

# Study on the Relationship between Resistance and Capacitance Parameters in Chua's Chaotic Circuit

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## Abstract

To see how the values of resistance and capacities in Chua's Chaotic Circuit determine the periodic bifurcation. By studying the relationship between capacitance value and resistance value when Chua's chaotic circuit produces period-doubling bifurcation and chaos, two capacitance values corresponding to the maximum change of R from 1P to chaos are found.

## Keywords

Chua's Chaotic Circuit, periodic bifurcation, circuit parameter.

## 1. Experimental Background

In the middle of the 20th century, the study of physics was no longer limited to simple linear systems but went deep into complex nonlinear systems. Complex nonlinear systems and their phenomena can be seen in the probability distribution of macroscopic quantum, astrophysics, earth sciences, life sciences, and human sociology. In the study of these phenomena, many Chaos and periodic bifurcation phenomena have been observed. [1], [2] The study of chaos in nonlinear systems is of great help to the development of science, especially physics. Intrinsic randomness is the essence of chaos, which leads to high sensitivity of nonlinear systems to initial values and some parameter values, and long-term unpredictability of system state. But this randomness and unpredictability are not undetectable, it also has certain rules. [3] Various chaos models have been proposed to study some properties of chaos and period-doubling bifurcation. Among them, the Chua's circuit was designed by Professor Chua (as shown in figure 1), it is a kind of can produce chaos and period-doubling bifurcation phenomenon of the circuit, the circuit for the third order RLC circuit, is composed of four linear elements (two capacitors and a resistor, an inductor) and a nonlinear resistor, including nonlinear resistance can be composed of two operational amplifiers. [3] Chua's circuit is simple in structure but has rich and complex chaotic dynamics characteristics. [4] It is often used in all kinds of chaos research and has great practical application value, such as in the application of secure communication. At present, how to control its period-doubling bifurcation and chaos has become an important problem.

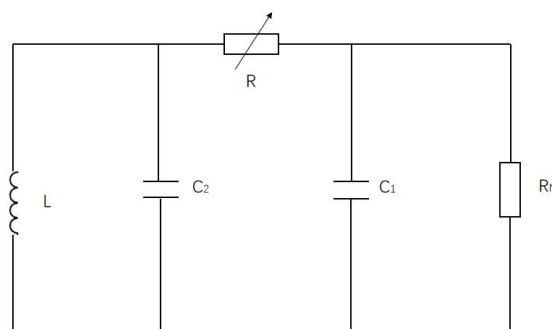


Figure 1. Schematic diagram of Chua's circuit

The nonlinear negative resistance RN in Figure 1 has the following concepts: In general, the resistance acts as a load, and the voltage at both ends of it is proportional to the current passing through it. The slope of its volt-ampere characteristic curve is positive, which is usually called positive resistance. The resistance is negative if the voltage at both ends of the resistance is inversely proportional to the current flowing through it, and the slope is negative. There is no natural negative resistance element in nature, only resistance in the circuit and current through the negative resistance will occur. Negative resistance means that the resistance plays a role in the power supply in the circuit, which can be regarded as the power supply in the circuit. [5] As shown in Figure 2, it can be composed of a dual operational amplifier connected to the power supply and a resistor.

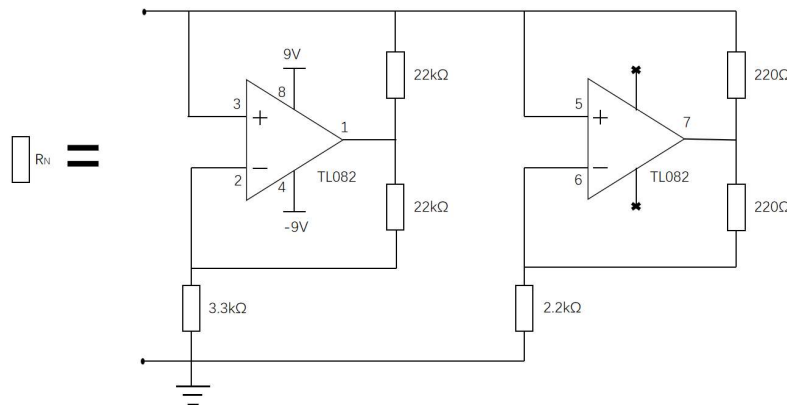


Figure 2. Schematic diagram of nonlinear negative resistance

## 2. Research Objectives

Control variable method is used to explore the chaotic circuit, to Mr. Li as waveform figure 1 cycle track formation, just appeared 2 periodic bifurcations and chaos of capacitance and resistance, the relationship between these two parameters through the control of one of the capacitance values, study another value of the capacitance and resistance tolerance of the corresponding optical system, analysis the data, get chaotic circuit, The relationship between capacitance parameters and resistance parameters in the generation of period-doubling bifurcation and chaos. Look for the two capacitance values corresponding to the maximum change in R between a single period and chaos. By determining the capacitance value, the chaotic circuit can enhance the ability to regulate periodic bifurcation by changing the variable resistance.

## 3. Experimental principle

According to the knowledge of chaos, for some nonlinear iterative formulas, the iterative results in the middle of them are random. However, the limit value of iteration has the following law. In general, parameters in a large range, the results of the iteration will lead to chaos. For example, large parameters tend to make the iteration chaotic rather than a limit value. However, when this parameter is small, the limit of the iteration is equal to 0. When the limit of iteration is 0 and chaos is involved, there is a period-doubling bifurcation. It's also called a limit bifurcation. Take period 2, which is the two limits, by which I mean that  $x_k, x_{k+2}, x_{k+4}, \dots, x_{k+2b}$  ( $b \in \mathbb{Z}$ ) and  $x_{k+1}, x_{k+3}, x_{k+5}, \dots, x_{k+2b+1}$  ( $b \in \mathbb{Z}$ ) tend to different limits when K is very large.  $\lim_{k \rightarrow \infty} x_{k+2b} \neq \lim_{k \rightarrow \infty} x_{k+2b+1}$ . The iteration limit is 0, and the period-doubling bifurcation or chaos is completely determined by the parameters of the iteration. For a particular parameter, when it is strictly increasing or decreasing, the limit of the iterative expression will appear as 0  $\rightarrow$  single periods (1P)  $\rightarrow$  2 periods (2P)  $\rightarrow$  4 periods (4P)  $\rightarrow \dots \rightarrow$  2N cycle (2nP)  $\rightarrow \dots$  And chaos.

For Chua's chaotic circuit, Kirchhoff's law is used to analyze the relation of current in the circuit, and the relation equation of each electrical quantity is obtained as follows.

$$C_1 \frac{dV_{C_1}}{dt} = \frac{1}{R} (V_{C_2} - V_{C_1}) - F(V_{C_1})$$
$$C_2 \frac{dV_{C_2}}{dt} = \frac{1}{R} (V_{C_1} - V_{C_2}) + I_L$$
$$V_{C_2} = L \frac{dI_L}{dt}$$

Where  $V_{C_1}$ ,  $V_{C_2}$  and are the potential difference between capacitors C1 and C2 respectively, and  $I_L$  is the current through inductor L.

The size of C1, C2, L and R are the parameters that determine the limits of  $V_{C_1}$ ,  $V_{C_2}$  and  $I_L$  in the iterative equation above, so chaos can be controlled by changing C1, C2 and R. Since the circuit is a closed loop, equivalent to an infinite iterative cycle,  $V_{C_1}$ ,  $V_{C_2}$  and eventually tend to the above situation, so we can observe the Rissaru graph of  $V_{C_1}$ ,  $V_{C_2}$  and through oscilloscope to determine whether their limit is 0, single period, multi-period or chaos.

#### 4. Experimental instruments and materials

Plug circuit board, Dupin wiring, patch capacitance (10nF, 100nF), inductance (10mH), ZX96 DC resistor, color ring resistor (220  $\omega$ , 2.2 K  $\omega$ , 3.3 K3, 22K  $\omega$ ), dual operational amplifier (TL082CP), 2 programmable linear power supply and digital storage oscilloscopes.

#### 5. Experimental steps

- 1). Connect the circuit on the circuit board according to Figure 1 and Figure 2.
- 2). Use an oscilloscope to measure the voltage of C1 and C2 with channel 1 and channel 2 respectively, with voltage as ordinate and time T as an absciss. Observe the waveforms of both.
- 3). Keep C1=10nF unchanged, and make C2 equal to 100, 110, 120, 130, 140, 150, 160, 170, 180, 190 and 200nF, respectively. First increase the R value, so that the oscilloscope screen does not display waveform, and gradually reduce the value of variable resistance R. At this time, a circle can be observed in the X-Y display. The Rissalu figure is a 1-period orbit, and the magnitude of the variable resistance R is recorded.
- 4). Keep C2=100nF and C1=10, 15, 20, 30, 40, 50, and 60nF respectively. Repeat Step 3.
- 5). Explore the corresponding relationship between the value of capacitor C2 and the value of resistance R when bifurcation occurs from one cycle to two cycles. Keep C1=10nF unchanged and make C2 equal to 100, 110, 120, 130, 140, 150, 160 and 170nF, respectively. By changing the value of variable resistance R, make the oscilloscope under the x-Y display, the Lisaru graph can obtain a 2-period orbit from the bifurcation of a single circle. Record the value of R just when the waveform changes from one period to two periods.
- 6). Measure the relationship between capacitance and resistance value when the rissalu graph appears chaos. C1 is fixed so that C2 is equal to 100, 110, 120, 130, 140, 150, 160, and 170nF respectively. By changing the value of variable resistance R, The Rissaru graph just changes from period-doubling bifurcation to chaos, and the VALUE of R at this time is recorded.
- 7). Using the obtained data, make C2 -- R curve and C1 -- R curve when no waveform output  $\rightarrow$  waveform appearance and C2 -- R curve when 1 cycle  $\rightarrow$  2 cycle. And explore their relationship.

#### 6. Data records

When C1=10nF, 1-period orbit just appears, and the relationship between C2 and variable resistance value R (1P).

Table 1. Experimental data of R with different values of C2 (0→1P)

C <sub>2</sub> (nF)	R <sub>1</sub> (Ω)	R <sub>2</sub> (Ω)	R <sub>3</sub> (Ω)	R <sub>4</sub> (Ω)	R <sub>5</sub> (Ω)	R <sub>6</sub> (Ω)	R (1P) (Ω)
100	1923.2	1923.4	1922.8	1922.8	1923.3	1923.5	1923.17
110	1958.1	1957.6	1957.9	1958.5	1957.5	1957.8	1957.90
120	1988.4	1988.3	1988.6	1988.6	1988.5	1987.9	1988.38
130	2015.7	2015.6	2015.5	2015.0	2015.2	2015.4	2015.40
140	2039.0	2039.6	2039.4	2039.5	2039.7	2039.4	2039.43
150	2060.2	2060.5	2060.4	2059.9	2060.0	2060.3	2060.22
160	2078.8	2078.6	2078.7	2078.9	2078.7	2078.6	2078.72
170	2096.3	2096.4	2096.2	2096.0	2096.1	2096.1	2096.18
180	2112.0	2112.3	2112.4	2112.3	2112.2	2112.1	2112.22
190	2126.7	2126.9	2126.8	2126.7	2126.8	2126.9	2126.80
200	2141.4	2141.6	2141.8	2141.9	2142.2	2142.3	2141.87

Where R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, R<sub>4</sub>, R<sub>5</sub> and R<sub>6</sub> are the values obtained by six measurements respectively. Average  $\bar{R}(1P) = \frac{1}{6}(R_1 + R_2 + R_3 + R_4 + R_5 + R_6)$  Using EXCEL, the average value of R measurement results, R, as the ordinate, and the size of C<sub>2</sub> as the abscissa, are used to make the figure r-C<sub>2</sub> as follows.

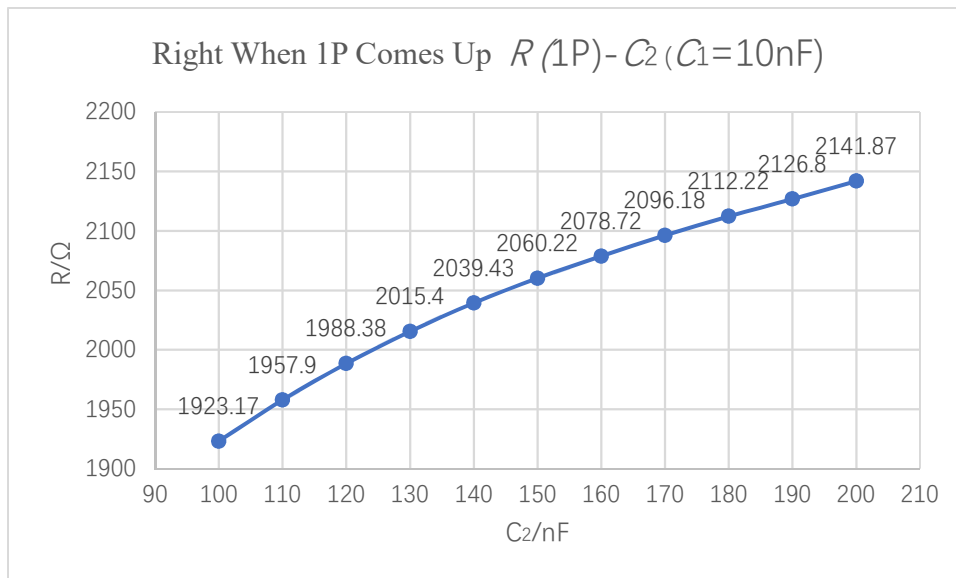


Figure 3. R (1P) -- C<sub>2</sub> curve (0→1P)

When C<sub>2</sub>=100nF, the relationship between C<sub>1</sub> and R just appears.

Table 2. Experimental data of R(1P) with different values of C1 (0→1P)

C <sub>1</sub> (nF)	R <sub>1</sub> (Ω)	R <sub>2</sub> (Ω)	R <sub>3</sub> (Ω)	R <sub>4</sub> (Ω)	R <sub>5</sub> (Ω)	R <sub>6</sub> (Ω)	R̄ (1P) (Ω)
10	1923.0	1923.3	1923.2	1923.4	1923.2	1923.2	1923.22
15	1536.6	1536.5	1536.5	1536.7	1536.8	1536.6	1536.62
20	1117.0	1116.8	1117.1	1117.3	1116.8	1116.9	1116.98
30	905.3	905.0	904.8	904.9	905.1	904.9	905.00
40	725.5	725.3	725.6	725.4	725.6	725.7	725.52
50	587.4	587.5	587.6	587.5	587.6	587.4	587.50
60	483.5	483.7	483.2	483.3	483.6	483.8	483.52
70	365.2	365.0	365.1	365.3	365.2	365.3	365.18

Using EXCEL, taking the average value R (1P) of R measurement results as the ordinate and the size of C<sub>1</sub> as the abscissa, the figure r-C<sub>1</sub> is made as follows.

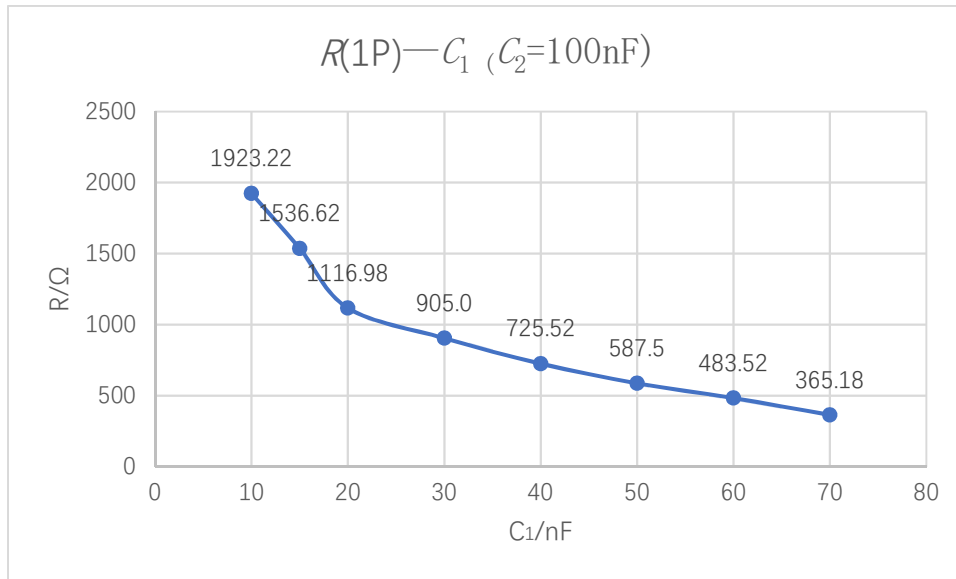


Figure 4. Curve of R(1P) -- C1 (0→1P)

Explore the corresponding relationship between the value of different capacitance C2 and the value of resistance R in case of bifurcation from one cycle to two cycles, when capacitor C1=10nF remains unchanged.

Table 3. Experimental data of R with different values of C2 (1P→2P)

C <sub>2</sub> (nF)	R <sub>1</sub> (Ω)	R <sub>2</sub> (Ω)	R <sub>3</sub> (Ω)	R <sub>4</sub> (Ω)	R <sub>5</sub> (Ω)	R <sub>6</sub> (Ω)	$\bar{R}(2P)$ (Ω)
100	1863.6	1861.8	1861.7	1862.9	1861.9	1863.8	1862.62
110	1896.5	1897.5	1898.2	1896.9	1897.4	1897.0	1897.25
120	1928.8	1927.9	1928.7	1929.3	1928.1	1929.1	1928.65
130	1963.5	1963.0	1962.3	1962.0	1962.4	1963.3	1962.75
140	1997.7	1998.3	1998.8	1998.1	1997.7	1997.5	1998.02
150	2039.0	2039.6	2040.4	2040.3	2040.0	2039.7	2039.83
160	2075.3	2075.0	2074.7	2074.4	2074.6	2074.8	2074.80
170	2095.9	2095.7	2096.0	2096.0	2095.9	2095.9	2095.90

Using EXCEL, the average value of R measurement results, R, as the ordinate, and the size of C2 as the abscissa, are used to make the figure r-C2 as follows.

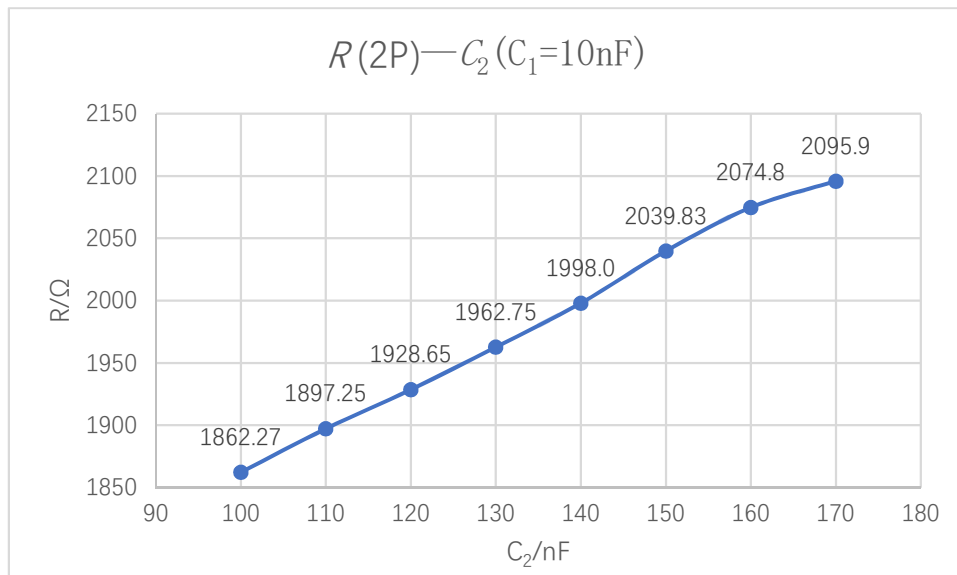


Figure 5. R (2P) -- C2 curve (0→1P)

After the experimental operation, it is found that when C1 is above 15nF, it is difficult to make two bifurcations in the cycle by adjusting the size of the variable resistor R because the adjustment range of R is very small from 1P to 2P.

Fix C1=10nF and explore the relationship between C2, which makes the system fall into chaos, and the variable resistance value R (C).

Table 4. The relationship between C2 and R when chaos occurs

C <sub>2</sub> (nF)	R <sub>1</sub> (Ω)	R <sub>2</sub> (Ω)	R <sub>3</sub> (Ω)	R <sub>4</sub> (Ω)	R <sub>5</sub> (Ω)	R <sub>6</sub> (Ω)	$\bar{R}(C)$ (Ω)
100	1808.6	1808.0	1807.9	1808.9	1808.3	1808.7	1808.40
110	1852.7	1853.6	1853.0	1854.0	1853.8	1854.0	1853.52
120	1897.0	1896.3	1896.9	1897.3	1896.5	1896.7	1896.78
130	1932.7	1932.1	1932.5	1931.5	1931.9	1932.9	1932.27
140	1964.9	1964.3	1965.3	1964.8	1964.0	1963.8	1964.57
150	1991.8	1990.8	1991.9	1991.0	1990.5	1991.9	1991.32
160	2048.3	2048.5	2046.5	2046.4	2046.5	2047.8	2047.13
170	2068.9	2068.0	2069.0	2069.3	2069.2	2069.9	2069.05

According to the data in Table 4, C2 as abscissa and R as ordinate were plotted, as shown in Figure 6.

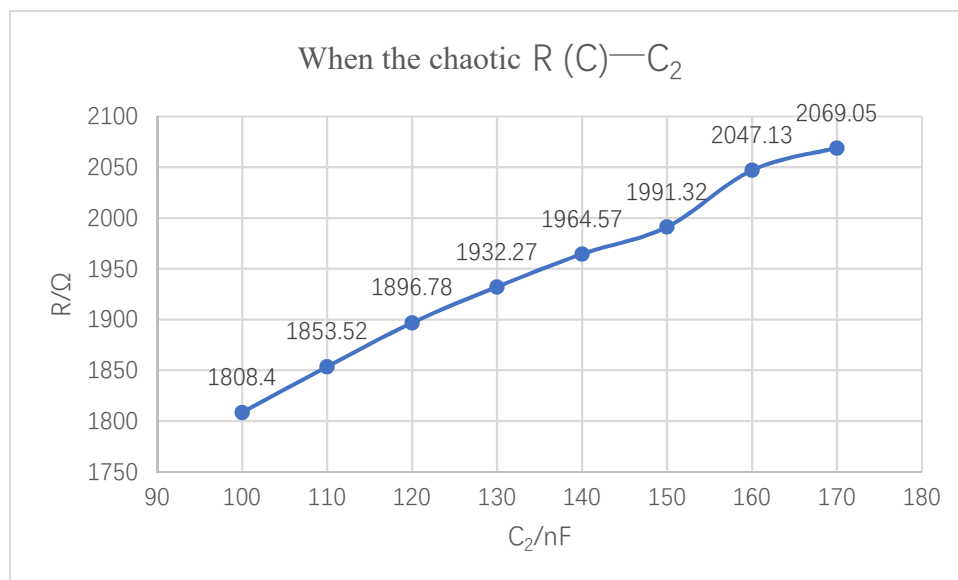


Figure 6. Graph of R(C) -- C2 in chaos

## 7. The data analysis

- 1). When C1 is fixed, when C2 increases, the waveform just appears, that is, the resistance R corresponding to a single period just increases with the increase of C2. And R (1) is approximately a function of C2.
- 2). C2 is fixed. When C1 increases, the waveform just appears, that is, the resistance R decreases with the increase of C2 in the period of 1. And R(1P) is approximately inversely proportional to C1.
- 3). When C1 is fixed, when C2 increases, the resistance R of the Risaru graph bifurcates from one period to two periods increases with the increase of C2. And, approximately, R(2P) is a function of C2 in the range from 100nF to 150nF.
- 4). When C1 is fixed, when C2 increases, the resistance R when the Rissalu graph is just chaotic increases with the increase of C2. And, approximately, R(C) is a function of C2 in the range of 100nF to 140nF.

5). Explore the difference of resistance values  $\delta R$  corresponding to 1P and 2P when  $C_1$  is fixed and  $C_2$  is different, where  $\delta R = R(1P) - r(2P)$ .

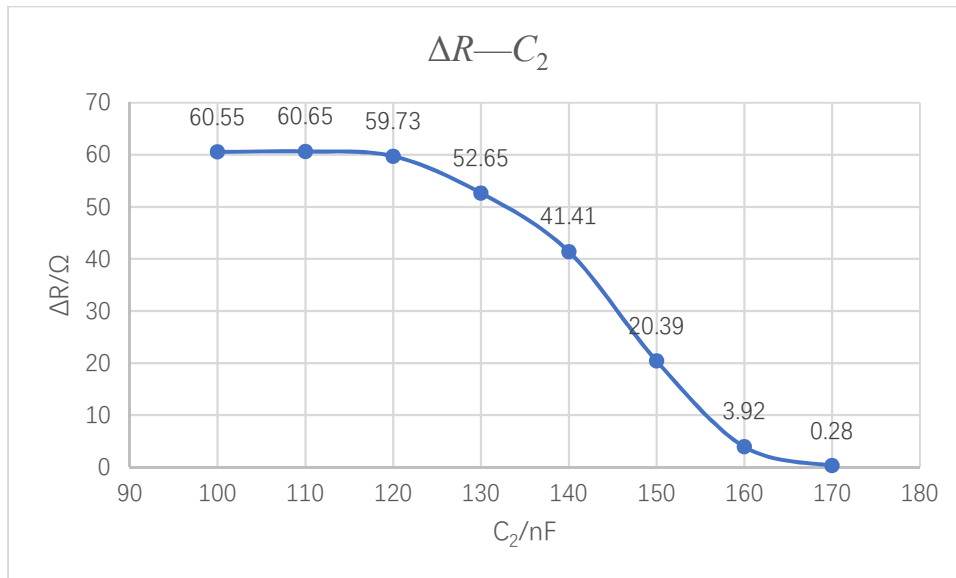


Figure 7.  $\delta R$  --  $C_2$  curve

From FIG. 7, we can see that when  $C_1$  is kept constant, the range of resistance variable is not very different between 1P and 2P when the size of  $C_2$  is approximately between 100-120nF. When  $C_2$  is greater than 120nF,  $C_2$  is larger and the range of resistance variable between 1P and 2P is smaller.

6). Explore the difference between the two resistance values  $\delta R'$  corresponding to 1P and chaos when  $C_1$  is fixed and  $C_2$  is different, where  $\delta R' = R(1P) - R(C)$ .

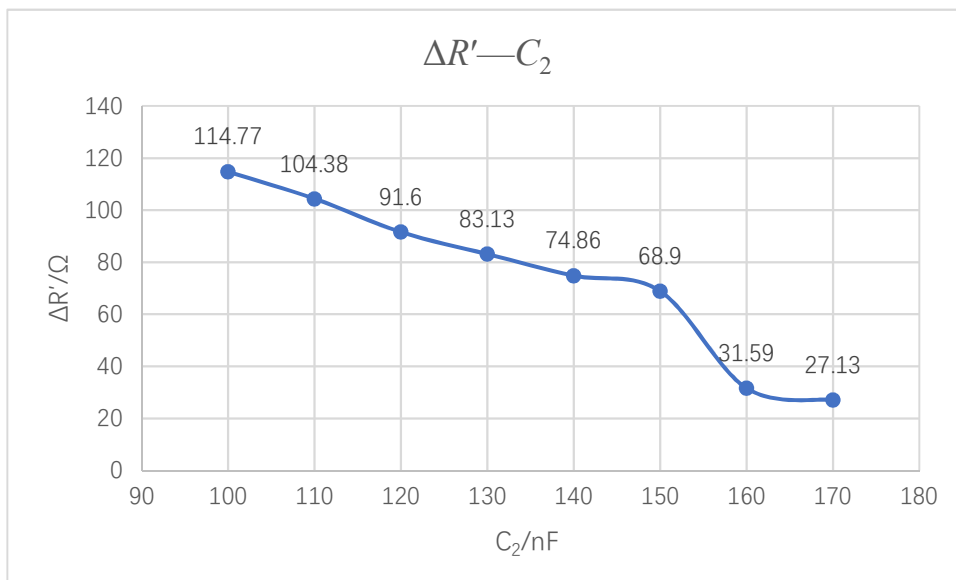


Figure 8. Curves of  $\delta R'$  and  $C_2$

According to FIG. 8, when  $C_1$  is kept constant, the larger  $C_2$  is, the smaller the adjustable resistance  $R$  can be adjusted between the occurrence of 1 cycle and chaos.

## 8. Conclusion

1). To maximize the control range of periodic bifurcation by changing the resistance in the chaotic circuit,  $C_1$  should be selected as near 10nF as possible.

- 2). To make the range of resistance R adjustable between the occurrence of 1 period and chaos as large as possible, C2 should be set to a smaller value.
- 3). To make the corresponding variable resistance value larger when 1P is generated, C2 should be between 100 and 110nF.

## 9. Error analysis

- 1). Measurement error of oscilloscope,
- 2). The waveform changes greatly, and the value error when judging the critical point of change,
- 3). Error caused by circuit contact resistance.

## 10. Innovative

By studying the relationship between capacitance value and resistance value when Chua's chaotic circuit produces period-doubling bifurcation and chaos, two capacitance values corresponding to the maximum change of R from 1P to chaos are found. Some data and qualitative analysis are provided for making a circuit that can control the output voltage period of the chaotic circuit in a large range.

## 11. Deficiencies and improvements

In this experiment, the pluggable circuit board is used as the circuit board to connect the components. There may be some problems caused by bad contact. If in the measurement, touching the circuit will make the waveform change, bring measurement error. If the use of welded circuit board, to take the way of welding to connect the line, reliability will be higher, the line should also be more stable.

## References

- [1] Yuying Liu, Zhonghui Jin, Shunye Li. Physics and engineering, 2018, 28(05): 59-62.
- [2] Shida liu, Fuming Liang, Shishi Liu et al. Chaos and Analysis in natural Science [M]. Beijing: Peking University Press, 2003.
- [3] Dianxue Wang. Study on the relationship between chaos phenomenon and nonlinear resistance voltammetry curve in CAI's circuit [J]. Journal of liaodong university (natural science edition), 2010, 17(04): 317-320.
- [4] Michael Peter Kennedy. Three Steps to Chaos-part ii iA Chua's Circuit Primer [J] IEEE Trans on CASI, 1993, 40(10):657-674
- [5] Shuhua Wu, Weide Meng, Zhichun Ma. Experimental and simulation research on CAI's circuit [J]. Journal of Shijiazhuang university, 2019, 21(06):27-33.