ISSN: 2414-1895 DOI: 10.6919/ICJE.202110_7(10).0049

The Mechanism of Individual Time-cost Heterogeneity Promotes Cooperation in Evolutionary Game

Hancheng Wang[1,*], Yansong Deng, Qinzhen Huang

[1]Key Laboratory of Electrical and Information Engineering, State Ethnic Affairs Commission, Southwest Minzu University, Chengdu, Sichuan, 610041, China.

*Corresponding author. E-mail: 168449569@qq.com

Abstract

Cost of time passing plays an important role when investigate the collective behaviour in real world. Each rational individual can get a more reasonable strategy by comprehensively considering the time-cost. Motivated by the fact, we here propose a mechanism with individual time-cost heterogeneity whose core lies in three aspects: 1. Each individual is given a time-cost parameter that takes into account the effect of time-cost on benefits when they interact with their neighbors. 2. The time-cost is divided into a fixed time-cost that maintains a fixed value and a unfixed time-cost where each individual time-cost is different for each round. 3. Four different update methods are used to study evolution cooperating with time-cost on the rule lattice. Simulation results show that the proposed mechanism effectively promotes cooperation in the snowdrift game. Moreover, high time-cost are more conducive to higher levels of cooperation. Within the same range of time-cost, fixed time-cost promote cooperation better than unfixed time-cost. And among the four update methods, the asynchronous update method is better than the synchronous update method, and the results of the asynchronous update method are similar but the process is different.

Keywords

Snowdrift Game; Time-cost; Cooperation; Evolutionary Game; Heterogeneity.

1. Introduction

Natural selection is the core idea of Darwin's theory, and the law of the jungle is a common phenomenon in the process of evolution [1, 2]. Individuals survive by adapting to nature, perish from not fitting to nature. However, from ants to bees, monkeys to wolves, and even human society, cooperative behavior is everywhere [3, 4, 5]. The choice of cooperation is the loss of their own interests, to benefit others [6, 7, 8, 9, 10]. So, the contradiction between individuals and groups formed a social dilemma [11, 12, 13, 14]. The study of evolutionary game provides a powerful mathematical framework for solving social dilemmas. Up to now, many social dilemma models have been evolved, such as snowdrift game, prisoner's dilemma, public goods game, etc [15, 16, 17, 18, 19, 20]. In order to understand the damage to individual benefits and promote the interests of groups of cooperative behavior [21].

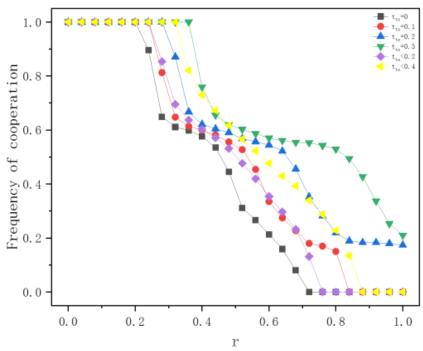
In the classical snowdrift game model, it is assumed that there is such a scenario: Two drivers are trapped on the road by a snowdrift in snowy days. At this time, drivers can choose to get off to shovel snow (cooperate) or stay in the car (defect). If both choose to cooperate, they will get benefit R, otherwise they will get benefit P; If one chooses to cooperate, one chooses to defect, the defector will get temptation T, and the cooperator will get the benefit S. where $T > R > S > P$, $2R = T + S$. Scholars have made great progress in the study of snowdrift game model in the past. In 2006, Nowak

 Volume 7 Issue 10, 2021
DOI: 10.6919/ICJE.202110_7(10).0049

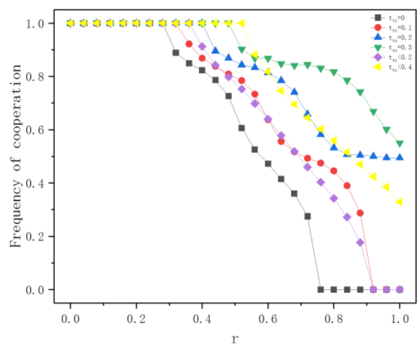
summarized the previous studies into five mechanisms [22, 23, 24, 25, 26, 27, 28]: Kinship selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection. For a long time, the study of evolutionary game has been carried out around these five mechanisms. In recent years, the focus of evolutionary game has gradually shifted to individual attributes and environmental rules. Among them, the mechanism of individual attributes is like memory mechanism [29, 30, 24, 31, 32, 33, 34, 35], reputation mechanism [36, 37, 38], individual learning ability mechanism [39, 40, 41], etc. Such as update delay [42], the three-strategy game [43] belongs to the mechanism of environmental rules. Research shows that the above mechanisms are effective in promoting cooperation. Based on the above, this paper tries to propose a rules including individual attributes and environment rule in the snowdrift game.

In this paper, we propose a new individual time-cost heterogeneity (ITCH) mechanism. In the snowdrift game model, two drivers are trapped on the road, except for the snowdrift in front of them, Time is constantly passing, and losing a lot of it will also take its toll. So, besides the cost of shoveling snow, we should also consider the time-cost. On the one hand, when two drivers make different choices, it takes different time to shovel snow. On the other hand, the respective time-costs of the two drivers are also different. In this paper, the simulation are carried out on four different update rules (synchronous update, sequential asynchronous update, asynchronous random non-repeat update, asynchronous random repeat update) in the rule network. The study found that the ITCH mechanism could effectively improve the level of group cooperation compared with the traditional mechanism. However, the results of the four update rules are different.

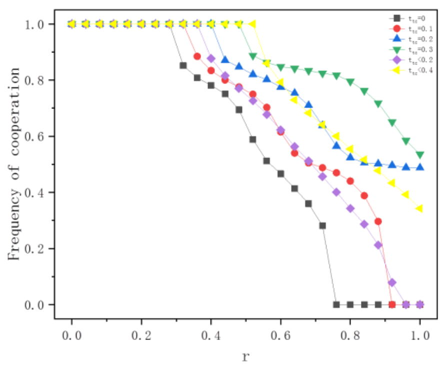
The rest of this paper is organized as follows: In Section 2, we introduce a model of ITCH mechanism. In Section 3, we show the simulation results and analyze it. In Section 4, we give the conclusion of this article.



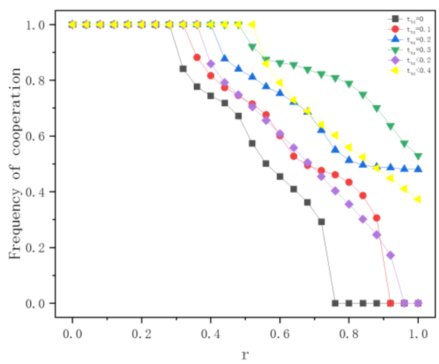
(a) Synchronous update.



(b) Sequential asynchronous update.



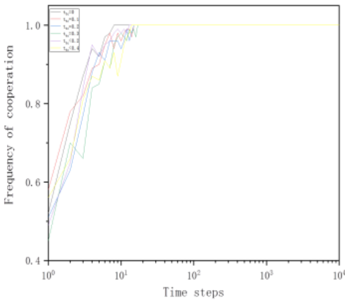
(c) Asynchronous random non-repeat update.



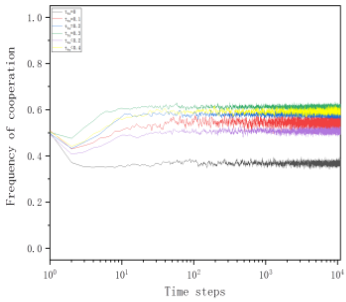
(d) Asynchronous random repeat update.

Figure 1. The level of cooperative F is represented by different t_{tc} with r, where k = 0.1 under the four update methods. When t_{tc} = 0 is the conventional model.

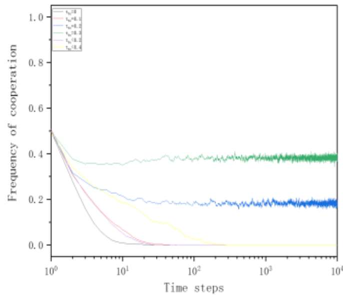
Volume 7 Issue 10, 2021
DOI: 10.6919/ICJE.202110_7(10).0049



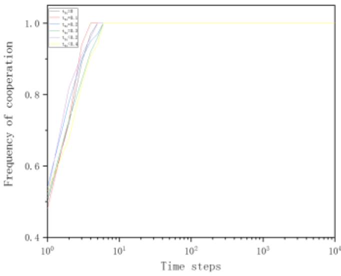
(a) r = 0.1



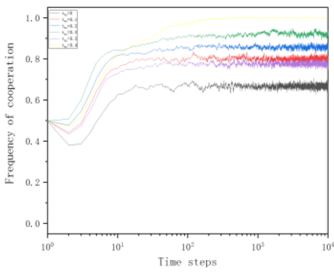
(b) r = 0.5



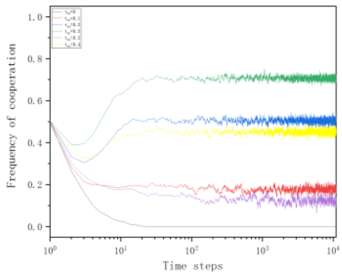
(c) r = 0.9



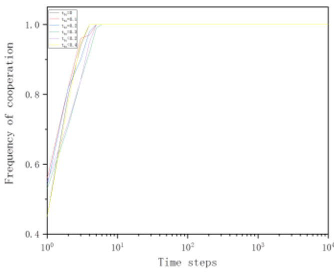
(d) r = 0.1



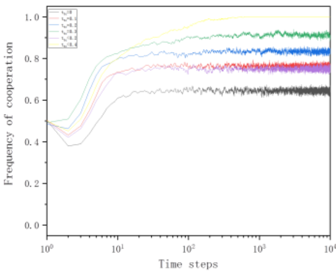
(e) r = 0.5



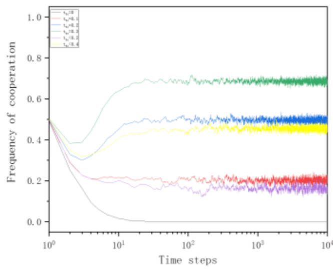
(f) r = 0.9



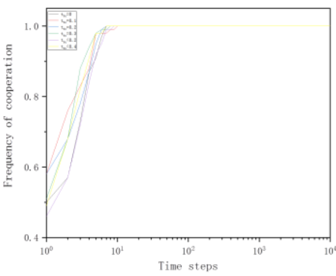
(g) r = 0.1



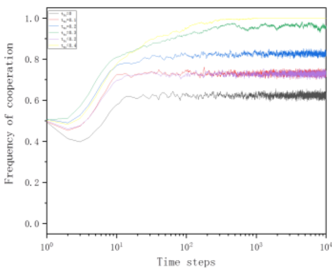
(h) r = 0.5



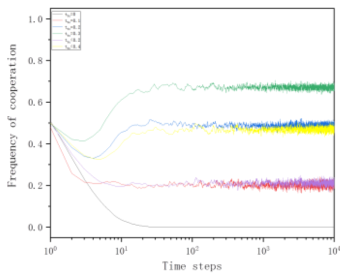
(i) r = 0.9



(j) r = 0.1



(k) r = 0.5



(l) r = 0.9

Figure 2. When r = 0.1, 0.5, 0.9, the time series evolution of the proportion by cooperators F on square lattices for different t_{tc}. Where (a), (b), (c) is synchronous update, (d), (e), (f) is 4 sequential asynchronous update, (g), (h), (i) is asynchronous random non-repeat update, (j), (k), (l) is asynchronous random repeat update.

2. Model

We construct a 100×100 regular network [44]. Each point represents an individual x_i, and the four surrounding points represent neighbors $y_{i,n}$, i=[1,10000], n e{1,2,3,4}. The article uses c to represent the cost of snow shoveling, and b to represent the individuals' benefit, so the benefit matrix become:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202110_7(10).0049

$R = b-c/2$, $T = b$, $S=b-c$, $P=0$, obey $T>R>S>P$. Let $R=1$, and we use a parameter r to represent the cost-benefit ratio of snow shoveling. Then the benefit matrix become: $R = 1$, $T=1+r$, $S = 1-r$, $P=0$. Then $r=c/2=c/(2b-c)$, where $0<r<1$. In the classical snowdrift game model, two drivers are trapped on the road by a snowdrift. In addition to considering the cost of snow shoveling, we should also consider the impact of the time passing. We introduce a parameter t_{tc} of time-cost (representing the proportion of time-cost to total cost). When one chooses to cooperate with another chooses to defect, the defector does not pay the cost of shoveling snow, but he waits for the same time on the road as the cooperator and expended the time-cost. Therefore, at this time, the benefits of cooperators and defectors should be changed as follows: $S=1-r-2rt_{tc}$, $T=1+r-2rt_{tc}$. When both choose to cooperate, the time spent on shoveling snow is reduced by half due to the two drivers shoveling snow together, so the benefit at this time become $R=1-rt_{tc}$. On the contrary, when both chose to defect, the two drivers remained stranded on the road at all. At this point, the benefit become: $P=0-4rt_{tc}$.

Each individual x_i interacts with neighbor $y_{i,n}$, and gets a benefit $P_{yi,n}$ every time, which is repeated four times among four neighbors. Then calculate the cumulative income Pxi,t of four games, where t means round. The formula is:

$$P_{xi,t}=P_{yi,n,t} \tag{1}$$

The ITCH mechanism includes fixed time-cost and unfixed time-cost. In the constructed grid network, it is assumed that in each independent simulation of evolutionary game, if the time-cost takes a fixed value, it is called fixed time-cost, if the time-cost takes an arbitrary value for each individual in every round in a certain set, it is called unfixed time-cost. The Fermi update function [45] is used to update the strategy of each round. the player who has a higher benefit can use the strategy by probability. The formula is:

$$w(S_{x,t+1} \leftarrow S_{y,t+1}) = \frac{1}{1+exp[-(P_{x_i,t}-P_{y_i,n,t}/k)]} \tag{2}$$

Where k represents the rational factor. The individual uses the strategy of comparing neighbors with him in the next round by a probability of W. $k\rightarrow 0$ means that the individual keeps absolute rationality. In every case, if $P_{yi,n,t}>P_{xi,t}$, the individual x will use the neighbor's strategy in the next round, and on the contrary, it will not change the strategy. $k\rightarrow\infty$, the individual will not be affected by the benefit and strategy, and each round of strategy will be randomly selected. So we choose a reasonable value and make $k=0.1$.

This article uses four kinds of update way to research, and we will introduce them below:

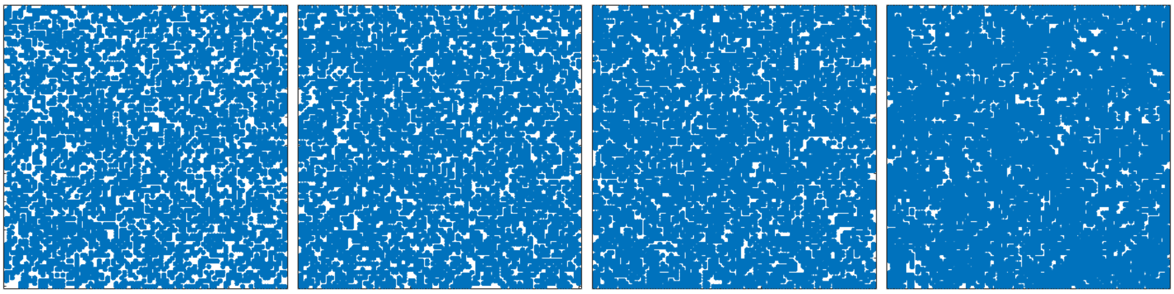
1. Synchronous update (when each round of games started, individuals $x_1, x_2 \ldots x_i$ simultaneously play games with their four neighbors $y_{i,n}$ and calculate the cumulative benefit, while using the Fermi rule update strategy)

2. Sequential asynchronous update (when each round of games is started, firstly, selecting an individual x_1, calculating the accumulated benefit P_{xl} after playing games with four neighbors $y_{i,n}$ of x_1, and then randomly selecting a neighbor to perform Fermi update to obtain a new strategy S_1' of x_1, which replaces S_1 in the original strategy. Then select $x_2, x_3 \ldots x_i$ in sequence, and repeat the above steps until all the individuals are updated)

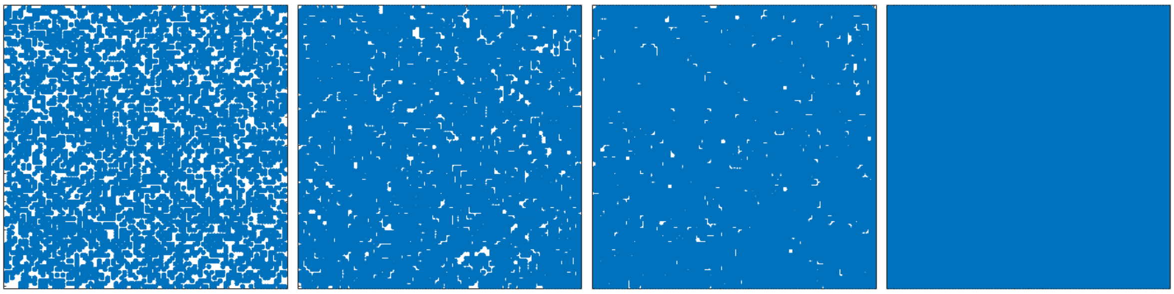
3. Asynchronous random non-repeat update (when each round of game started, we randomly select an individual x in the individual set x_i, calculating the accumulated benefit P_x after playing games with four neighbors y_n of x, and then randomly selecting a neighbor to perform Fermi update to obtain a new strategy S' of x, which replaces S in the original strategy. On the next selection, x is removed from the set and then randomly selected from the set, repeat that above steps until all individual are updated)

4. Asynchronous random repeat update (when each round of game started, we randomly select an individual x in the individual set x_i, calculating the accumulated benefit P_x after playing games with four neighbors y_n of x, and then randomly selecting a neighbor to perform Fermi update to obtain a new strategy S' of x, which replaces S in the original strategy. This process is repeated i rounds)

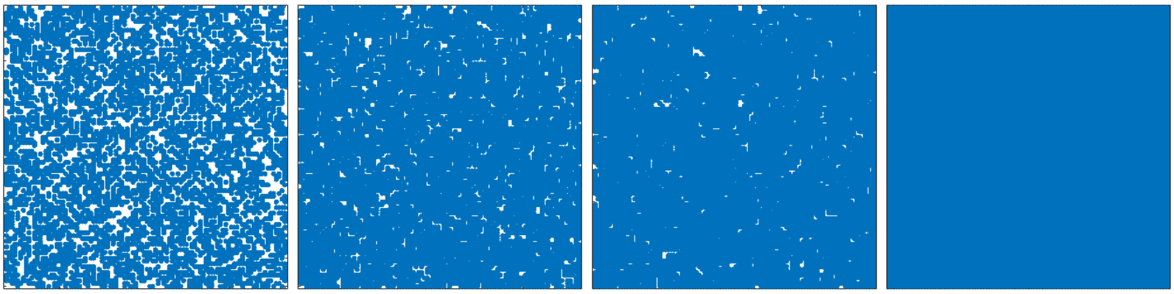
Volume 7 Issue 10, 2021
DOI: 10.6919/ICJE.202110_7(10).0049



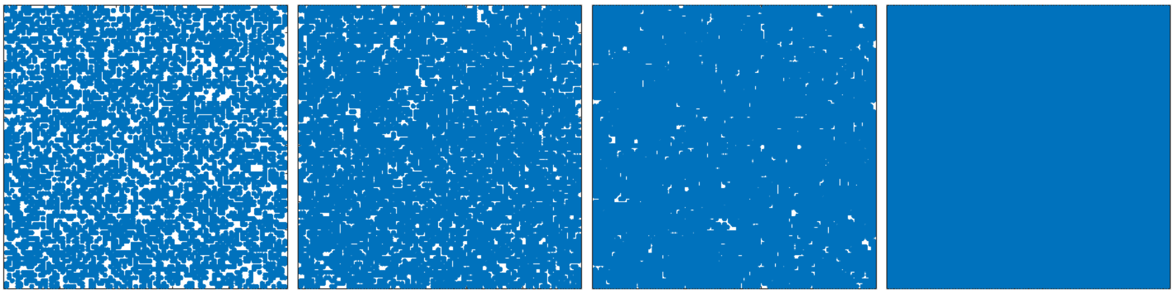
(a) Synchronousupdate. Timestep = 1, 5, 10, 50.



(b) Sequentialasynchronousupdate. Timestep = 1, 5, 10, 50.



(c) Asynchronousrandomnon−repeatupdate. Timestep = 1, 5, 10, 50.



(d) Asynchronousrandomrepeatupdate. Timestep = 1, 5, 10, 50.

Figure 3. A snapshot diagram of the characteristics of cooperators (blue) and defectors (white) at different update methods and time steps for t_{tc} = 0.36 on SDG.

3. Simulation and discussion

The simulation is carried out on a 100×100 regular network. The evolution process starts from the random state. When t =0, the probability of each individual choosing cooperation or defection during initialization is 0.5. In the traditional model, the level of group cooperation decreases with the increase of cost until it disappears. We discuss the impact of t_{tc} with different time costs on evolutionary cooperation by four update rules. The cooperation level F is calculated by the 103 rounds evolution cooperation proportion value after 10^4 rounds of evolution game, and in order to ensure the accuracy, each point in the simulation is realized by independent simulation of an average of 20 times.

We started to discuss the cooperative behavior of the ITCH mechanism in the snowdrift game for different values of time-cost. In Fig. 1, the ITCH mechanism effectively promotes cooperation in all

ISSN: 2414-1895 DOI: 10.6919/ICJE.202110_7(10).0049

four update methods compared to the corresponding traditional model (t_{tc}=0). First, we observe Fig. 1(a), in the synchronous update method, and find that the higher cooperation level is maintained up to r=0.24 at t_{tc}=0, and then F starts to decline and dies out completely at r=0.72. In contrast, $t_{tc}\neq 0$ can maintain F at high cooperation levels for a longer period of time, and in particular, cooperation levels with larger t_{tc} have less impact on cost-benefit ratio increase. Considering the same range of fixed time-cost t_{tc}=0.2, and unfixed time cost t_{tc}<0.2, the cooperation level of fixed value is better than that of unfixed value. Then observe the other three update methods, in Fig. 1(b), (c), (d), similarly high cooperation levels F are achieved in the case of $t_{tc} \neq 0$. The three models have similar variations for r value. It is noteworthy that at r=0.76, the level of cooperation disappears early. However, the model still maintains a high level of cooperation at t_{tc} 0. In particular, cooperators survival at t_{tc} = 0.2, 0.3 and t_{tc} < 0.4, up to r = 1.

From the above description, it is clear that the ICTH mechanism effectively promotes cooperation, and for larger t_{tc}, the level of cooperation is higher and decreases more slowly at the same r value. Comparing the four figures reveals that the results of the three asynchronous update methods are similar and better than the synchronous update for improving the cooperation level under any of the same conditions. The reason for its occurrence is because during the update process, the synchronous update causes all the individuals to be updated at the same time, all the strategies and benefits are changed at the same time, and no new strategies are added when calculating the next round of strategies. However, unlike asynchronous updating, in which individuals are updated one by one in a round, the updated individual strategies and benefits are based on the optimal strategies derived in the previous round, and this new strategy is added to the round for further computation.

In order to further verify the results and explain the causes leading to these phenomena, the time series evolution and micro-evolutionary processes of the parts are observed below.

In the synchronous update described in Fig. 2(a), for small cost-benefit ratio r=0.1, the cooperation level F grows rapidly for all cases and reaches a stable peak after a short period. For a middle value of r=0.5. In Fig. 2(b), all values fall for a few cycles before rising. It is clearly seen that t_{tc}=0,0.1 as well as t_{tc} < 0.2 are smaller than the original value when reaching stability, and t_{tc}=0.2,0.3 and t_{tc}<0.4 start to rise after just a few decreasing cycles until stabilization. When the temptation is high r = 0.9, in Fig. 2(c), t_{tc}=0.2, 0.3 still can keep fewer cooperators survive and form a stable trend. And other cases after several cycles cooperation has died out. The evolution of the cooperative behavior is then observed for sequential asynchronous update. For small r = 0.1 Fig. 2(d), F peaks within a few cycles for all t_{tc}. For a medium size r=0.5 (Fig. 2(e)), the cooperation level increases according to the size of t_{tc} and remains at a high level (F>0.5). Increasing to r = 0.9, we find that both t_{tc}=0.2,0.3 and t_{tc}<0.4 remain at high F remain stable, while t_{tc}=0.1 and t_{tc}<0.2 decrease, but remain above t_{tc}. Finally, the evolution of cooperative behavior is observed for asynchronous random non-repeat update and asynchronous random repeat update. In Fig. 2(g)−(k), they are all similar to sequential asynchronous update after stabilization, and in Fig. 2(l), t_{tc}=0.1 and t_{tc}<0.2, t_{tc}=0.2 and t_{tc}<0.4 almost overlap. In addition, the three asynchronous update methods are differentiated during the tolerance period (trying to tolerate individual choice defect) and the expansion period (when cooperators form tight clusters and convert neighboring defectors). As shown in Fig. 2, asynchronous random repeat update show the largest fluctuations in the tolerance period, especially larger fluctuations in Fig. 2(j) and Fig. 2(l), followed to asynchronous random non-repeat update, and sequential asynchronous update are better than both.

In Fig. 3, we observe the micro-evolutionary processes of the four update methods at r =0.36 and t_{tc} = 0.2. With the inclusion of the ITCH mechanism, the network environment is suitable for the survival of cooperators. However, in the synchronous update, both cooperators and defectors form clusters at the same time to defend each other against invasion. With this update mechanism, the defectors successfully resist the expansion of the cooperators. In contrast, in sequential asynchronous update and random non-repeatable update, the cooperators quickly form small clusters and swallow the defectors in a very quick turn. In the asynchronous random repeat updates, although the result is the same as the other two updates, the process is greater for the cooperators to form clusters and engulf the betrayers than in the first two asynchronous updates.

4. Conclusions

In this paper, we propose an ICTH mechanism based on the snowdrift game by considering both the heterogeneity of individuals' own attributes and the environmental rules (time-cost) in the evolutionary game. The effects of fixed time-cost and unfixed time-cost on the snowdrift game are explored. And four types of update methods: Synchronous update, sequential asynchronous update, asynchronous random non-repeat update, and asynchronous random repeat update are used for simulation. The simulation results show that the ICTH mechanism effectively promotes cooperation. Under the same conditions, the high time-cost are more conducive to higher levels of cooperation. And within the same range of time-cost, fixed time-cost promote cooperation better than unfixed time-cost. This suggests that time-cost have great relevance in the study of cooperation in real societies, and that groups bearing the same losses promote cooperation more than individual heterogeneity. Among the four update methods, synchronous update has the advantage of simplicity, while asynchronous update has to select the individuals gradually in each cycle. And the asynchronous update results in better than the synchronous update. In the asynchronous update method, the evolutionary process and results were similar for sequential asynchronous update and asynchronous random non-repeat update by the different update order, and the evolutionary process was different for asynchronous random repeat update, but the results were the same. It indicates that the order has almost no effect on group cooperation influence, and the selection method has no effect on group cooperation when the period is long enough.

Acknowledgments

This work was partially supported by National Science Foundation (62073270, 61673016, 61703353), Innovation Research Team of the Education Department of Sichuan province (15TD0050), and the Graduate Innovative Research Project of Southwest Minzu University, Project number: CX2020SZ90.

References

[1] CARMAN, TAYLOR, Darwinian theory., TLS (5580) (2010) 6–6.

[2] Whitfield, John, Bushmeat: The law of the jungle, Nature 421 (6918) (2003) 8–9.

[3] L. A. Dugatkin, Cooperation in animals: An evolutionary overview, Biology and Philosophy 17 (4) (2002) 459–476.

[4] T. Clutton-Brock, Cooperation between non-kin in animal societies, Nature 462 (7269) (2009) 51–57.

[5] R. Gadagkar, P. A. Gowaty, Survival strategies: Cooperation and conflict in animal societies., quarterly review of biology 81 (4) (2000) 132.

[6] R. Axelrod, W. D. Hamilton, The evolution of cooperation, Quarterly Review of Biology 79 (2) (1981) 135–160.

[7] Xu-Wen, Wang, Sen, Nie, Luo-Luo, Jiang, Bing-Hong, Wang, Shi-Ming, Chen, Cooperation in spatial evolutionary games with historical payoffs, Physics Letters A.

[8] G. Szabo, G. Fath, Evolutionary games on graphs, Physics Reports 446 (4) (2007) 97–216.

[9] R. C. Lewontin, Evolution and the theory of games, Journal of Theoretical Biology 1 (3) (1961) 382–403.

[10] J. M. Smith, G. R. Price, The logic of animal conflict, Nature 246 (5427) (1973) 15–18.

[11] P. Kollock, Social dilemmas: The anatomy of cooperation, Annual Review of Sociology 24(1998)183–214.

[12] T. Cheon, Altruistic duality in evolutionary game theory, Physics Letters A 318 (4-5) (2003) 327–332.

[13] W. X. Wang, J. Ren, G. Chen, B. H. Wang, Memory-based snowdrift game on networks, Physical Review E 74 (5 Pt 2) (2006) 056113.

[14] F. F. A. B, X. C. A. B, L. L. A. B, L. W. A. B, Social dilemmas in an online social network: The structure and evolution of cooperation, Physics Letters A 371 (12) (2007) 58–64.

[15] T. Wu, F. Fu, L. Wang, Coevolutionary dynamics of aspiration and strategy in spatial repeated public goods games, New Journal of Physics 20 (6) (2018) 063007.

[16] T. Wu, L. Wang, Adaptive play stabilizes cooperation in continuous public goods games, Physica A-statistical Mechanics and Its Applications 495 (2018) 427–435.

[17] Q. Su, A. Li, L. Wang, Evolutionary dynamics under interactive diversity, New Journal of Physics 19 (10) (2017) 103023.

[18] Z. Wu, Z. Rong, M. Z. Q. Chen, Diverse roles of the reduced learning ability of players in the evolution of cooperation, EPL 110 (3) (2015) 30002.

[19] F. Fu, M. A. Nowak, C. Hauert, Invasion and expansion of cooperators in lattice populations: Prisoner's dilemma vs. snowdrift games, Journal of Theoretical Biology 266 (3) (2010) 358–366.

[20] J. Vukov, G. Szabo, A. Szolnoki, Cooperation in the noisy case: Prisoner's dilemma game on two types of regular random graphs, Physical Review E 73 (6) (2006) 067103.

[21] C. Taylor, D. Fudenberg, A. Sasaki, M. A. Nowak, Evolutionary game dynamics in finite populations, Bulletin of Mathematical Biology 66 (6) (2004) 1621–1644.

[22] M. A. Nowak, Five rules for the evolution of cooperation, Science 314 (5805) (2006) 1560–1563.

[23] W. D. Hamilton, The genetical evolution of social behaviour. i., Journal of Theoretical Biology 7 (1) (1964) 1–16.

[24] P. A. M. Van Lange, K. Visser, Locomotion in social dilemmas: How we adapt to cooperative, tit-for-tat, and noncooperative partners., Journal of Personality and Social Psychology 77 (4) (1999) 762–773.

[25] J. E. Bone, B. Wallace, R. Bshary, N. J. Raihani, The effect of power asymmetries on cooper- ation and punishment in a prisoner's dilemma game, PLOS ONE 10 (1).

[26] R. Albert, A. Barabasi, Statistical mechanics of complex networks, Reviews of Modern Physics 74 (1) (2001) 47–97.

[27] M. A. Nowak, S. Bonhoeffer, R. M. May, Spatial games and the maintenance of cooperation, Proceedings of the National Academy of Sciences of the United States of America 91 (11) (1994) 4877–4881.

[28] M. Nakamaru, H. Matsuda, Y. Iwasa, The evolution of cooperation in a lattice-structured population, Journal of Theoretical Biology 184 (1) (1997) 65–81.

[29] F. Shu, Y. Liu, X. Liu, X. Zhou, Memory-based conformity enhances cooperation in social dilemmas, Applied Mathematics and Computation 346 (2019) 480–490.

[30] F. Shu, X. Liu, K. Fang, H. Chen, Memory-based snowdrift game on a square lattice, Physica A-statistical Mechanics and Its Applications 496 (2018) 15–26.

[31] F. Shu, M. Li, X. Liu, Memory mechanism with weighting promotes cooperation in the evolutionary games, Chaos Solitons and Fractals 120 (2019) 17–24.

[32] W. X. Wang, J. Ren, G. Chen, B. H. Wang, Memory-based snowdrift game on networks, Physical Review E 74 (5 Pt 2) (2006) 056113.

[33] S. Qin, Y. Chen, X. Zhao, J. Shi, Effect of memory on the prisoner's dilemma game in a square lattice, Physical Review E Statistical Nonlinear and Soft Matter Physics 78 (4).

[34] J. Wang, L. N. Liu, E. Z. Dong, L. Wang, An improved fitness evaluation mechanism with memory in spatial prisoner's dilemma game on regular lattices, Communications in Theoretical Physics 59 (003) (2013) 257–262.

[35] Y. Dong, H. Xu, S. Fan, Memory-based stag hunt game on regular lattices, Physica A: Statistical Mechanics and its Applications 519 (2019) 247–255.

[36] Q. A. Ji, A. Ct, A. Yz, B. Xw, C. Jby, Reputation evaluation with tolerance and reputation- dependent imitation on cooperation in spatial public goods game, Chaos, Solitons and Fractals 131.

[37] J. He, J. Wang, F. Yu, L. Zheng, Reputation-based strategy persistence promotes cooperation in spatial social dilemma, Physics Letters A 384 (27) (2020) 126703.

[38] Q. Pan, L. Wang, M. He, Social dilemma based on reputation and successive behavior, Applied Mathematics and Computation 384 (2020) 125358.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202110_7(10).0049

[39] T. Wu, H. H. Wang, J. Yang, L. Xu, Y. Li, J. Zhang, The prisoners dilemma game on scale-free networks with heterogeneous imitation capability, International Journal of Modern Physics C29(09)(2018)1850077.

[40] A. Szolnoki, M. Perc, Coevolution of teaching activity promotes cooperation, New Journal of Physics (3).

[41] J. Guan, Z. Wu, Y. Wang, Effects of inhomogeneous activity of players and noise on cooperation in spatial public goods games, Physical Review E 76 (5 Pt 2) (2007) 056101.

[42] D. Wang, X. Shuai, Q. Pan, J. Li, X. Lan, M. He, Long deliberation times promote cooperation in the prisoner's dilemma game, Physica A: Statistical Mechanics and its Applications 537.

[43] A. Szolnoki, M. Perc, G. Szab, Phase diagrams for three-strategy evolutionary prisoner's dilemma games on regular graphs, Phys Rev E Stat Nonlin Soft Matter Phys 80 (5 Pt 2) (2009) 056104.

[44] W. B. Du, X. B. Cao, M. B. Hu, H. X. Yang, Z. Hong, Effects of expectation and noise on evolutionary games, Physica A: Statistical Mechanics and its Applications 388 (11) (2009) 2215–2220.

[45] C. Y. Xia, L. Wang, J. Wang, J. S. Wang, Behavior of collective cooperation yielded by two update rules in social dilemmas: Combining fermi and moran rules, Communications in Theoretical Physics.