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Methods for solving combinatorial optimization problems: a review

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Abstract

With the emergence of new fields of social networks and biological networks and the advancement of theoretical computer science, the research on combinatorial optimization problems has experienced explosive growth. Combinatorial optimization problems are an important branch of computer science and operations research, which mainly studying the solution methods of discrete structure optimization problems. The research of combinatorial optimization problem involves the fields of information technology, communication network, computer vision and so on. In this paper, we introduced typical combinatorial optimization problems: minimum vertex cover problem, minimum weighted vertex cover problem, maximum clique problem, and summarized various algorithms for solving combinatorial optimization problems. Finally, the shortcomings of each algorithm are shown.

Keywords

Combinatorial optimization problems, Minimum vertex cover problem, Minimum weighted vertex cover problem, Maximum clique problem.

1. Introduction

In the real world, there are a lot of NP-hard [1] optimization problems. The characteristic of NP-hard problems is that as the scale of instances increases, the number of feasible solutions and the degree of solution tend to increase exponentially. As an important branch of operations research, the combinatorial optimization problem has important practical significance in the fields of logistics scheduling, route planning, transportation, and wireless communication [2-3].

Many real-life combinatorial optimization problems can be modeled into graphs, and then heuristic algorithms for solving optimization problems in graph theory can be designed. For example, the minimum vertex cover problem. The purpose of the minimum vertex cover problem is to find the minimum set of vertices covering all edges in the graph [4]. We can model many problems in reality as minimum vertex cover problems. For example, in urban surveillance systems, designers need to choose suitable locations to install cameras at intersections. We assume that a camera placed at an intersection can monitor the road where the intersection is located. The more cameras mean the higher the cost. The designer hopes to use the least number of cameras to cover the entire city. This is a typical example of the minimum vertex cover problem. It has a wide range of applications in network traffic engineering [5], signal transmission problems in wireless sensor networks [6], and coding theory [7]. The minimum weighted vertex cover problem is an extension of the minimum vertex cover problem. In order to obtain a high-quality solution in a limited time, researchers often need to design some efficient algorithms.

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2. Definition

Definition 1 (Combination Optimization Problem): The combinatorial optimization problem is to find the maximum value or minimum value of the objective function under given constraints. The mathematical model is as follows:

$$\begin{aligned} &\min f(x) \\ &s.t. \quad g(x) \geq 0 \\ &\qquad\ x \in D \end{aligned} \tag{1}$$

Among them, $f(x)$ is the objective function, $g(x) \geq 0$ indicates the constraint condition, and D is the domain of the variable x. Maximum value and minimum value can be converted into each other.

Definition 2 (Vertex Cover): Given an undirected graph $G = (V, E)$, V represents the vertex set, and E represents the edge set. The vertex cover is a vertex subset of the vertex V, so that any vertex of any edge in the graph is in the subset.

Definition 3 (Minimum Vertex Cover Problem): The minimum vertex cover problem is to find the minimum set of vertices that can cover all edges in the graph.

Definition 4 (Minimum Weighted Vertex Cover Problem): The minimum weighted vertex cover is to find the minimum vertex cover in the graph while ensuring that the sum of the weights is the minimum.

Definition 4 (Maximum Clique Problem): Given an undirected graph $G = (V, E)$, the maximum clique problem is to find a complete subgraph with the largest number of vertices in the graph G.

Fig. 1 shows examples of vertex cover, minimum vertex cover and minimum weighted vertex cover. Fig. 1 (a) V_1 = {1,2,3,4} is a vertex cover, Fig. 1 (b) V_2 = {2,3,4} is a minimum vertex cover, Fig. 1 (c) V_3 = {1,2,4} is a minimum weighted vertex cover, where black means covering and white means not covering. Fig. 2 shows example of maximum clique. V_4= {1,2,4} is the maximum clique.

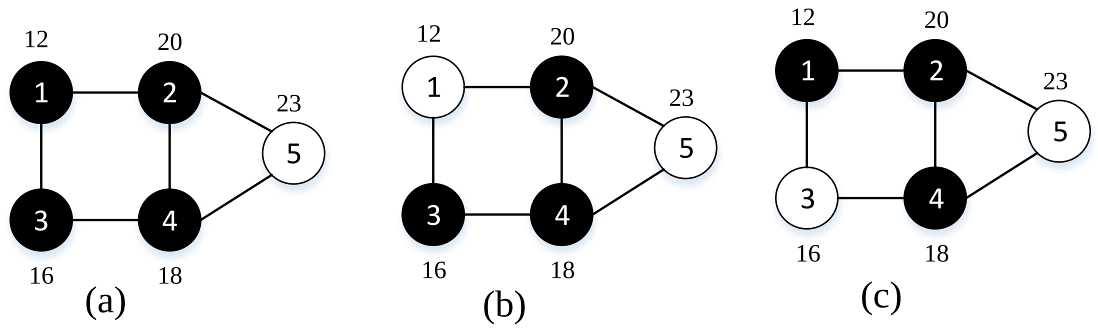

Figure 1. Shows the vertex cover, minimum vertex cover and minimum weighted vertex cover

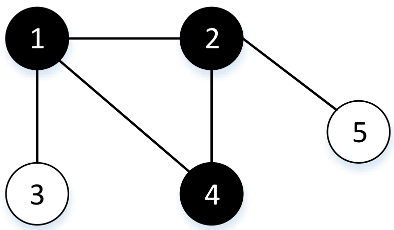

Figure 2. Shows the maximum clique

3. Optimization Algorithm for Solving Combinatorial Optimization Problems

Combinatorial optimization problem is a problem of finding the optimal objective from a limited set of objectives. Common combinatorial optimization problems include the minimum vertex cover problem, the maximum clique problem and so on. At present, the algorithms for solving combinatorial optimization problems are mainly divided into exact algorithms and heuristic algorithms.

3.1 Exact algorithm

The exact algorithm searches the entire solution space of the problem through the system to ensure that the final solution is the global optimal solution. At present, the research of exact algorithm mainly focuses on exact index algorithm and fixed parameter algorithm. In [8], the author used divide-and-conquer analysis to give an exact index algorithm with a time complexity of 0*(1.2210n), which derived a centralized improved algorithm [9-10]. The fixed parameter algorithm is mainly a method of solving the vertex cover through parameter setting. Among them, the difference between the lower limit of linear programming (LP) and the best value of integer programming (IP) is the latest applied parameter, and some research results have been achieved [11-13]. At present, the low-complexity algorithm is based on the branch reduction framework and uses a simple lower limit of LP to prune the search. In 1990, Carraghan and Pardalos [14] proposed a simple and easy-to-understand algorithm for the maximum clique problem based on the branch and bound method. The algorithm simply uses the number of vertices contained in the subgraph to estimate the maximum clique of the subgraph and excludes those search subtrees that are impossible to find the optimal solution to reduce the actual query space. In 2003, Tomita [15] proposed the MCQ algorithm. The MCQ algorithm adopts the sequential vertex coloring method. The candidate vertices are arranged in a pre-arranged order on each node of the search tree, and each vertex is assigned the minimum color number it can obtain one by one. In 2007, Konc [16] proposed the MaxCliqueDyn algorithm, which continued the basic framework of the MCQ algorithm and improved the order of the vertices. In 2010, Li and Quan [17] introduced MaxSAT inference technology to the maximum clique problem to improve the previous estimation based on coloring and proposed an efficient MaxCLQ algorithm. In 2015, Segundo [18] integrated the idea of MaxSAT reasoning into the BBMC algorithm and proposed an improved BBMCX algorithm.

For solving combinatorial optimization problems, when the scale of the feasible solution is within the controllable range, an exact algorithm can be used to find the optimal solution. However, as the scale of the problem gradually increases, when the amount of calculation and storage space required to solve the optimal solution of this type of problem increases exponentially, it becomes almost impossible to use an exact algorithm to find the optimal solution.

3.2 Heuristic Algorithm

Heuristic algorithms are methods to solve problems by summarizing past experience information and analyzing experiments. Heuristic algorithm is based on intuitive foundation rather than mathematical theory itself. Its purpose is to construct a fast and efficient solution mode and provide a high-quality solution within an acceptable calculation time.

In [19], the author proposed a genetic algorithm to solve the minimum vertex cover problem of complex networks. In [20], the author proposed a memory-based optimal response algorithm to solve the minimum vertex cover problem. This algorithm uses the game to select the optimal strategy in memory to improve local information, but the memory length often affects the quality of the algorithm. In [21], the author proposed a local search algorithm with two-stage exchange and edge weighting with forgetting. For local search algorithms employ edge weighting techniques and have poor performance on instances with structures that defeat greedy heuristics. In [22], the author proposed a vertex weighting scheme TwMVC algorithm to address this shortcoming. In [23], the author proposed a new membrane evolution algorithm that uses four operators abstracted from the life cycle of living cells to evolve to solve the minimum vertex cover problem. In [24], the author proposed an ant colony optimization algorithm, which can find an approximate solution to the minimum weighted vertex cover problem, but it often falls into a local optimum. In [25], the author improved the ant colony optimization algorithm by introducing a pheromone correction heuristic strategy and using the information of the current best solution to exclude some components that are considered bad in the solution, so as to avoid the algorithm search falling into local optimization. In [26], the author proposed a multi-start iterative tabu search algorithm, which uses a multi-start mechanism and a tabu strategy and incorporates a novel neighbor construction process and rapid evaluation strategy to solve the minimum weighted vertex cover problem. In [27], the author proposed a framework based on asymmetric games. According to the strict Nash equilibrium of asymmetric games, it is an intermediate state between the weighted vertex cover state and the minimum weighted vertex cover state to solve the minimum weighted vertex cover problem. In [28], the author studied the exploration of larger neighborhoods in local search algorithms to minimize vertex cover. In [29], the author proposed an improved memetic algorithm to solve the partial vertex cover problem. In [30], the author proposed a new hybrid genetic algorithm for finding the maximum clique in a graph.

For solving combinatorial optimization problems, when the scale of the problem gradually becomes larger, the use of heuristic algorithm to solve can often get a satisfactory solution in an effective time, but the stability of the solution quality and search efficiency and the optimality of the solution cannot be proved.

4. Conclusion

Most combinatorial optimization problems are NP-Hard problems. The common feature of these problems is that as the parameters or scale of the problem increase, the number of feasible solutions increases exponentially or higher, which leads to the complexity of solving combinatorial optimization problems. In this paper, we focus on the exact algorithms and heuristic algorithms to solve the minimum vertex cover problem, the minimum weighted vertex cover problem, and the maximum clique problem. Finally, the advantages and disadvantages of the two types of algorithms are analyzed.

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