

Based on Genetic Algorithm to Determine the Best Three-dimensional Geometric Shape of a Sandcastle

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Abstract

Where there are beaches in the world, children and adults are happily building sandcastles. We all hope that sandcastle can be more stable so that it can exist for longer. However, tides and waves can cause damage to these buildings. Therefore, it is very interesting and meaningful to establish an optimal three-dimensional sandcastle foundation model. In this paper, we predict based on empirical knowledge that the best three-dimensional geometric shape of the sand castle may resemble a round table or an oval table. The numerical simulation of the spatial flow field with Fluent software verifies our preliminary idea. Then, based on the knowledge of hydrostatics and dynamics, a basic model of sand castle erosion by seawater was established. Next, we consider the sandcastle's own ability to resist damage, and use stiffness to measure this ability. Finally, we use genetic algorithms to find the approximate slope angle of the optimal three-dimensional geometry with a sand castle volume of 0.1m, which is about 30 degrees.

Keywords

Fluent software, The best three-dimensional geometric shape, Genetic algorithm, Kinetic model.

1. Introduction

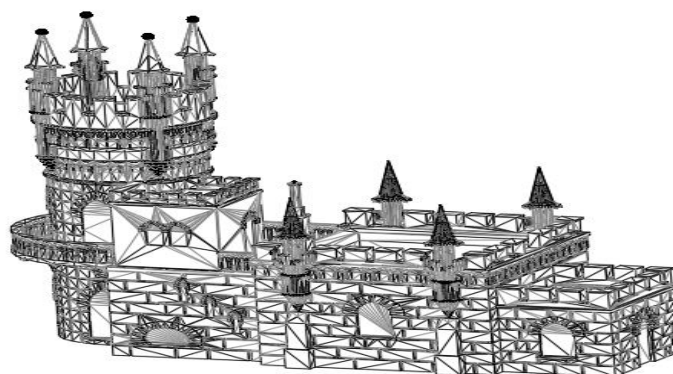


Figure 1. The hand drawing of a castle

Among the various entertainment activities on the beach, sandcastles are an indispensable form. No matter who you are, you will be attracted by its fascinating appearance, interesting functions, and the inner logic of mathematics. This is what we are focusing on in this paper. First of all, given that the castle was built by the sea, there is no doubt that it will be affected by the surrounding environment,

including tides and waves. More importantly, considering these factors, what we should do now is to build a sandcastle that can stand the longest time in such an environment. At the same time, we can know from the problem that the sandcastle foundation is the core that affects the life of the castle. Therefore, it is very important to establish an optimal sandcastle foundation.

2. Assumptions

After a thorough analysis of the problem, to simplify our model, we propose the following reasonable assumptions.

- 1) We believe that after the sand and water are mixed uniformly, the overall bonding situation is better, and there are no special locations that will greatly affect the overall stability of the sandcastle.
- 2) Because sandcastles are piled on the beach, and excessive waves and tides are only occasional, we believe that the flow of seawater is a stable laminar flow, and the speed is small and constant. We mainly consider the effects of tides and waves on sand. External factors of the fort.
- 3) Analyzing the state of sand particles from a microscopic perspective is very complicated, so we consider the macroscopic perspective and use the sandcastle foundation as a whole to simplify the model.
- 4) Because the waves and tides are highly random, we assume that as long as the sandcastle's ability to resist erosion is stronger, it is considered to last longer.

3. Numerical Model Analysis of the Fluid Velocity

Most of the sandcastle shapes you usually see are cylinders, indicating that the cylinders have a certain strength to resist the erosion of seawater and tides. We all know that the resistance of fish swimming in the water and rockets in the air at very high speed is very small compared to other objects of the same volume and different shapes, indicating that the ellipse may be a good alternative for the sandcastle foundation. Then inspired by the knowledge of fluid mechanics, using Fluent software to do numerical simulation analysis of the circular and elliptical flow fields[1], the plane position z (height) is the spatial flow field distribution map (velocity distribution cloud map) at the centroid height.

It can be concluded from the Figure 2(The left is a round table, the right is an oval table) that the velocity of a fluid with a certain flow rate does not change much after it passes around an oval table, indicating that the fluid is not much resisted by the obstacle, which is known according to the force interaction. Therefore, we decided to proceed from the three-dimensional geometric shapes of the round table and oval table to find the optimal basic shape of the sandcastle.

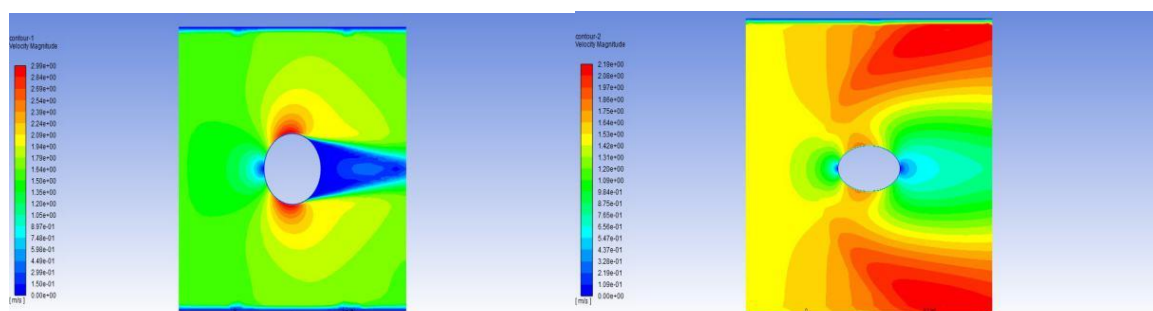


Figure 2. Cloud diagram of velocity distribution in space

4. Seawater-sandcastle kinetic Model

4.1 Pressure of a Stationary Liquid on a Curved Surface

Finding the total pressure of the static liquid acting on the curved surface belongs to the summation of the space force system. Because the direction of the force at different points is different, the total

pressure cannot be obtained by directly integrating the area like a flat wall. The force at each point should be decomposed first, that is, it should be converted into two sets of parallel The sum problem of the force system, and then add up to find the resultant force. The calculation steps of the total pressure of the static liquid acting on the surface are as follows.

- 1) Decompose total pressure into horizontal component force F_{sx} and a vertical component force F_{sz} .
- 2) Calculation of horizontal component force.
- 3) Determine the volume of the pressure body.
- 4) Calculation of vertical component force, the direction is determined by virtual and real pressure bodies.
- 5) Calculation of total pressure.

4.2 The Dynamic Pressure of Seawater Flow on Sandcastle Foundation

When the fluid of velocity v hits a stationary object, Newton first proposed that resistance is proportional to the square of velocity. According to the knowledge of the interaction force, the resistance of the object to the fluid is equal to the dynamic pressure of the fluid on the object [2]. Here ρ is the fluid density and σ is the cross-sectional area of the object perpendicular to the direction of motion, and the calculation of F_d is shown in formula (1)

$$F_d = \frac{d(mv)}{dt} = (\rho w v \sigma) v = \rho w \sigma v^2 \tag{1}$$

4.3 Sandcastle's Stiffness

When an object is subjected to external forces, its ability to resist deformation is called stiffness [3]. From a macro perspective, the elastic modulus of an object is a measure of its ability to resist shear deformation. The relationship between stiffness F_r and elastic modulus E_c is as shown in formula (2)

$$F_r = A_x E_c \tag{2}$$

Where, A_x represents the cross-sectional area of the object and the size of E_c can be set as 3.5MPa.

4.4 Sandcastle Foundation Force Model

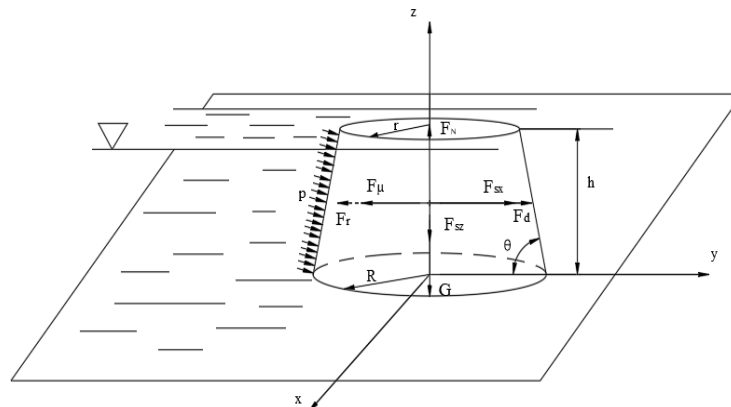


Figure 3. Sandcastle force analysis

From the force analysis chart of the sandcastle foundation, We can get the horizontal total pressure F on the sand castle as shown in formula (3).

$$F = \mu(F_{sz} + G) + F_r - F_d - F_{sx} \tag{3}$$

Where, G is the gravity of the sandcastle.

5. Genetic Algorithm to Optimize the Best Geometric Shape

Find out the optimal 3-dimensional geometry of the sandcastle foundation, so that the sandcastle foundation will persist for the longest time under the action of waves and tides. Based on empirical knowledge, we get the following assumptions.

1) Sandcastles are kept at the same distance from the seawater on the same beach, and the seawater on the coast is relatively stable, and the speed does not change much. In order to simplify the problem, we think that the seawater can be regarded as a laminar flow state, the speed v of the seawater is a constant value, and the value is 1m/s.

2) It is mentioned in the title that the type and quantity of sand used to build the sandcastle are the same, and the same water-sand mixture ratio. We checked the data to understand that the sand on the beach is similar to the type of medium sand used in construction [3]. Therefore, the sand density in this question is $\rho = 1350\text{kg} / \text{m}^3$, and the foundation volume is a fixed value V.

3) The quality of the sand castle foundation is very important for the construction of sand castles, so before finding the best 3-dimensional geometric shape of the sand castle foundation, we take the upper and bottom area of the geometric shape as a fixed value $S1=0.04\text{m}^3$, in order to Continue to build on the basis of the sandcastle.

The previous paper mentioned the two three-dimensional geometric shapes of the round table and oval table as the basis of the sandcastle. Therefore, in order to find the best three-dimensional geometric shape[4], we used genetic algorithms to change the bottom surface area S2 and height h of the geometry. Optimize the three-dimensional geometric model, that is, find the maximum resultant force F. The linear programming model of this problem is shown in the following formula (4).

$$\begin{cases} \max F(x1, x2) \\ x1 > r, a_1 \\ V_i = 0.1 \\ V_1 = \frac{1}{3} \pi h_1 (R^2 + r^2 + Rr) \\ V_2 = \frac{1}{3} \pi h_2 (a_1 b_1 + a_2 b_2 + \sqrt{a_1 b_1 a_2 b_2}) \end{cases} \tag{4}$$

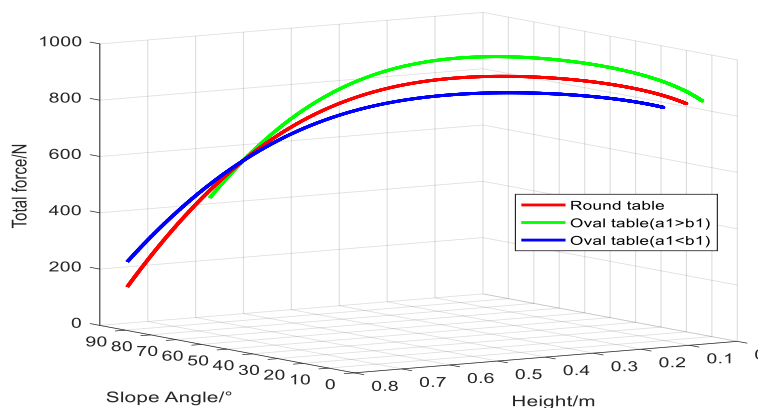


Figure 4. The relationship between total force and slope angle and height

We carry out the optimization of two shapes of a round table and an oval table. The oval table includes two cases. One is that the interface through the short axis is perpendicular to the direction of seawater flow ($a_1 > b_1$) and the other is the interface with the long axis. Perpendicular to the direction of seawater flow ($a_1 < b_1$). The above figure simulates the relationship between the combined force and the slope angle and height. It can be seen that in three cases, the combined external force shows a trend of increasing first and then decreasing as the slope angle decreases. There is an optimal solution, that is, the most excellent geometry. The following table 1 shows the size of the geometry for the optimal solution in three cases.

Table 1. The size of the geometry for the optimal solution in three cases

	Upper Surface(m)	Under Surface(m)	Height(m)	Slope Angle(°)	Total Force(N)
Round table	$r=0.20$	$R=0.54$	0.21	31.9	911.6
Oval table ($a_1 > b_1$)	$a_1=0.24$ $b_1=0.16$	$a_2=0.67$ $b_2=0.44$	0.22	27.0	972.9
Oval table ($a_1 < b_1$)	$a_1=0.16$ $b_1=0.24$	$a_2=0.45$ $b_2=0.67$	0.21	37.2	861.6

From the results in the table, we can know that we can get that the round table and oval table have good effects as the foundation of the sand castle. When the slope angle is about 30 degrees, the effect of resisting seawater erosion is the best.

References

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