

Theory and Calculation of Advanced Analysis of Metal Structure

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Abstract

With the progress of technology and the improvement of industrial manufacturing standard, the common steel structure design method has been unable to meet the requirements, so we need to learn new analysis theory and apply it. This chapter mainly introduces the theory of advanced analysis of metal structure. According to the regulations of "steel structure design standard-2017", it mainly introduces the current second-order analysis of structure in China, and gives several examples.

Keywords

Advanced analysis, Metal structure, Second-order analysis.

1. Introduction to the theory of advanced analysis of metal structure

At present, the general steps for the safety design of building steel structures in the world are as follows: firstly, the displacement and internal forces of each component under various loads and their combination are calculated by the first-order or second-order elastic method [1], that is, the elastic analysis of the whole structure; Then the internal forces obtained from the structural analysis are applied to the limit state equations of the components for the component design, that is, the inelastic design of a single component. If the member satisfies the prescribed limit state equations, the structure design is considered to meet the requirements of the specification. This design method is essentially a structural design based on the ultimate bearing capacity of components, which has the following four defects:

1. The calculation model of internal force of structure is not consistent with that of component bearing capacity.
2. The calculation mode of overall structural instability is inconsistent with the actual instability state.
3. The current design method cannot accurately predict the failure mode and ultimate bearing capacity of the structural system.
4. The ultimate reliability level of bearing capacity of different structures is inconsistent.

Advanced analysis refers to any method that can accurately track the whole process of plastic gradual change of each component in the structure and accurately predict the failure mode and ultimate load of the structural system and its components, without checking each component one by one according to the standard formula [2].

Higher analysis method advocated full consideration in the structure analysis structure affects the performance of the various nonlinear factors (geometric nonlinearity, material nonlinearity, nonlinear, etc.), direct calculation or the ultimate bearing capacity calculation of the structure of the whole, to completely abandon the calculating equation for the length and the artifacts related concept, namely from component calculation steps, creates a more unified structure reliability, this will greatly simplify the design process, improve the efficiency of structural design.

Structural analysis can be divided into geometric nonlinear analysis, double nonlinear analysis and triple nonlinear analysis.

The advanced analytical theory of two-dimensional structures has been developed and can be used in the design of general frame structures. However, the problems related to the analysis of 3d frame structure, such as the theoretical and mathematical expressions, and the accuracy and applicability of the solution, have not been well solved.[3]

2. Examples of structure second-order analysis

When the influence of structural deformation on geometric relations is neglected, the structural analysis of equilibrium equation is established on the basis of the geometric relations before structural deformation, which is called first-order analysis. When considering the influence of structural deformation on geometric relations, the structural analysis of equilibrium equation is established on the basis of the geometric relations after structural deformation, which is called second-order analysis. At present, China's latest standard only considers the second order analysis of the structure for the higher analysis.

Here is a typical second-order nonlinear case:

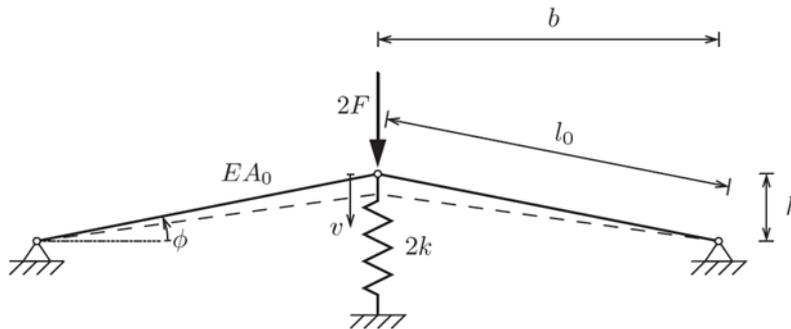


FIG. 1.1 planar flat truss structure

The flat truss structure shown in figure 1.1. Considering the structural element equilibrium after deformation, the internal force expression of symmetrical flat half truss can be derived as follows:

$$F_{int} = -A\sigma \sin \phi - F_s \tag{1.1}$$

Where, σ_s is the axial stress of the member, F_s is half of the spring force, and ϕ is the Angle between the truss member and the horizontal plane in the deformed structure. Under the assumption of small strain, the difference between the cross-sectional area in the current configuration and that in the original configuration can be ignored. Similarly, the length of the rod in the original configuration:

$$l_0 = \sqrt{b^2 + h^2} \tag{1.2}$$

And the bar length in the current configuration:

$$l = \sqrt{b^2 + (h - v)^2} \tag{1.3}$$

In the denominator of the strain expression, the difference between can be ignore:

$$\varepsilon = \frac{l - l_0}{l_0} \tag{1.4}$$

Similarly, when calculating the inclination Angle ϕ :

$$\sin \phi = \frac{h - v}{l} \approx \frac{h - v}{l_0} \tag{1.5}$$

The dimensions b and h definitions are shown in figure 1.1. If the vertical displacement v is in the positive direction, take half of the spring force, and can get:

$$F_s = -kv \tag{1.6}$$

Where k is the elastic stiffness, and the axial stress of the bar is:

$$\sigma = E\varepsilon \tag{1.7}$$

E is the young's modulus. In the bending equilibrium equation (1.1), the replacement stress σ , spring force F_s and Angle ϕ are obtained:

$$F_{int}(v) = -EA_0 \sin \phi \frac{l-l_0}{l_0} + kv \tag{1.8}$$

Formula (1.8) indicates that the internal force of the structure is a nonlinear function of the vertical displacement v. In general, if the external force $F_{ext}^{t+\Delta t}$ at the time $t + \Delta t$ is given, the displacement v must be calculated under the following conditions:

$$F_{ext}^{t+\Delta t} - F_{int}^{t+\Delta t} = 0 \tag{1.9}$$

The correct value of v was calculated repeatedly by the newton-raphson iterative method:

$$F_{ext}^{t+\Delta t} = F_{int}(v_j) + \frac{dF_{int}}{dv} dv + \frac{1}{2} \frac{d^2 F_{int}}{dv^2} dv^2 + O(dv^3) \tag{1.10}$$

Where j is the number of iterations, take the linear approximation, we have the iterative correction of displacement v:

$$dv = \left(\frac{dF_{int}}{dv} \right)_j^{-1} (F_{ext}^{t+\Delta t} - F_{int}(v_j)) \tag{1.11}$$

When the convergence condition $\|F_{ext}^{t+\Delta t} - F_{int}^{t+\Delta t}(v_j)\| < \varepsilon$ is satisfied, the iteration process stops, ε is a very small number. The derivative $\frac{dF_{int}}{dv}$ is the tangential stiffness modulus, which can be obtained by formula (1.8) :

$$\frac{dF_{int}}{dv} = \frac{A_0 \sin^2 \phi}{l_0} \left(E + \frac{dE}{dl} (l-l_0) \right) + \left(k + \frac{dk}{dv} v \right) + \frac{A_0 \sigma}{l_0} \tag{1.12}$$

FIG. 1.2 shows the results of the truss with different values of spring stiffness. The figure directly reflects the application of the internal force expression (1.8) and is combined with the equilibrium condition of (1.9). The iterative process can only be applied when there is a large value of spring stiffness, that is, a load-displacement curve without a local maximum.

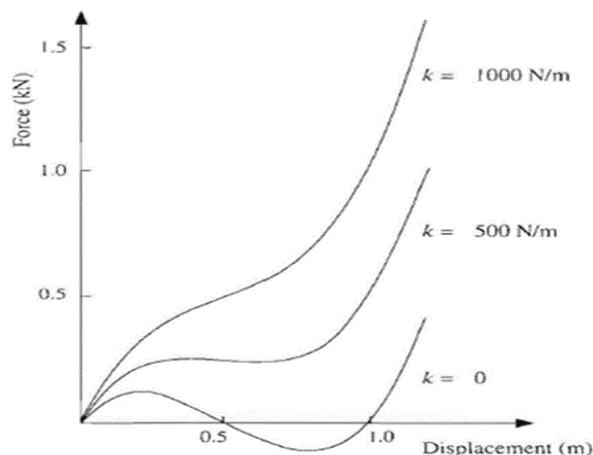


FIG. 1.2 load-displacement curve of flat truss structure with different values of spring stiffness k. The above is a typical example of structure analysis which uses the geometric relation after the structure is stressed and deformed to establish the equilibrium equation. The newton-raphson iterative method is also a common method for solving this type of equation.

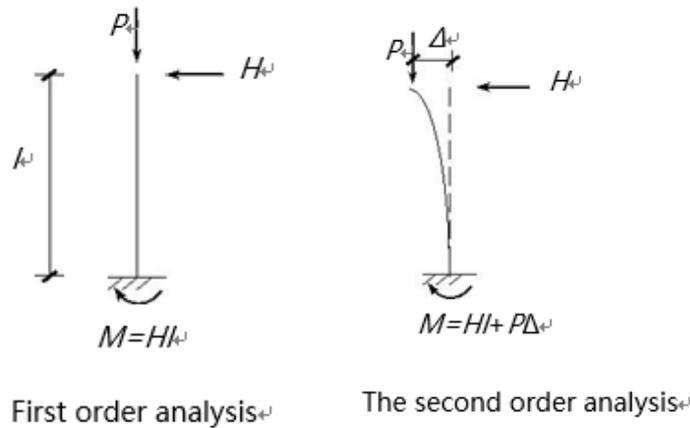
3. Provisions of the national standard on second-order analysis of structures

In steel structure design standards-2017, [4] there are two definitions of structure height analysis:

3.1 First-order analysis and second-order analysis

(1) In the first-order analysis, the influence of structural deformation on internal forces is not considered, and the internal forces and displacements are analyzed according to the undeformed structural equilibrium condition.

(2) In order to consider the influence of structural deformation on the internal force, the second order analysis analyzes the internal force and displacement of the structure according to the structural equilibrium condition of deformation, which is also called the analysis considering the p-Δ effect.



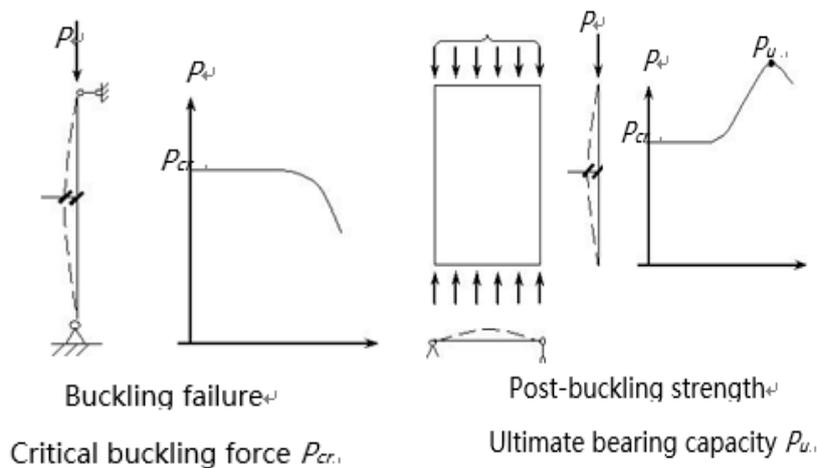
3.2 Buckling and post-buckling

(1) buckling

The deformation of the whole structure or component under external load suddenly changes from the original equilibrium state to another equilibrium state, which is called the buckling of the whole structure or component.

(2) buckling strength and post-buckling strength

The bearing capacity of a structure or component after buckling has two possibilities according to the specific situation of the structure or component. One is that the structure or component has reached the maximum bearing capacity when buckling occurs, and the occurrence of buckling indicates the structure or component failure. The other is that when buckling occurs, the structure or member does not reach the maximum bearing capacity, but still has subsequent bearing capacity, that is, the strength after buckling.



4. Method of second-order structure analysis

The calculation method of second order analysis is complex, including analysis by elastic stage (second order elastic analysis) and analysis by elastic-plastic stage (second order inelastic analysis). According to the importance and complexity of the structure, as well as the accuracy requirements in different design stages, some approximate simplified methods can be used in the analysis and calculation.

Approximate method:

1. Effective length method (a method combining the elastic analysis of first-order internal force with the checking of components)
2. Hypothetical horizontal load method
3. Amplification factor method
4. Merchant-Rankine-Wood simplification method

Precise method:

The accurate method can obtain all the reliable parameters and data, which needs to be solved iteratively by using finite element. Firstly, the element stiffness equation is established in the element coordinate system (local coordinate system). Then it is converted to the unit stiffness equation in the global coordinate system. Reassembled into the overall stiffness equation of the structure; Finally, the structural nonlinear equation is solved. And get the effect.

In the second order analysis of the structure, the key is to establish the nonlinear stiffness equation of the whole structure. There are two methods to establish the nonlinear stiffness equation of beam-column element: beam column approach; The other is the finite element approach.

The following is a general approach to solving problems using the finite element method:

1. Determine what the underlying assumptions are for the problem to be solved.
2. Choose the appropriate finite deformation theory of 3d continuum to describe the coordinates of object motion.
3. Choose an appropriate method to describe the strain state of the object.
4. According to the method selection and theoretical description above, the virtual work increment equation is established.
5. According to the structure characteristics, build the appropriate displacement interpolation function.
6. The ratio of force increment to displacement increment is used to obtain the stiffness equation.
7. Establishment and solution of structural nonlinear equilibrium equation.

Here is just a general summary of the use of finite element method to solve the structure of the second order analysis of the general ideas, the specific content

And methods need more in-depth study to master.

5. Summary and prospect

Higher analysis can be expected to represent the structure of the latest technology will become the 21st century basic structural engineers design tools, set up in the structure of the overall limit state and structure of the overall structure of the integrated design method of ultimate bearing capacity as the goal, will eventually replace the existing steel structure design based on component bearing capacity limit state method, it is an inevitable trend in the development of structural design.

As mentioned above, many problems have not been solved when advanced analysis enters into the design of 3d frame structure. After the frame structure, there is still research on buckling and post-buckling strength of plate and shell structure. Structural analysis, an ancient discipline, needs a new generation of researchers to continue to explore as computing hardware is updated and research methods are innovated.

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