetary orbit of planet P and the X-Y plane versus time and the inclination between the planetary orbit of planet Q and the X-Y plane versus time.

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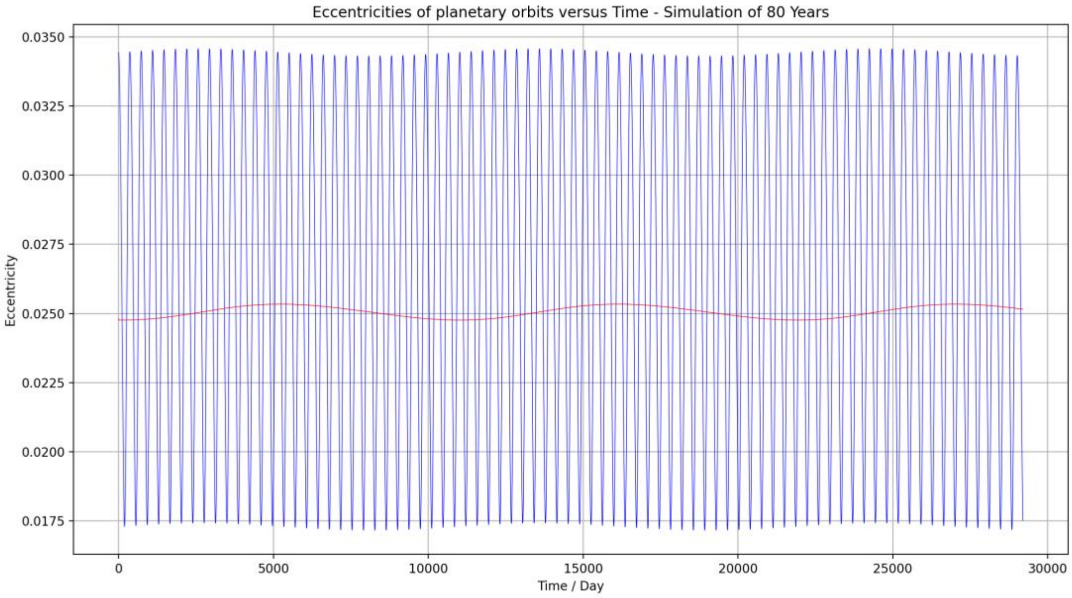

Fig 7. Results of simulation on system Y in a 80 year trial.

System Y is a three-body system with arbitrarily chosen primary of mass equals 1M⊙, secondary of mass 1MEarth, and perturber of mass 1MSaturn. The inner orbit has a=1AU and the outer orbit has a=9.582AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. The graph shows the eccentricity of the planetary orbits of planet P and planet Q versus time, respectively.

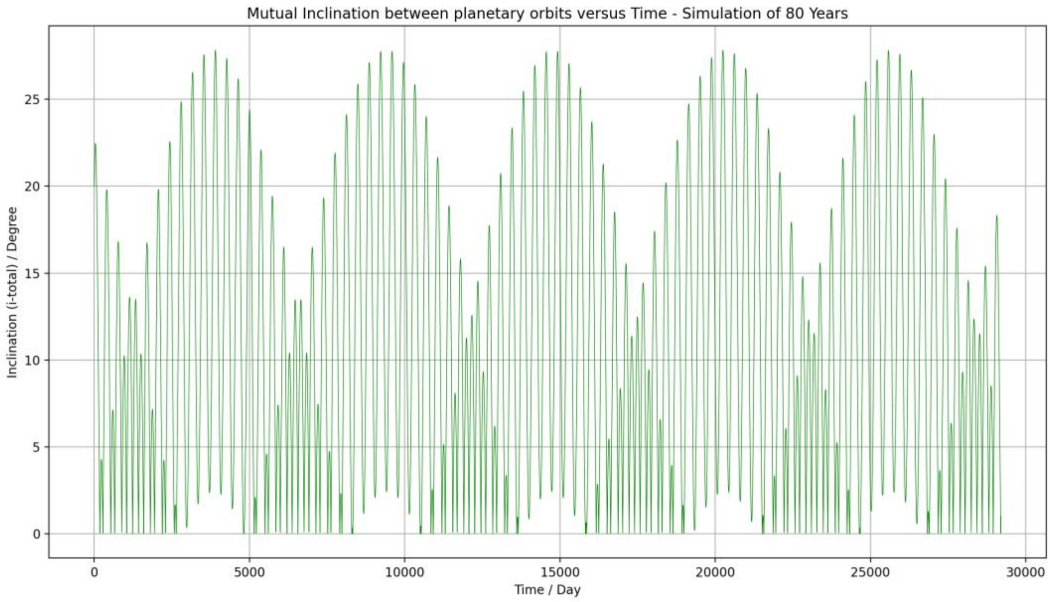

Fig 8. Results of simulation on system Y in a 80 year trial.

System Y is a three-body system with arbitrarily chosen primary of mass equals 1M⊙, secondary of mass 1MEarth, and perturber of mass 1MSaturn. The inner orbit has a=1AU and the outer orbit has a=9.582AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. The graph shows the mutual inclination between the two planetary orbits, defined in equation (36).

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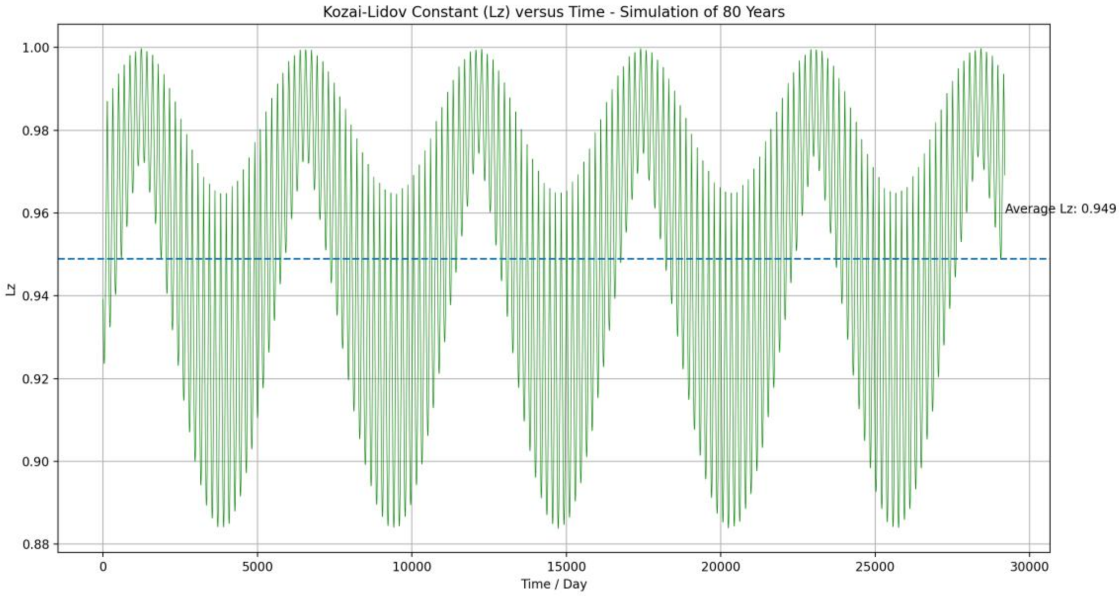

Fig 9. Results of simulation on system Y in a 80 year trial.

System Y is a three-body system with arbitrarily chosen primary of mass equals 1M⊙, secondary of mass 1MEarth, and perturber of mass 1MSaturn. The inner orbit has a=1AU and the outer orbit has a=9.582AU. The initial mutual inclination between the two orbits is 20 degrees. The initial eccentricity of inner orbit e1= 0.03 and that of the outer orbit e2= 0.05. The graph shows the Kozai-Lidov constant, which is defined in equation (28), versus time. The blue dashed line shows the average Lz over the trial time length.

5. Analysis

From the part 4, we obtained four graphs from each simulation model (system X and system Y), each graph containing inclination, eccentricity, mutual inclination, and L_z over time. In analysis section, the aim is to use our simulated results to evaluate and prove Kozai-Lidov Mechanism's effectiveness in the two constructed models.

From equation(8) in part 2:

$$L_z = \sqrt{1 - e_1{}^2} \cdot \cos(i)$$

We know the orbit-averaged equations of motion for the secondary has a conserved quantity: L_z.

For the System X used in simulation part, we graphed L_z over time, and calculated the average value of L_z with respect to time span of 50 years, because the revolution period of Jupiter is 11.8618 years, a relatively long time period.

For the System Y used in simulation part, we graphed L_z over time, and calculated the average value of L_z with respect to time span of 80 years, because the revolution period of Saturn is 29.67 years, a relatively long time period.

Average value of L_z in the System X are approximately all equal to 0.95, which attests the equation(8).

Average value of L_z in the System Y are approximately all equal to 0.95, which attests the equation(8).

And thus proves the Kozai-Lidov Mechanism's effectiveness in relating the inclination and the eccentricity in hierarchical three-body systems.

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6. Conclusion

The result obtained from above simulation of two constructed three-body systems using gravitational force between planet and star generally matches the equation of Kozai-Lidov Mechanism, showing that the step-by-step simulation of two typical hierarchical three-body gravitational interaction cases invented in this work verified the interrelation of the orbit's inclination and eccentricity. The analysis part shows that the change of inclination and eccentricity with respect to time of the particular system follows the relationship predicted by the Kozai-Lidov equations. This mechanism is useful in explaining peculiar inclination or eccentricity of exoplanets in multi-star system and explaining the existence of Double Blackholes or neutron-stars merging: under the third body's perturbation, the orbit eccentricity of the two-body increases, which results in a decrease of energy at the perigee. Besides, high eccentricities under Kozai-Lidov effects can result in tidal evolution leading to tight inner binary. Switching to a different setting, this can result in mergers, collisions, tidal disruption events, and supernovas. [33] This mechanism can be applied to the real world for helping modify the orbit of earth's satellite if it is perturbed by the moon.

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