

# Modeling and Simulation of Ship Motion for Loading and Unloading Containers on the Shore

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## Abstract

To establish a ship model on the shore, assuming that the ship is moving on a micro-radial wave, first establish a six-degree-of-freedom ship motion equation under unconstrained conditions, and then consider the shore constraint conditions to simplify the ship's motion on the regular wave into pitch and lift Shen motion, combined with slice theory to calculate hull force. The Simulink tool of Matlab is used to simulate the built model, and the movement law of shore container ship is obtained.

## Keywords

Micro-radial waves; Regular waves; Pitch and heave motion; Simulation.

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## 1. Introduction

At present, many literatures have studied the mathematical model of ship motion. Zhang Xianku et al. [1] established a responsive nonlinear ship motion model. Considering that the ship motion will be disturbed by the outside world, a mathematical model of the external disturbance force was established as the input of the ship motion model to obtain the motion response. Du Jialu et al. [2] established the ship's rolling and pitching motion models, combined with adaptive nonlinear control, to achieve real-time control of the ship. Ma Guoqin [3] used a three-dimensional time-domain method to build a ship's nonlinear motion model in waves, and conducted a scaled-down ship model comparison experiment. The results showed a high degree of agreement and achieved the nonlinear motion response of the real ship. Zhang Xiufeng et al. [4], according to the idea of the separation model (MMG), superimposed the interference force and moment of the wave on the ship to the separate ship motion mathematical model, and established a six-degree-of-freedom mathematical model of the ship motion under the action of regular waves.

In this paper, after establishing the six-degree-of-freedom motion equation of the ship, combined with the situation of loading and unloading containers on the shore, considering constraints, the ship motion on the regular wave is simplified into pitch and heave motion, a new ship motion mathematical model is established, and then Matlab / Simulink The tool completes the simulation. The simulation results help to improve the efficiency of grabbing containers, which in turn improves the economic efficiency of the port.

## 2. Organization of the Text

### 2.1 Establishing six degrees of freedom unconstrained ship motion equation

#### 2.1.1 Establish a coordinate system

When studying the movement of ships, two kinds of coordinate systems are commonly used [5]: inertial coordinate system and appendage coordinate system, as shown in Figure 1. The inertial coordinate system is also called the geodetic coordinate system, which is used as a reference for

reference, usually expressed by  $o_0x_0y_0z_0$ . The origin can be selected at a certain point on the sea, and the positive direction of the  $x_0$ -axis is north, the positive direction of the  $y_0$ -axis is east, and the  $z_0$ -axis points vertically downward to the center of the earth. The attached coordinate system is also called the on-board coordinate system. Its origin is located at a specified point  $O$  in the ship (taken at the center of gravity of the ship). It is generally indicated by  $oxyz$  and specifies that the  $x$ -axis points to the bow, the  $y$ -axis points to the starboard side, and the  $z$ -axis points to the ship keel. Obviously, the attached coordinate system is non-inertial and can move arbitrarily in space with the ship. When analyzing the movement of ships, the coordinate system of appendages is widely used. When discussing the space trajectory of ships, the inertial coordinate system is used.

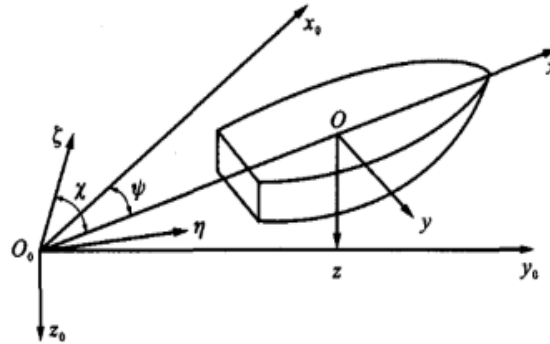


Fig. 1 Coordinate System

2.1.2 The establishment of six degrees of freedom ship motion equation

Assuming that the ship is a rigid body and moves on small waves, the mutual coupling between the six degrees of freedom of ship motion is ignored. The origin of coordinates is taken at the center of gravity of the ship, then  $x_G = y_G = z_G = 0$ . The following six-degree-of-freedom motion equation for ships is available.

$$\begin{cases} m(\dot{u} - vr + wq) = X_\Sigma \\ m(\dot{v} - wp + ur) = Y_\Sigma \\ m(\dot{w} - uq + vp) = Z_\Sigma \\ I_{xx}\dot{p} + (I_{zz} - I_{yy})qr = K_\Sigma \\ I_{yy}\dot{q} + (I_{xx} - I_{zz})rp = M_\Sigma \\ I_{zz}\dot{r} + (I_{yy} - I_{xx})pq = N_\Sigma \end{cases} \quad (1)$$

In the above equation,  $m$  is the mass of the ship;  $I_{xx}$ ,  $I_{yy}$ ,  $I_{zz}$  are the rotational inertia around the  $x$ -axis, the  $y$ -axis and  $z$ -axis respectively;  $u, v, w, p, q, r$  are the vertical, horizontal, and vertical velocities and the angular velocity around the  $x$ -axis,  $y$ -axis, and  $z$ -axis.  $X_\Sigma, Y_\Sigma, Z_\Sigma, K_\Sigma, M_\Sigma, N_\Sigma$  are the external force and external moment acting on the direction of six degrees of freedom of the hull. The external forces and moments acting on the ship can be divided into fluid power, paddle force, rudder force, wave force, wind force, namely

$$\begin{cases} X_\Sigma = X_H + X_P + X_R + X_{WAVE} + X_{WIND} \\ Y_\Sigma = Y_H + Y_P + Y_R + Y_{WAVE} + Y_{WIND} \\ Z_\Sigma = Z_H + Z_P + Z_R + Z_{WAVE} + Z_{WIND} \\ K_\Sigma = K_H + K_P + K_R + K_{WAVE} + K_{WIND} \\ M_\Sigma = M_H + M_P + M_R + M_{WAVE} + M_{WIND} \\ N_\Sigma = N_H + N_P + N_R + N_{WAVE} + N_{WIND} \end{cases} \quad (2)$$

The subscripts  $H, P, R, WAVE, WIND$  in the above formula are the bare hull, propeller, rudder, wave, wind.

## 2.2 Establishment of the equation of ship's motion on shore

### 2.2.1 Ship motion model in regular wave

In order to build a ship movement model on the shore, the following assumptions need to be made:

- 1) In the mooring environment on the shore, the waves are slightly divergent, and the ship sway is small;
- 2) Since the container ship is roped, only pitching and heave motions are considered at this time;
- 3) The wave load is calculated by slice theory [6], and the hull is a slender body.

The transformation relationship between the ship coordinate system and the inertial coordinate system is:

$$\begin{cases} \xi = x_0 \cos \mu + y_0 \sin \mu \\ \eta = y_0 \cos \mu + x_0 \sin \mu \\ \zeta = z_0 \end{cases} \quad (3)$$

Regular wave surface motion equation:  $\zeta(\xi, \eta, t) = a \cos(k\xi + \omega t)$ . Let the angle between the  $O\xi$ -axis and the  $Ox_0$ -axis be  $\mu$ , and this angle is the absolute wave direction angle. In the equation,  $a$  is the wave amplitude,  $k$  is the number of waves, and  $\omega$  is the wave angular frequency.

$a = \frac{1}{2} \times 0.17 \lambda^{3/4}$ ,  $\omega = \frac{2\pi}{T}$ , Where  $\lambda$  is the length of the wave, and the number of waves  $k$  can be obtained by the following equation:  $k = \frac{2\pi}{\lambda}$ .  $T$  is the wave motion period, which can be obtained from the empirical formula:  $T = 0.8\sqrt{\lambda}$ .

According to the famous Froude-Krylov hypothesis, the presence of the ship in the regular wave does not affect the pressure distribution in the wave, then the partial contact pressure between the hull and the water flow:

$$\Delta p(\xi, \zeta, t) = -\rho g a e^{-k\zeta} \cos(k\xi - \omega t) \quad (4)$$

Where  $\rho$  is the density of water and  $g$  is the acceleration of gravity, which is taken as  $9.8 \text{ m/s}^2$ .

In order to facilitate the subsequent integration calculation, the above formula is converted to the coordinate system of the ship for discussion:

$$\Delta p(x, z, t) = -\rho g a e^{-kz} \cos(kx \cos \varphi - ky \sin \varphi - \omega_e t) \quad (5)$$

Where  $\varphi$  is the angle of the encounter wave;  $\omega_e$  is the frequency of interaction between the wave and the hull, that is, the frequency of encounter.

Using the slice theory, we can know the wave interference force of the heave motion of the ship and the interference torque of the pitch motion, namely:

$$\begin{cases} F = F_{zc} \cos \omega_e t + F_{zs} \sin \omega_e t \\ M = M_{\theta c} \cos \omega_e t + M_{\theta s} \sin \omega_e t \end{cases} \quad (6)$$

Where  $F_{zc}, F_{zs}, M_{\theta c}, M_{\theta s}$  are wave force (moment) coefficients.

Establish the mathematical model of the ship's pitch and heave motion in the regular wave:

$$\begin{cases} (m + A_{zz})\ddot{Z}_G + B_{zz}\dot{Z}_G + C_{zz}Z_G + A_{z\theta}\ddot{\theta} + \\ B_{z\theta}\dot{\theta} + C_{z\theta}\theta = F_{zc} \cos \omega_e t + F_{zs} \sin \omega_e t \\ (I_{\theta\theta} + A_{\theta\theta})\ddot{\theta} + B_{\theta\theta}\dot{\theta} + C_{\theta\theta}\theta + A_{\theta z}\ddot{Z}_G + \\ B_{\theta z}\dot{Z}_G + C_{\theta z}Z_G = M_{\theta c} \cos \omega_e t + M_{\theta s} \sin \omega_e t \end{cases} \quad (7)$$

Where  $Z_G$  is the heave of the hull and  $\theta$  is the pitch angle of the hull;  $A_{zz}, B_{zz}, A_{z\theta}, B_{z\theta}, A_{\theta\theta}, B_{\theta\theta}, A_{\theta z}, B_{\theta z}$  are hydrodynamic coefficients;  $C_{zz}, C_{z\theta}, C_{\theta\theta}, C_{\theta z}$  are hydrostatic coefficients;  $m$  is the mass of the hull;  $I_{\theta\theta}$  is the longitudinal moment of inertia of the hull.

### 2.2.2 Calculate the wave interference force and moment of the hull

Since the water flow is in direct contact with the hull surface, the outer surface of the hull is integrated by the method of Gauss 's theorem projection, and the force and moment acting on the hull are

$$\begin{cases} \mathbf{F} = -\iint_S \Delta p \mathbf{n} dS \\ \mathbf{M} = -\iint_S \Delta p (\mathbf{r} \times \mathbf{n}) dS \end{cases} \quad (8)$$

namely

$$\begin{cases} Z = -\iiint_V \frac{\partial \Delta p}{\partial z} dV \\ M = -\iiint_V \frac{\partial \Delta p}{\partial z} x dV \end{cases} \quad (9)$$

Where  $S$  is the part of the surface area of the hull that is immersed in water;  $\mathbf{n}$  is the unit normal vector at each point on the surface of the hull;  $V$  is the part of the volume of the hull that is immersed in water;  $\mathbf{r}$  is the vector diameter.

The contact pressure between the hull and the water increases with the depth of the immersion. In the coordinate system of the ship, the pressure gradient is

$$\frac{\partial \Delta p}{\partial z} = \rho g a e^{-kz} \cos(kx \cos \varphi - ky \sin \varphi - \omega_e t) \quad (10)$$

Combining the above types, the available hull forces and moments are

$$\begin{cases} Z = -\rho g a k \int_L \frac{\sin(0.5kB(x) \sin \varphi)}{0.5kB(x) \sin \varphi} e^{-k(\xi_G + x\theta + d(x))} \square \\ A(x) \cos k(\xi_G + x \cos \varphi - ct) dx \\ M = \rho g a k \int_L \frac{\sin(0.5kB(x) \sin \varphi)}{0.5kB(x) \sin \varphi} e^{-k(\xi_G + x\theta + d(x))} \square \\ xA(x) \cos k(\xi_G + x \cos \varphi - ct) dx \end{cases} \quad (11)$$

Considering that the hull is a complex curved structure, to ensure a certain accuracy, it is necessary to use simple calculations. Assuming that the ship is a regular hexahedron, its draft  $d(x)$ , ship width  $B(x)$ , and cross-sectional area  $A(x)$  are no longer affected by the change of  $x$  and are constant, then  $L$  and  $B$  can be considered to replace  $A(x)$  and  $B(x)$ , that is

$$\left\{ \begin{array}{l} Z = \rho g a k e^{-0.5kd} B d \frac{\sin(0.5kBL \sin \varphi)}{0.5kBL \sin \varphi} \int_{-0.5L}^{0.5L} \cos(kx \cos \varphi - \omega_e t) dx \\ M = \rho g a k e^{-0.5kd} B d \frac{\sin(0.5kBL \sin \varphi)}{0.5kBL \sin \varphi} \int_{-0.5L}^{0.5L} x \cos(kx \cos \varphi - \omega_e t) dx \end{array} \right. \quad (12)$$

Where  $\omega_e$  is the encounter frequency and  $\omega_e = \omega(1 + \frac{\omega}{g} v \cos \varphi)$ ;  $v$  is the ship speed;  $\varphi$  is the wave angle.

### 2.3 Simulation Research

Choose an example, substitute the ship parameters, use Matlab / Simulink to calculate the model, and find the change of heave amount and the change of pitch angle.

#### 2.3.1 Parameter calculation

In this paper, a container ship is used as an example for calculation. First, the empirical formula  $x_b = (B - A) / C$  is used to calculate the location of the buoyancy center of the hull. Then divide the hull into 21 equally-spaced cut planes, numbered sequentially from the stern to the bow, then the formula for calculating each coefficient is as follows:

$$\left\{ \begin{array}{l} A = l(\sum_{i=1}^{11} x_i A_i - \frac{x_1 A_1 + x_{11} A_{11}}{2}) \\ B = l(\sum_{i=11}^{21} x_i A_i - \frac{x_{11} A_{11} + x_{21} A_{21}}{2}) \\ C = l(\sum_{i=1}^{21} A_i - \frac{A_1 + A_{21}}{2}) \end{array} \right. \quad (13)$$

Where  $l$  is the distance between adjacent cut planes of the hull;  $i$  is the number of each cut plane ( $i = \{1, 2, \dots, 21\}$ ); the area  $A_i$  of each cut plane is known;  $x_i$  is the distance between each cut plane and the geometric center of the hull.

#### 2.3.2 Simulation calculation

Use the Matlab / Simulink toolbox to build a block diagram and run calculations based on the listed differential equations, as shown in Figure 2.

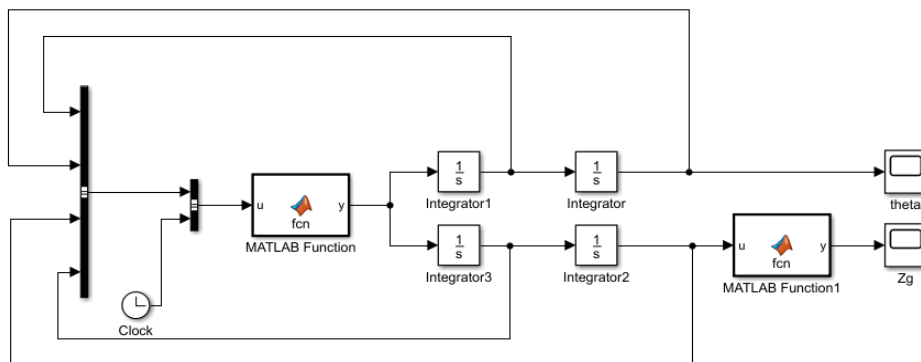


Fig. 2 Simulation block diagram

The Matlab function in the block diagram is the embedded Matlab programming language. This function:

$$\begin{aligned}
 f_1 &= -B_{zz}\dot{Z}_G - C_{zz}Z_G - B_{z\theta}\dot{\theta} - C_{z\theta}\theta + F_{zc} \cos \omega_e t + F_{zs} \sin \omega_e t \\
 &= -B_{zz}\dot{Z}_G - C_{zz}Z_G - B_{z\theta}\dot{\theta} - C_{z\theta}\theta + \rho g a k e^{-0.5kd} B d \frac{\sin(0.5kBL \sin \varphi)}{0.5kBL \sin \varphi} \int_{-0.5L}^{0.5L} \cos(kx \cos \varphi - \omega_e t) dx \\
 f_2 &= -B_{\theta\theta}\dot{\theta} - C_{\theta\theta}\theta - B_{\theta z}\dot{Z}_G - C_{\theta z}Z_G + M_{\theta c} \cos \omega_e t + M_{\theta s} \sin \omega_e t \\
 &= -B_{\theta\theta}\dot{\theta} - C_{\theta\theta}\theta - B_{\theta z}\dot{Z}_G - C_{\theta z}Z_G + \rho g a k e^{-0.5kd} B d \frac{\sin(0.5kBL \sin \varphi)}{0.5kBL \sin \varphi} \int_{-0.5L}^{0.5L} x \cos(kx \cos \varphi - \omega_e t) dx \\
 \begin{bmatrix} \ddot{\theta} \\ \ddot{Z}_G \end{bmatrix} &= \begin{bmatrix} A_{z\theta} & m + A_{zz} \\ I_{\theta\theta} + A_{\theta\theta} & A_{\theta z} \end{bmatrix}^{-1} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}
 \end{aligned} \tag{14}$$

This function has 5 inputs and 2 outputs, which are  $\dot{Z}_G, Z_G, \dot{\theta}, \theta, t$  and  $\ddot{Z}_G, \ddot{\theta}$  respectively. The output shows the value of the hull sag  $Z_G$  and the hull pitch angle  $\theta$ .

Substitute the initial condition value and set the simulation time for 100 seconds, and then obtain the change in the heave amount of the hull as shown in Figure 3, and the change in the pitch angle of the hull as shown in Figure 4.

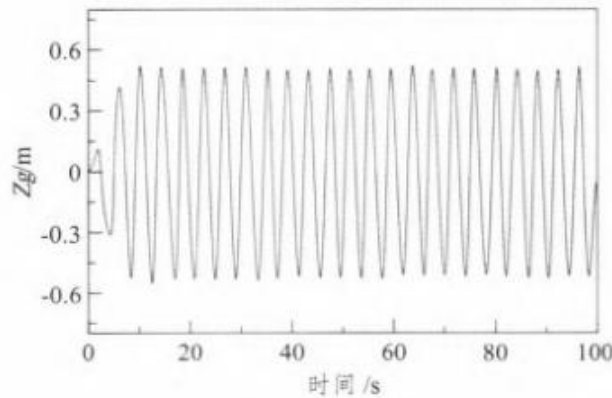


Fig. 3 Heave amount

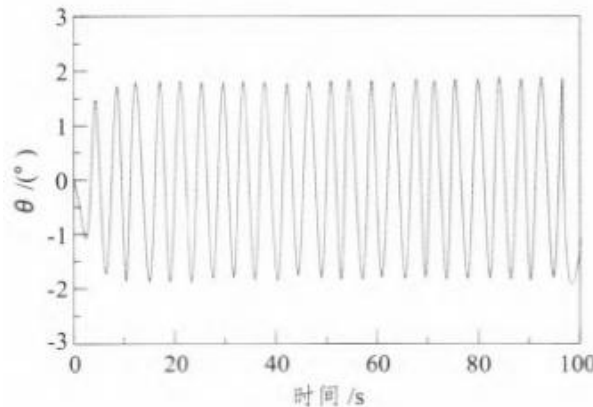


Fig. 4 Pitch angle

By analyzing the law of the simulation curve, it can be seen that the amplitude of the heave motion of the ship on the shore on the regular wave is small but its frequency is large, and the pitch angle changes with time to a sinusoidal function, the pitch angle is small but its frequency is large. The results obtained in this paper are similar to the existing literature.

### 3. Conclusion

This article first establishes the ship motion equation from the hull unconstrained condition, then considers the shore constraints, and combines the slicing theory to obtain the movement law of the shore container ship on the regular wave, which is similar to the results discussed in the existing literature and reasonable. Pave the way for research on improving the efficiency of handling containers and improving the economic benefits of ports.

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