Impact of the Attribute-updating Mechanism on the Evolution of Cooperation in Scale-free Networks

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Abstract

The diversity of individual attributes plays an important role in the evolution of social dilemmas. Inspired by this fact, we propose a new attribute- updating mechanism which is related with the willingness-rate value and the alternation period to update their strategies. We explore from the following aspects: At the beginning, we regard strategy-updating rules as individual attributes with the conformity attribute and the benefit-driven attribute. Besides, we also investigate that after each individual adheres to his own attributes in Monte Carlo Steps (MCS), it will determine whether to update its attribute with a certain probability. In other words, once the update conditions are met, all individuals will have the right to choose whether to update their attributes or not. From our numerical results, we find that the attribute-updating mechanism can promote cooperation in scale-free networks. Specifically, the cooperation level becomes higher as the alternation period decreases or the willingness-rate value increases. At the same time, the moderate conformity proportion can enhance the reciprocity of networks.

Keywords

Attribute-updating; Scale-free Networks; Snowdrift Game; Evolutionary Cooperation.

1. Introduction

Cooperation behaviors in various aspects have continued to strengthen in recent years, which is obviously contrary to Darwin's theory of evolution. In order to understand this phenomenon, there are growing appeals for scientists to use evolutionary theory, especially classic evolution models, which widely is considered to be a good way to study the influence of individual choice on cooperation. Nevertheless, the foremost problems are the facts that how to strengthen cooperation among the diversity of individual attributes is vital, meanwhile, how to deal with the problem has received substantial interest for a lot of researchers.

In terms of the classic snowdrift model [1-6], it can be de- scribed as: When both players choose to cooperate in front of the snowdrift dilemma, they have to bear the labor cost c, but can also obtain the benefit b. Finally, both players get the benefit R = b - (c/2). If both players choose to defect, both benefits are P =0. However, any of them chooses to cooperate and the other makes a defection decision, the former obtains the benefit S=b-c in the end, and the latter gets the benefit T=b. Among them, T>R>S>P and 2R=T+S. So we can conclude that the defectors always can obtain the highest benefits in the population to some extent. Besides, there are prisoner's dilemma game [7-10] and other evolution models [11,12]

There has been less previous evidence [13-16] for the diversity of individual attributes considered by only a few studies, which has shown the results can significantly improve the level of cooperation

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[17-20]. In 2015, Szolnoki and Perc published a paper [21], they mentioned the conformity, as one of the individual attributes has attracted researchers' attention in recent years [22-25], is that individuals learn the strategies frequently selected by their neighbors as their own strategies, which is different from the strategy of higher benefits [26]. Their research simultaneously indicates that after introducing the moderate proportion of conformity individuals into the population, cooperation can be enhanced in evolutionary game.

In the above mentioned works, the conformity attribute is an effective mechanism to promote cooperation in fierce competition. Whereas, we have developed a generic method to solve a variety of problems. We will now demonstrate our method for some specific problems. Combined with many previous studies [27-29] that used Fermi function to update individual strategies, we take benefit-driven and conformity-driven strategy-updating rules as two different attributes of individuals. Meanwhile, as the labor cost increases, individuals can choose whether to update their attributes in order to promote cooperation in the population. To examine the impact of our method on cooperation, we build a spatial network model to truly reflect social relationships. In reality, the distribution of social networks is similar to small-world networks [30, 31] and scale-free networks [32-36]. As a result, we adopt a model that new nodes are often connected to nodes with higher degrees in scale-free networks [37-39]. To conclude, with this aim in mind, in this paper we present a new method for evolutionary game. We introduce two attributes into scale-free networks and use an attribute-updating method of standard Monte Carlo Steps (MCS): Monte Carlo time t is divided into N attribute retention periods, at the same time, each t includes s Monte Carlo time steps. Each step s mainly completes three procedures in the evolution process including the interaction of individuals with its neighbors, the strategy-updating rules and the attribute-updating rule.

After defining the problem, we explain the goals of the thesis. Then, we summarize the main contribution of this thesis. The key contribution of this work is that individual heterogeneity is taken into account, that is to say, it provides the option that individuals consider whether to update their attributes.

The organizational structure of this paper is as follows: In the second part, we describe the evolutionary game model and the two mechanisms used by us. Then, the third part gives the simulation results and discussion. Finally, the fourth part draws the conclusion of this article.

2. Model

Each individual simultaneously decides to cooperate (*C*) or to defect (*D*) with the same probability. For the classic snowdrift game model SG, the payoff matrix is expressed as follows:

Table 1. The payoff matrix of SG model

	C	D
C	b-(c/2)	b-c
D	b	0

Among them, T=b is the temptation benefit of the defector when its opponent decides to cooperate. R = b-(c/2) represents the reward as they choose mutual cooperation and P = 0 is the punishment as they both defect. The cooperator obtains the sucker's benefit S = b-c when the opponent chooses to defect. In order to facilitate the calculation, we simplify SG as: R=1, S=1-r, T =1+r and P = 0, which r ($0 \leq r \leq 1$) represents the ratio of labor costs.

In order to realistically reflect the real social networks relationship, we use the BA scale-free networks in the spatial network structure, which is a model with two characteristics of growth and preferential connection proposed by Barabasi and Albert in October 1999: (1) Growth: First, m0 points connect with each other to form the fully-connected network. Then, a new node is added as time increases, and the new node is connected with the existed node to form m ($m \leq m0$) edges. (2) Preferential connection:

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The relationship between the probability p of a new node connecting to the node i and the degree of the node i is:

$$p_i = \frac{k_l}{\sum_l k_l} \tag{1}$$

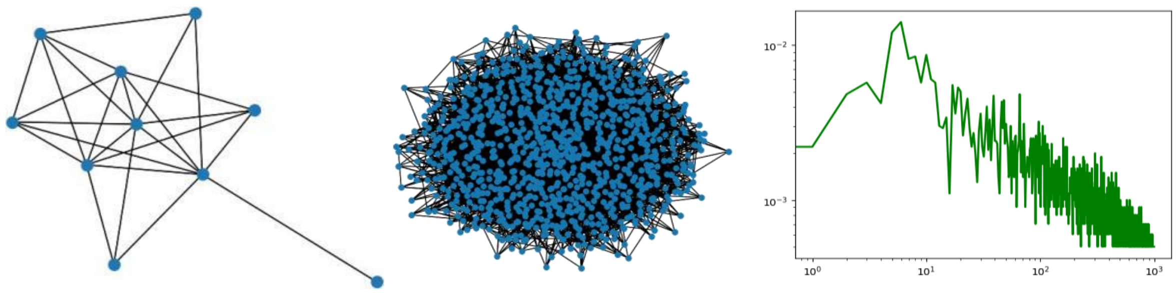
Among them, k_l is the degree of the node i, l means all existing nodes.

The regular networks, random networks and small-world networks. In terms of degree distribution random networks and small-world networks have the same Poisson distribution shape that looks like an arch with a peak value. Unlike the mentioned networks, we use a BA model whose degree distribution follows the power law distribution. It presents the power-law distribution of 1000 individuals forming scale-free networks in Figure 1(c): As the number of individuals increases, the probability that new nodes connect with existed nodes becomes smaller.

Each point *n* represents an individual in scale-free networks, where all individuals constitute the population. We randomly assign individuals in the population to conformity attribute and benefit-driven attribute respectively at the ratio of *Pc* and 1-*Pc*. The individual attribute-updating method uses standard Monte Carlo Steps (MCS): Monte Carlo time *t* is divided into *N* attribute retention periods, at the same time, each *t* includes *s* Monte Carlo time steps. Each step *s* mainly completes the following procedures in the evolution process:

First, the interaction between individuals and its neighbors: The individual *i* plays with its neighbors *j* in the benefit-driven attribute, then we

calculate the cumulative benefits are $\sum_i$ and $\sum_j$ respectively. For the conformity attribute, we separately calculate the number of cooperation and defection strategies which are chosen by the neighbors *j* of the individual *i*. Then, the individual *i* learns the neighbors' strategy with the largest number as its optimal strategy.



(a) (b) (c)

Figure 1. (a) mainly shows the two major characteristics of scale-free networks, which composed by 1000 individuals is showed in (b). (c) is the power-law distribution of scale-free networks.

Secondly, the strategy-updating rules: If the individual *i* is the benefit- driven attribute, the individual *i* updates its strategy according to the fol- lowing Fermi function:

$$P(s_i \leftarrow s_j) = G\,\frac{1}{1+\exp[(\sum_i - \sum_j)/k]} \tag{2}$$

In the formula, P(si←sj) represents the probability that the individual i learns the strategy of its neighbors j. ∑i and ∑j are the cumulative benefits of individual i and its neighbors j respectively. k is an irrational factor that values 0.1 in this paper. $G(0 \le G \le 1)$ is the willingness-rate value. If the individual *i* is the conformity driven attribute, then the strategy-updating rule is based on the following function:

$$P_{i\leftarrow j} = G\,\frac{1}{1+\exp[(N_{si}-d_h)/k]} \tag{3}$$

Where Pi←j means the probability that the individual i learns its neighbors' strategies with the largest number. Nsi represents the same number of individuals i's strategy as that of its neighbors j. dh is half of the individual i degree.

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Third, the attribute-updating rule: If the update condition t =sN is met, the individual will update its attributes. When the attribute retention period N is only considered, the attribute-updating frequency becomes faster as N is smaller, which is in line with common sense. Whereas individuals have the right to choose whether to update their own attributes. This right is represented by the probability G in the homogeneous rule (G is abbreviated as the willingness-rate value). There are three situations for constant G(0≤G≤1). (1) The population is divided into two fixed sub- groups when G = 0, their attributes are not updated as time increases. (2) For G = 1, the benefit-driven attribute of individuals will be updated to the conformity attribute. At the same time, the attribute of individuals is updated from the conformity attribute to the benefit-driven attribute. (3) If 0<G<1, the individual's attribute is updated with the size of G when the condition t = sN is met.

3. Simulation results and discussion

All simulation results are obtained on BA scale-free networks where the number of individuals is n = 1000, the newly added sides m and the fully-connected nodes m_0 are both set as 5. The results are performed with the Monte Carlo Steps t = 10000 and small steps s = 100 until stabilization evolution time is reached. To research the influence of individual attribute- updating mechanism on the cooperation level, we mainly analyze from the following aspects and use fc instead of the cooperator frequency.

We set the initial values Pc and G to be 0.5, as well as mainly analyze that the population cooperation level changes with r and t under different attribute retention periods N in Figure 2. We discover that f_c is the highest when N = 1 for r<0.6 and the smallest for N = 30 in Figure 1(a). But under the same r, fc falls with the increase of N. Meanwhile, the population was divided into two subgroups of the conformity subgroup and the benefit-driven subgroup until $N \rightarrow o$, in which their attributes are not updated. Now, we can investigate the reasons for the above results: The defectors occupy most of the space as r increases in the population. However, the smaller N values are, the faster attribute-updating frequency becomes after N is introduced, which causes the expansion of clusters formed by the co-operators becomes sooner to resist the defectors. The increasing number of the cooperators' clusters can promote the cooperation level. (b) detailedly describes the change of the cooperation level with different N and r at each t. For N = 1 and r = 0.2, f_c is the highest under the entire t. On the contrary, f_c is the lowest when N = 30 and r = 0.8. However, the cooperation level does not change with t for all individuals with the conformity attribute, which remains at around 0.5. This means that f_c only increases when the individual attributes are updated. In summary, it is more beneficial to the improvement of the cooperation level as N becomes smaller in each Monte Carlo time t.

To illustrate the relationship between the willingness-rate value and the level of cooperation, we study the evolutions of G and f_c in Figure 3. All individuals choose to update their attributes starting with the initial values N=5 and Pc=0.5 for G=1 when t= sN is satisfied in Figure 3(a), where the population cooperation level is the highest value. For 0<G<1, individuals determine whether to update their attributes with the given probability G. Interestingly, we have the results that when G=0.2, individuals are more willing to keep their attributes unchanged than to update their attributes, which means f_c is the lowest level. In order to figure out how f_c evolves over time, we obtain Figure 3(b) to describe in detail how different values of G and r affect the cooperation level. When G=1 and r is small, f_c moderately increases and stabilizes in the end as t becomes larger. Meanwhile, we find that the cooperation level fluctuates around 0.5, on account of all individuals with the conformity attribute.

The specific reasons for the above results can be explained as follows. Not all individuals update their attributes when t = sN is satisfied. Although the proportion Pc of individuals with the conformity attribute is set as 0.5 in the population, individuals with the benefit-driven attribute can choose to cooperate or defect in order to obtain higher benefits. With the increase of r, more individuals are inclined to defect in the dilemma. In this way, the cooperators pay for their labour, but the benefits are the same as the defectors' or less. What's more, the cooperators also defect over time. Then, the defectors in the population are in dominant advantage with time going on, which is not in favour of improving the cooperation level.

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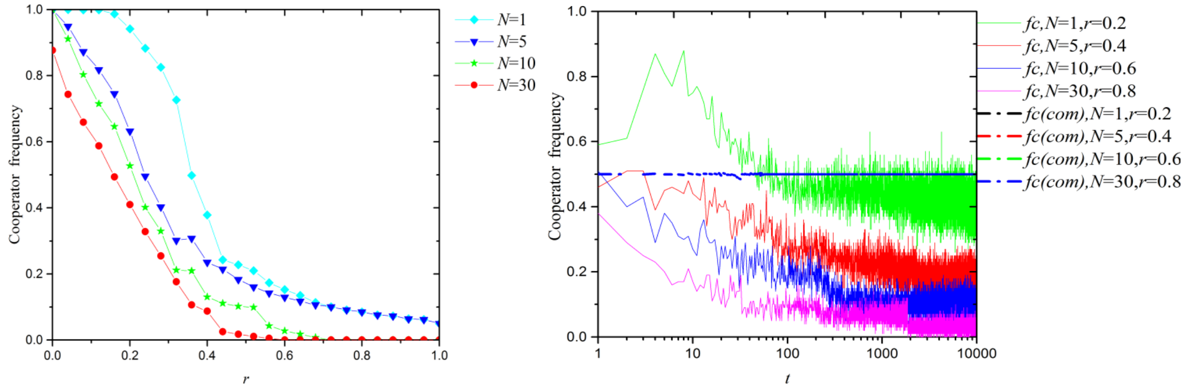
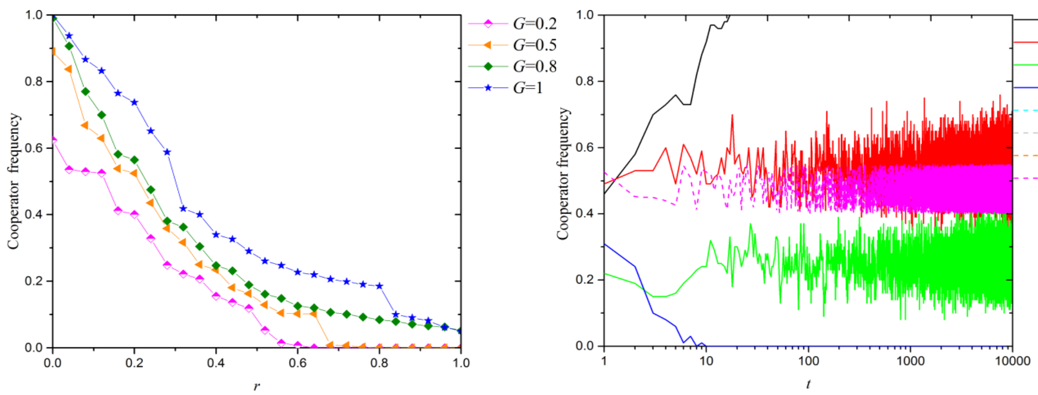


(a) (b)

Figure 2. (a) mainly analyzes the cooperation level vs r for different N. The cooperation level compares with that for all individuals with conformity attribute vs t in (b), under different N and r.

Whereas, individuals with the conformity attribute learn the most number of strategies chosen by their neighbors and there are no more defectors as r increases. Therefore, f_c becomes stronger as G becomes greater. Namely, the chance of the defectors forming clusters is extremely low, which will leave the greater space to survive for the cooperators and simultaneously improve the cooperation level.

Furthermore, we notice that the proportion of the conformity attribute also affects the cooperation level. We acquire the results by setting the initial value $N = 5$ and $G = 0.5$, and discover that f_c is the lowest level when $Pc = 0$ for all individuals with the benefit-driven attribute. As Pc reaches 0.8, which obviously improves the cooperation level. However, f_c doesn't fluctuate widely with the increase of r remaining basically around 0.5 when $Pc = 1$. Individuals with the conformity attribute have nothing to do with the labor cost ratio r, on account of only selecting the most number of the strategy chosen by their neighbors. Therefore, we can conclude that the cooperation level improves as Pc increases within the range of $0 < Pc < 1$. Besides, compared with Figure 4, we can find the macro-scopical snapshots show the distribution of the cooperator clusters and defector clusters in Figure 5. As Pc increases, clusters formed by the cooperators significantly increase for the same values t and their boundaries become smoother, which explains why the introduced conformity attribute can enhance the reciprocity of the networks to promote the cooperation level. Conversely, the cooperator clusters are broken and eventually stabilize as the values t increase for the same value of Pc. The reason is that the cooperator clusters with the conformity attribute work together with the benefit-driven cooperator clusters to resist the invasion of the defector clusters. By this means, the defectors are at the inferiority position. Specifically, either decreasing N or increasing G can improve the frequency of attribute-updating to promote cooperation.



(a) (b)

Figure 3. (a) mainly is the cooperation level vs r for different G. (b) depicts the cooperation level compares with that for all individuals with conformity attribute vs t, under different G and r.

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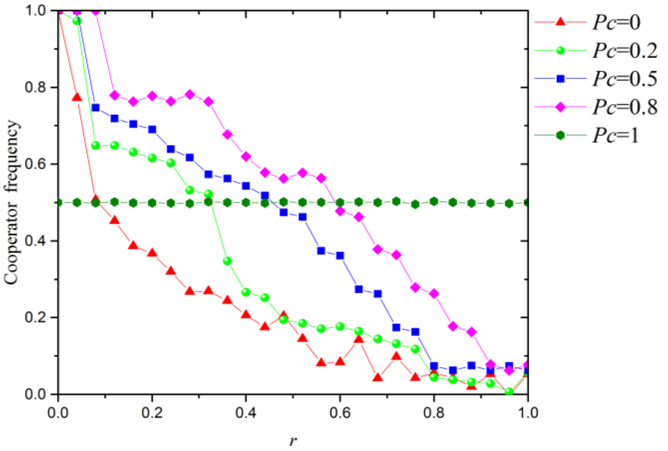
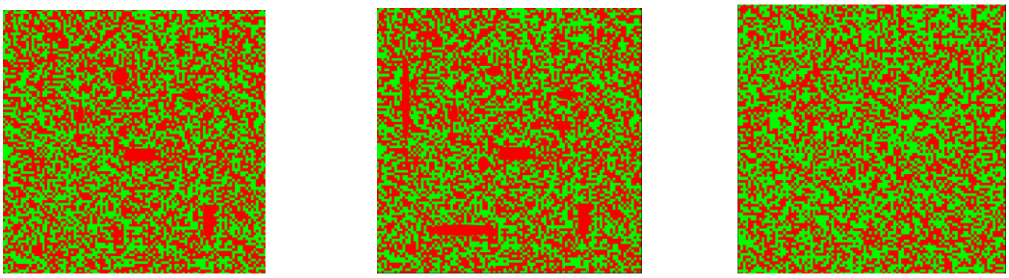
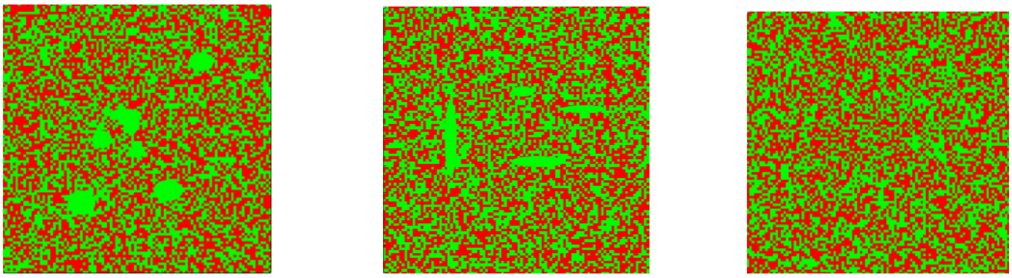


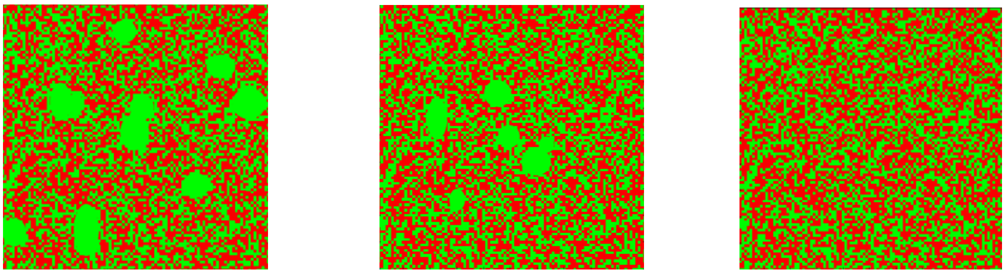
Figure 4. It mainly describes the cooperation level vs *r* for different *Pc*.



(a) Pc=0.2,t=10 (b) Pc=0.2,t=100 (c) Pc=0.2,t=1000



(d) Pc=0.5,t=10 (e) Pc=0.5,t=100 (f) Pc=0.5,t=1000



(g) Pc=0.8,t=10 (h) Pc=0.8,t=100 (i) Pc=0.8,t=1000

Figure 5. Under different values Pc and t, the evolution snapshots of the cooperators and defec- tors, where red pixels mean the defectors and green pixels represent the cooperator

4. Conclusion

In summary, we have researched evolutionary games on scale-free networks with the attribute-updating mechanism affecting the cooperation level. Networks of two characteristics with growth and the preferential connections are exploited to demonstrate the interaction among individuals in the real world. Each player can have two strategy-updating rules that are considered as their attributes. They can update their attributes when the update condition is met. We investigate whether the attribute retention period, the update willingness probability and the conformity proportion affect the cooperation level or not. It is worth pointing out that the cooperation level can be improved within a small range of N and there is a moderate value of G leading to the highest cooperation level. Though the cooperation level can be enhanced as the conformity proportion increases, the excessive proportion will make no difference for the cooperation. Moreover, the introduced conformity rule will obviously enhance the network reciprocity resulting in frequent alternations of the strategy-updating rules. Our research reveals that the introduction of attribute-updating mechanism in scale-free networks obviously promotes the cooperation level. However, it is possible that further building a model of evolutionary game can strengthen the cooperation of more complicated attributes evolution.

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