

# The Analysis and Experiment of the 1st-order Integrator

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## Abstract

Integrator is a kind of circuit, which is used to accumulate the input signal and calculate the result. It usually uses some resistors, capacitors and operational amplifiers. In this paper, the basic concept of the 1st-order integrator is first introduced, then the theoretical analysis, software simulation and practical experiment are carried out. Finally, how to build, design, test and characterize a 1st-order integrator is understood. It is also pointed out that in application, a 1st-order integrator circuit is typically used to eliminate the integral compensation in control according to feedback.

## Keywords

1st-order integrator, Integral, Op-amp.

## 1. Introduction

Integral circuit is a circuit that makes the time integral value of output signal and input signal proportional. It can convert pulse wave input into triangle wave output based on the principle of capacitor charging and discharging [1]. In this lab, the simplest integral circuit, also known as 1st-order integrator, is studied, which is composed of a resistor  $R$ , a capacitor  $C$  and an operational amplifier, as shown in Fig. 1. In general, a resistor can be connected in series at the in-phase input and a large resistor can be connected in parallel on the capacitor.

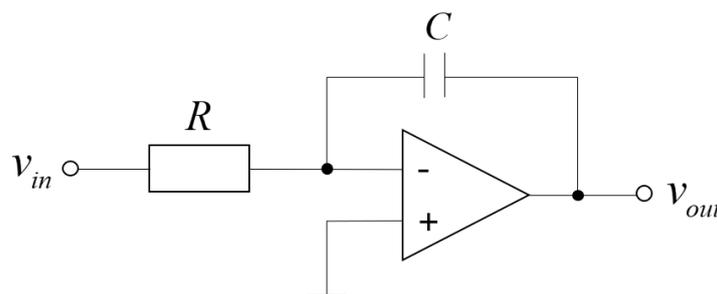


Fig. 1 The 1st-order integrator

## 2. Theoretical analysis

For the 1st-order integrator shown in Fig. 1, given a  $5.1\text{k}\Omega$  resistor, a  $1\mu\text{F}$  capacitor and a LM324 op-amp to design an 1st-order integrator circuit, get

$$v_o(t) = -v_c(t) = -\frac{1}{C} \int_{-\infty}^t i_c(t) dt = -\frac{1}{RC} \int_{-\infty}^t v_i(t) dt = -\frac{1}{5.1 \times 10^{-3}} \int_{-\infty}^t v_i(t) dt \quad (1)$$

The input and output are shown in Fig. 2.

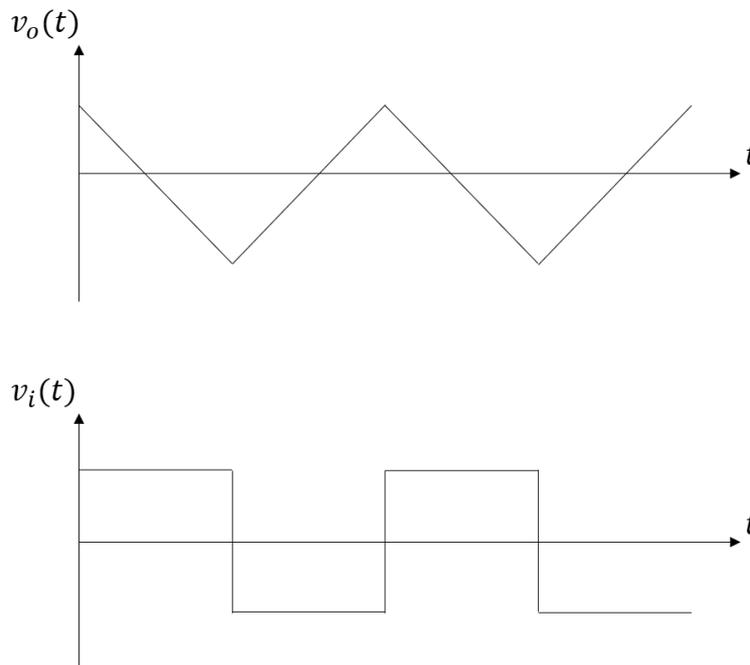


Fig. 2 The theoretical input and output

### 3. Software simulation

Use Multisim 14.0 to simulate the 1st-order integrator circuit, given a 5.009kΩ resistor, a 0.991μF capacitor and a LM324 op-amp to design an 1st-order integrator circuit, input has the frequency of 150Hz, the duty cycle of 50% and the amplitude of 1V, get

$$v_{o_{max}} = v_{max} - v_{DC} = v_{max} - \frac{v_{max} + v_{min}}{2} = 10.497 - \frac{10.497 + 9.382}{2} = 0.33V \quad (2)$$

Compare it with theoretical  $v_{o_{max}} = 0.65V$ , it is concluded that this difference may be due to the difference of resistance and capacitance and the influence of LM324. The circuit and result are shown in Fig. 3.

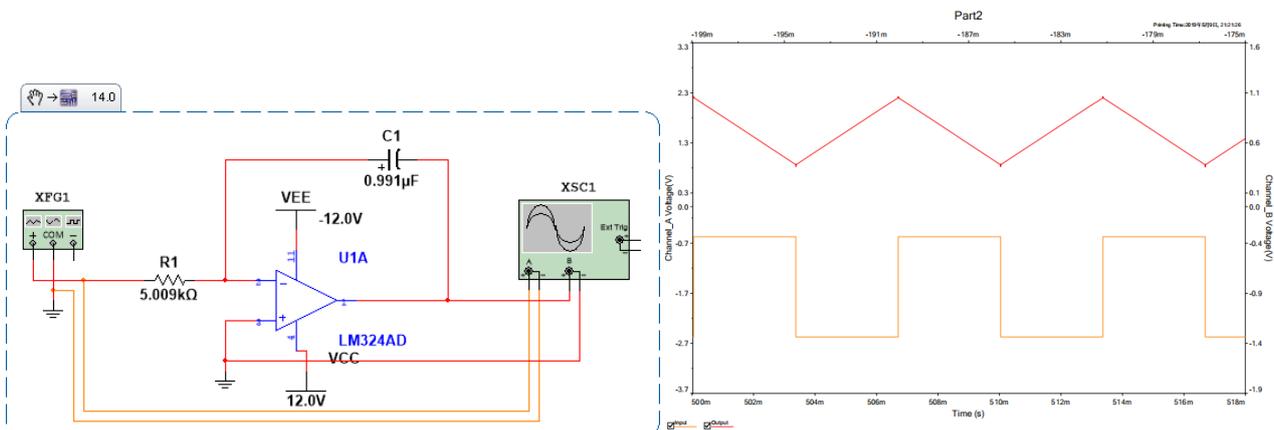


Fig. 3 The simulated circuit and result

### 4. Practical experiment

In order to build the 1st-order integrator circuit on a bread board, observe the output waveform, and measure the maximum value of the output voltage, the procedure includes the following steps:

- (i) Read the Wheatstone bridge circuit shown in Fig. 3.
- (ii) Find a 5.1kΩ resistor and a 1μF capacitor, and measure them by using the Digital Multimeter, get R = 5.1kΩ, C = 1μF.
- (iii) Place each element and connect the circuit correctly as shown in Fig. 4.

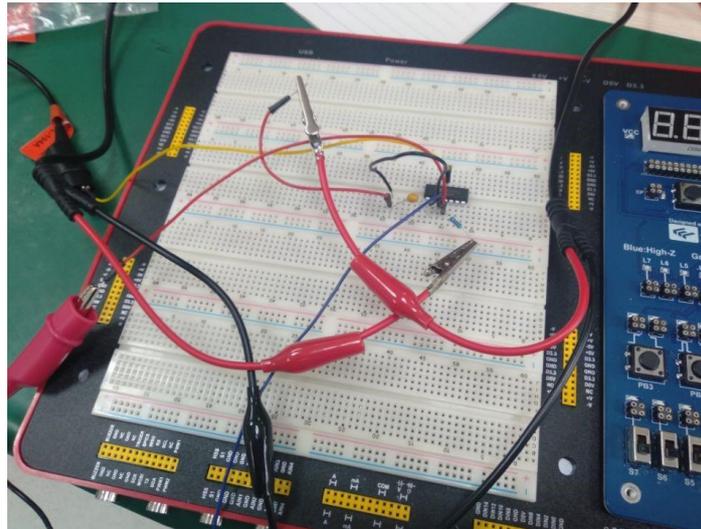


Fig. 4 The actual circuit

- (iv) Select Pulse Function from Function Waveform Generator, set the frequency is 150Hz, the duty cycle is 50% and the amplitude is 1V.
- (v) Adjust Digital Oscilloscope to show completely output waveform as shown in Fig. 5, measure the maximum value of the output voltage, get

$$V_{o\max} = V_{\max} - V_{DC} = V_{\max} - \frac{V_{\max} + V_{\min}}{2} = -9.960 - \frac{-9.960 - 10.860}{2} = 0.45V \quad (3)$$



Fig. 5 The actual result

Compare it with theoretical  $v_{o\max} = 0.65V$ , it is concluded that this difference may be due to the difference of resistance and capacitance and the influence of LM324. In order to make the input of the op-amp more balanced, a resistor can be connected in series at the in-phase input. In order to prevent the low frequency signal gain is too large, a large resistor can be connected in parallel to the capacitor to limit the low frequency gain and DC gain.

## 5. Conclusion

In this lab, it helps to understand how to build, design, test and characterize a 1st-order integrator, and study its configuration along with theory and principle. For integral circuit, it is mainly used in waveform transformation, elimination of offset voltage in amplifier circuit and integral compensation in feedback control [2], because of the characteristics of capacitor and operational amplifier.

## References

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