ISSN: 2414-1895 DOI: 10.6919/ICJE.202005_6(5).0021

Synchronous Control of Double-container Overhead Crane Based on PI Terminal Sliding Mode

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Abstract

In order to solve the synchronization problem of double-container overhead crane, the mean deviation coupling synchronization control strategy is adopted. The combination of PI control and non-singular fast terminal sliding mode control is applied to the tracking control of the double-container overhead crane system. For overhead crane system, this method enhances its robustness, reduces its steady-state error, and it can quickly converge in a limited time. Meanwhile, the super-twisting algorithm effectively suppresses the chattering effect in sliding mode control. Finally, the stability of the controller is verified by Lyapunov theory. The simulation results show the good performance of the proposed controller.

Keywords

Synchronous control; Double-container overhead crane; Terminal sliding mode; Super-twisting control.

1. Introduction

Compared with the traditional overhead crane, the double-container overhead crane improves the loading and unloading efficiency of the automated terminal and saves industrial energy consumption. However, the double-container overhead crane system has characteristics such as time-varying and nonlinear. It will be subject to system parameter perturbations and uncertain interference during operation [1]. During the work of the overhead crane, the potential energy load driven by the two spreaders will also change. These problems bring difficulties to the synchronous and coordinated control of the double-container overhead crane.

In order to solve the above problems, a combination of synchronous control and tracking control is usually used to control the double-container overhead crane. Synchronous control strategies are designed to resolve errors between double spreaders. The most commonly used strategies are master-slave control, deviation coupling control, cross coupling control. Reference [2] used a master-slave synchronous control strategy to control the overhead crane. However, the master-slave synchronous control strategy ignores the coupling between spreaders and cannot guarantee the accuracy of synchronization. In order to improve this situation, most algorithms now use cross-coupling synchronization control strategies [3-4]. However, with the increase of the number of motors in the cross-coupling control, the complexity of the system control structure also increases. Based on this, reference [5] proposes a mean deviation coupling strategy to control multiple motors synchronously, reducing the complexity of the system and ensuring the performance of synchronous control. The tracking control strategy aims to control the tracking error when a single spreader is working. The commonly used tracking control methods include PID control, sliding mode control, and robust control [6-8]. Sliding mode control is widely used in industrial production because it has the advantages of fast response speed and strong robustness to the uncertainty of the system. In order to

ISSN: 2414-1895
DOI: 10.6919/ICJE.202005_6(5).0021

ensure that the system can converge in a finite time and solve the singularity problem in sliding mode control, non-singular fast terminal sliding mode control is usually selected [9]. In order to further improve the robustness of the system, this paper chooses to use a combination of PI control and sliding mode control. For the chattering problem in sliding mode control, reference [10] proposed a super-twisting algorithm to solve the chattering in the approaching phase of sliding mode control. This paper is devoted to the design of the synchronous control and tracking control sliding mode controller of the double-container overhead crane. Meanwhile controller improve the chattering effect in the sliding mode control, to ensure the synchronization and tracking control performance of the system. It can also ensure the two containers of overhead crane run at the same speed and position.

For the tracking control problem of non-linear systems, this paper presents a PI non-singular fast terminal sliding mode controller. Non-singular fast terminal sliding mode control can improve the response speed of the system, make the system converge in effective time, and solve the singularity problem of terminal sliding mode. The application of PI control and super-twisting algorithm effectively improves the chattering effect of sliding mode control.

2. Model of the Double-container Overhead crane

The double-container overhead crane is composed of spreaders, lifting ropes, rotating shafts and driving motors (as shown in Fig. 1). Each lifting system is equipped with a driving motor, a lifting rope and a spreader. The spreader is driven by an induction motor to complete the loading and unloading of the container. During the operation of the double spreader, if the two drive motors are affected by the disturbance, the speed and position of the two drive motors will change, which will cause the two spreaders to fail to run synchronously.

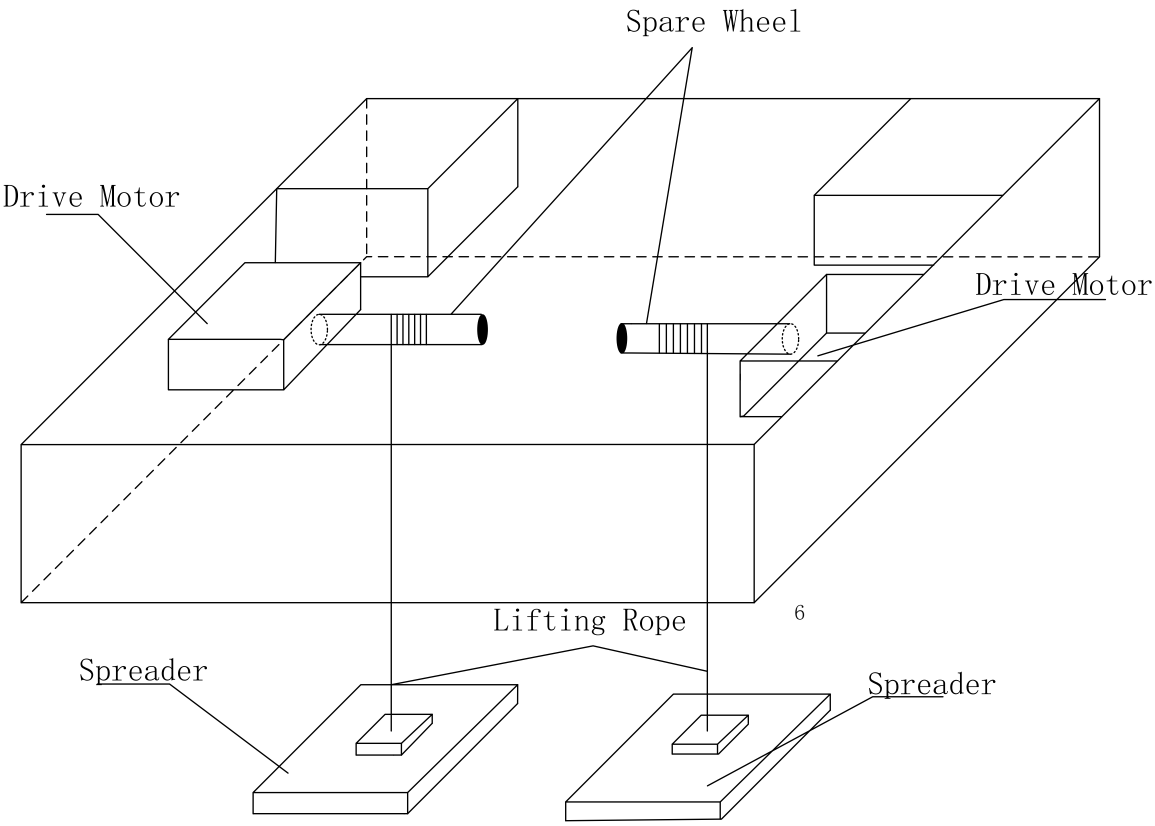

Fig.1 Double-container overhead crane structure diagram

The mathematical model of the speed and acceleration of an induction motor is expressed as follows:

$$J_i\ddot{\theta}_i(t) + B_i\dot{\theta}_i(t) + T_{L_i} + d_r = K_T i_{qsi} \tag{1}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202005_6(5).0021

where J_i is the moment of inertia of the drive motor, B_i is the viscous friction coefficient of the drive motor, θ_i is the angular position of the drive motor, T_{Li} is the load torque, i_{qsi} is the control input of the drive motor, and K_T is the control for the electromagnetic torque coefficient, d_r is the external disturbances.

The dynamic model Eq. (1) of spreader i can be simplified as:

$$\ddot{\theta}_i = A(t)\dot{\theta}_i + L_1(t)u_i + L_2(t)\left(T_{L_i} + d_r\right) \tag{2}$$

where$A(t) = -B_i/J_i$, $L_1(t) = K_T/J_i$, $L_2(t) = -1/J_i$.

Further simplify Eq. (2) is:

$$\ddot{\theta}_i = A(t)\dot{\theta}_i + L_1(t)u_i + d_i \tag{3}$$

where$d_i = L_2(t)\left(T_{L_i} + d_r\right)$, $A(t)$, $L_1(t)$, $L_2(t)$ are bounded nonlinear time-varying functions.

3. Controller Design

The two spreaders of overhead crane controller proposed in this paper consists of a tracking error controller and a synchronization error controller. The mean deviation coupling strategy defines the synchronization error of the two spreaders. The PI nonsingular fast terminal sliding mode control is applied to the control of synchronization error and tracking error at the same time. The super-twisting algorithm further improves the chattering situation of the sliding mode in the approaching stage.

Define the mean deviation coupling error $e_i(t)$ and tracking error $e_{ref,i}(t)$ of spreader i as:

$$e_i(t) = \theta_i(t) - \textstyle\sum_{j=1}^{n} \theta_j(t)/n \tag{4}$$

$$e_{ref,i}(t) = \theta_d(t) - \theta_i(t) \tag{5}$$

where $\theta_i(t)$ is the actual position of the driving motor of the spreader i, and $\theta_d(t)$ is the expected position of the driving motor of the spreader i. i is the number of spreaders, $n = 1,2,3\ldots\ldots$

According to Eq. (3) (4) (5), the second derivative of the mean deviation coupling error and tracking error is calculated as:

$$\ddot{e}_{ref} = \ddot{\theta}_d - A(t)\dot{\theta}_i - L_1(t)u_i - d_i \tag{6}$$

$$\begin{aligned}\ddot{e}_i &= A(t)\dot{\theta}_i + L_1(t)u_i + d_i - \sum_{j=1}^{n}\left(A(t)\dot{\theta}_j + L_1(t)u_j + d_j\right)/n \\ &= A(t)\dot{e}_i + L_1(t)u_i^* + \left(d_i - \textstyle\sum_{j=1}^{n} d_j/n\right)\end{aligned} \tag{7}$$

where $u_i^* = u_i - \sum_{j=1}^{n} u_j/n$.

3.1 PI non-singular fast terminal sliding mode controller

In order to satisfy the design requirements of traditional sliding mode control, the non-singular fast terminal sliding mode surface is designed as follows:

$$\begin{aligned} s_1 &= e_{ref} + \chi_1{}^{-1}\dot{e}_{ref}^{p/q} + \chi_3{}^{-1}e^{g/h}{}_{ref} \\ s_2 &= e_i + \chi_2{}^{-1}\dot{e}_i^{p/q} + \chi_4{}^{-1}e_i^{g/h}\end{aligned} \tag{8}$$

where s_1 is the tracking error sliding mode surface, and s_2 is the synchronization error sliding mode surface.χ_1, $\chi_2, \chi_3, \chi_4 \in R^+$,$p, q, g, h \in N$. When $e = 0$, $\dot{e} \neq 0$, $1 < p/q < 2$,$g/h > p/q$

The PI-nonsingular fast terminal sliding mode surface $s_{PI-TSM1}$ and $s_{PI-TSM2}$ of tracking error and synchronization error are proposed as follows:

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$$s_{PI-TSM1} = \varpi_1 s_1 + \varpi_2 \int s_1$$

$$s_{PI-TSM2} = \varpi_1 s_2 + \varpi_2 \int s_2 \tag{9}$$

Where ϖ_1, ϖ_2 are PI coefficients. This sliding form surface combines the forms of PI and non-singular fast terminal sliding, with the advantages of both. The sliding mode surface proposed in this paper can make the system's instantaneous response faster, have lower steady-state error, and can guarantee finite-time convergence.

From Eq. (6) (7) (9), the first derivative of $s_{PI-TSM1}$ and $s_{PI-TSM2}$ are obtained:

$$\dot{s}_{PI-TSM1} = \varpi_1 \frac{d}{dt}\left(e_{ref} + {\chi_1}^{-1}\dot{e}_{ref}^{p/q} + {\chi_3}^{-1}e^{g/h}{}_{ref}\right) + \varpi_2\left(e_{ref} + {\chi_1}^{-1}\dot{e}_{ref}^{p/q} + {\chi_3}^{-1}e^{g/h}{}_{ref}\right)$$

$$= \Omega_{ref}\left(e_{ref}\right) + \kappa_{ref}\left(e_{ref}\right) + \varpi_1\left(p\dot{e}_{ref}^{p/q-1}\left(\ddot{\theta}_d - A(t)\dot{\theta}_i - L_1(t)u_i - d_i\right)/q\chi_1\right) \tag{10}$$

Among them $\Omega_{ref}\left(e_{ref}\right) = \varpi_1\dot{e}_{ref}\left(ge_{ref}^{g/h-1}/h\chi_3 + 1\right)$, $\kappa_{ref}\left(e_{ref}\right) = \varpi_2\left(e_{ref} + {\chi_1}^{-1}\dot{e}_{ref}^{p/q} + {\chi_3}^{-1}e^{g/h}{}_{ref}\right)$。

$$\dot{s}_{PI-TSM2} = \varpi_1 \frac{d}{dt}\left(e_i + {\chi_2}^{-1}\dot{e}_i^{p/q} + {\chi_4}^{-1}e^{g/h}{}_i\right) + \varpi_2\left(e_i + {\chi_2}^{-1}\dot{e}_i^{p/q} + {\chi_4}^{-1}e^{g/h}{}_i\right)$$

$$= \Omega_i(e_i) + \kappa_i(e_i) + \varpi_1\left(\dot{e}_i^{p/q-1}p/q\chi_2\left(A(t)\dot{e}_i + L_1(t)u_i^* + d_i - \textstyle\sum_{j=1}^{n} d_j/n\right)\right) \tag{11}$$

where $\Omega_i(e_i) = \varpi_1\dot{e}_i\left(e_i^{g/h-1}g/h\chi_4 + 1\right)$,$\kappa_i(e_i) = \varpi_2\left(e_i + {\chi_2}^{-1}\dot{e}_i^{p/q} + {\chi_4}^{-1}e_i^{g/h}\right)$.

The approach law of tracking error and synchronization error of spreader i are as follows:

$$\dot{s}_{PI-TSM1} = -\rho_1\, sgn\left(s_{PI-TSM1}(t)\right) - \psi_1 s_{PI-TSM1}$$

$$\dot{s}_{PI-TSM2} = -\rho_2\, sgn\left(s_{PI-TSM2}(t)\right) - \psi_2 s_{PI-TSM2} \tag{12}$$

Where ρ_1 and ψ_1 are control gains , $sgn(\cdot)$ is a sign function.

Note 1: When the PI sliding surface converges to zero, the nonsingular fast terminal sliding mode surface also converges to zero. For any initial state, $e_{ref}(0) \neq 0$, $e_i(0) \neq 0$. The finite time $t_{s,ref}$ and $t_{s,i}$ of the system convergence are:

$$t_{s,ref} = \frac{p/q\left|e_{ref}\right|^{1-p/q}}{\chi_1(p/q-1)} \cdot \Lambda\left(p/q, \frac{p/q-1}{(\lambda-1)p/q}; 1 + \frac{p/q-1}{(g/h-1)p/q}; -\chi_1\left|e_{ref}(0)\right|^{g/h-1}\right)$$

$$t_{s,i} = \frac{p/q|e_i|^{1-p/q}}{\chi_2(p/q-1)} \cdot \Lambda\left(p/q, \frac{p/q-1}{(\lambda-1)p/q}; 1 + \frac{p/q-1}{(g/h-1)p/q}; -\chi_2|e_i(0)|^{g/h-1}\right) \tag{13}$$

where $\Lambda$is a Gaussian hypergeometric function.

The control laws of tracking error and synchronization error of spreader i are calculated by formulas Eq. (6), (7), (12):

$$u_{ref} = {L_1}^{-1}(t)\left(-A(t)\dot{\theta}_i + \ddot{\theta}_d - q\chi_1(-\rho_1\, sgn(s_{PI-TSM1}) - \psi_1 s_{PI-TSM1}\right.$$

$$\left. -\Omega_{ref}\left(e_{ref}\right) - \kappa_{ref}\left(e_{ref}\right)\big) / \varpi_1 p\dot{e}_{ref}^{p/q-1}\right) \tag{14}$$

$$u_i^* = {L_1}^{-1}(t)\left(-A(t)\dot{\theta}_i + q\chi_2(-\rho_2\, sgn(s_{PI-TSM2})\right.$$

$$\left. -\psi_2 s_{PI-TSM2} - \Omega_i(e_i) - \kappa_i(e_i)\big)/\varpi_1 p\dot{e}_i^{p/q-1}\right) \tag{15}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202005_6(5).0021

3.2 Adaptive super-twisting algorithm

The driving motor described by Eq. (3) is a model of parameter uncertainty. In order to make the system converge in a finite time and reduce the chattering situation of sliding mode control, this paper uses a combination of super-twisting algorithm and sliding mode control. Further, the adaptive super-twisting algorithm improves the control accuracy and suppresses chattering.

According to the sliding mode control of the tracking error described by Eq. (10), the super-twisting algorithm is defined as:

$$\begin{cases} \dot{s}_{PID-TSM1} = -\hat{k}_1 |s_{PI-TSM1}|^{1/2} sgn\left(s_{PI-TSM1}(t)\right) + \upsilon_1 \\ \dot{\upsilon}_1 = -\hat{k}_3 \, sgn\left(s_{PI-TSM1}(t)\right) \end{cases} \tag{16}$$

where $\hat{k}_1, \hat{k}_3$ are the adaptive gains of the super-twisting algorithm.

According to Eq. (14) (16), the control input of spreader i is calculated as follows:

$$u_{ref} = {L_1}^{-1}(t)\big(-A(t)\dot{\theta}_i + \ddot{\theta}_d - q\chi_1\big(-\hat{k}_1|s_{PI-TSM1}|^{1/2} sgn\left(s_{PI-TSM1}(t)\right)$$

$$+\upsilon_1 - \Omega_{ref}\left(e_{ref}\right) - \kappa_{ref}\left(e_{ref}\right)\big)/\varpi_1 p\dot{e}_{ref}^{p/q-1}\big) \tag{17}$$

According to Lyapunov's theorem, the stability equation of the tracking error of spreader i is:

$$V_{ref} = \frac{1}{2} e_{ref}^2 \tag{18}$$

The derivative of V_{ref} is calculated as follows:

$$\dot{V}_{ref} = \frac{1}{2}\left(\frac{\partial V_{ref}}{\partial e_{ref}} \frac{\partial e_{ref}}{\partial \theta_i} \frac{\partial \theta_i}{\partial u_{ref}}\right)\left[\frac{\partial u_{ref}}{\partial \hat{k}_1}\frac{\partial \hat{k}_1}{\partial t} + \frac{\partial u_{ref}}{\partial \upsilon_1}\frac{\partial \upsilon_1}{\partial t}\right]$$

$$= \left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right)\left[\frac{\partial u_{ref}}{\partial \hat{k}_1}\frac{\partial \hat{k}_1}{\partial t} + \frac{\partial u_{ref}}{\partial \upsilon_1}\frac{\partial \upsilon_1}{\partial t}\right] \tag{19}$$

From Eq. (17), $\frac{\partial u_{ref}}{\partial \hat{k}_1}$ is calculated as follows:

$$\frac{\partial u_{ref}}{\partial \hat{k}_1} = \frac{{L_1}^{-1}(t)q\chi_1}{\varpi_1 p\dot{e}_{ref}^{p/q-1}} |s_{PI-TSM1}|^{1/2} sgn\left(s_{PI-TSM1}(t)\right) \tag{20}$$

From Eq. (16) (20), equation (19) can be further simplified as follows:

$$\dot{V}_{ref} = \left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right)\left[\frac{\partial u_{ref}}{\partial \hat{k}_1}\frac{\partial \hat{k}_1}{\partial t} + \frac{\partial u_{ref}}{\partial \upsilon_1}\frac{\partial \upsilon_1}{\partial t}\right]$$

$$= \frac{{L_1}^{-1}(t)q\chi_1}{\varpi_1 p\dot{e}_{ref}^{p/q-1}} |s_{PI-TSM1}|^{1/2} sgn\left(s_{PI-TSM1}(t)\right)\dot{\hat{k}}_1\left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right)$$

$$+\frac{{L_1}^{-1}(t)q\chi_1}{\varpi_1 p\dot{e}_{ref}^{p/q-1}}\hat{k}_3\, sgn\left(s_{PI-TSM1}(t)\right)\left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right) \tag{21}$$

According to Lyapunov's theorem, in order to ensure the stability of the system, the stability equation is required to be not positive. Then the update laws of $\hat{k}_1$and $\hat{k}_3$ are:

$$\dot{\hat{k}}_1 = \frac{\varpi_1 p\dot{e}_{ref}^{p/q-1} k_0}{{L_1}^{-1}(t)q\chi_1} s_{PI-TSM1}(t)\, sgn\left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right) \tag{22}$$

where k_0 is a constant.

$$\dot{\hat{k}}_3 = \varpi_1 p\dot{e}_{ref}^{p/q-1} s_{PI-TSM1}(t)\, sgn\left(e_{ref}\frac{\partial \theta_i}{\partial u_{ref}}\right)/{L_1}^{-1}(t)q\chi_1 \tag{23}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202005_6(5).0021

Similar to the tracking error controller, the super-twisting algorithm in the synchronization controller of spreader i is designed as follows:

$$\begin{cases} \dot{s}_{PID-TSM2} = -\hat{k}_2|s_{PI-TSM2}|^{1/2}\, sgn\big(s_{PI-TSM2}(t)\big) + \upsilon_2 \\ \dot{\upsilon}_2 = -\hat{k}_4\, sgn\big(s_{PI-TSM2}(t)\big) \end{cases} \tag{24}$$

where $\hat{k}_2$ and $\hat{k}_4$ are the adaptive gains

According to Eq. (15) (24), the control law of the synchronization error controller can be designed as follows:

$$\begin{aligned} u_i^* = {L_1}^{-1}(t)\big(-A(t)\dot{\theta}_i + \big(q\chi_2\big(-\hat{k}_2|s_{PI-TSM2}|^{1/2}\, sgn\big(s_{PI-TSM2}(t)\big) \\ +\upsilon_2 - \Omega_i(e_i) - \kappa_i(e_i)\big)/\varpi_1 p\dot{e}_i^{\,p/q-1}\big) \end{aligned} \tag{25}$$

Similar to the tracking error controller, the Lyapunov equation of the synchronization error controller and its derivative are calculated as follows:

$$V_i = \frac{1}{2}e_i^2 \tag{26}$$

$$\begin{aligned} \dot{V}_i &= \frac{1}{2}\left(\frac{\partial V_i}{\partial e_i}\frac{\partial e_i}{\partial \theta_i}\frac{\partial \theta_i}{\partial u_i^*}\right)\left[\frac{\partial u_i}{\partial \hat{k}_2}\frac{\partial \hat{k}_2}{\partial t} + \frac{\partial u_i^*}{\partial \sigma \upsilon_2}\frac{\partial \upsilon_2}{\partial t}\right] \\ &= \left(e_i\frac{\partial \theta_i}{\partial u_i^*}\right)\left[\frac{\partial u_i^*}{\partial \hat{k}_2}\frac{\partial \hat{k}_2}{\partial t} + \frac{\partial u_1^*}{\partial \upsilon_2}\frac{\partial \upsilon_2}{\partial t}\right] \end{aligned} \tag{27}$$

$\dot{V}_i$ can be simplified by calculation as follows:

$$\begin{aligned} \dot{V}_i &= \left(e_i\frac{\partial \theta_i}{\partial u_i^*}\right)\left[\frac{\partial u_i^*}{\partial \hat{k}_2}\frac{\partial \hat{k}_2}{\partial t} + \frac{\partial u_i^*}{\partial \upsilon_2}\frac{\partial \upsilon_2}{\partial t}\right] \\ &= -\frac{{L_1}^{-1}(t)q\chi_2}{\varpi_1 p\dot{e}_i^{\,p/q-1}}|s_{PI-TSM2}|^{1/2}\, sgn\big(s_{PI-TSM2}(t)\big)\,\dot{\hat{k}}_2\left(e_i\frac{\partial \theta_i}{\partial u_1^*}\right) \\ &\quad -\frac{{L_1}^{-1}(t)q\chi_2}{\varpi_1 p\dot{e}_i^{\,p/q-1}}\dot{\hat{k}}_4\, sgn\big(s_{PI-TSM1}(t)\big)\left(e_i\frac{\partial \theta_i}{\partial u_i^*}\right) \end{aligned} \tag{28}$$

The update laws of $\hat{k}_2$ and $\hat{k}_4$ can be calculated as follows:

$$\dot{\hat{k}}_2 = \frac{\varpi_1 p\dot{e}_i^{\,p/q-1}k_0}{{L_1}^{-1}(t)q\chi_2}s_{PI-TSM2}(t)\, sgn\left(e_i\frac{\partial \theta_i}{\partial u_i^*}\right) \tag{29}$$

$$\dot{\hat{k}}_4 = \frac{\varpi_1 p\dot{e}_i^{\,p/q-1}}{{L_1}^{-1}(t)q\chi_2}s_{PI-TSM2}(t)\, sgn\left(e_i\frac{\partial \theta_i}{\partial u_i^*}\right) \tag{30}$$

3.3 Proof of System Stability

Theorem 1: Consider the double spreader overhead crane system described in Eq. (1), and the sliding mode surface $s_{PI\text{-}TSM1}$ based on PI and non-singular fast terminal sliding mode theory proposed in Eq. (8). If under the control of Eqs. (17) and (25) the supercoil control law and $\rho_1 \geq L_2(t)T_{Li,max}$ then the stability of the system and the convergence of the sliding surface are guaranteed.

Proof: Consider the following Lyapunov function:

$$V = V_0 + V_i + V_{ref} = \frac{1}{2}(s_{PI-TSM1})^2 + V_i + V_{ref} \tag{31}$$

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V_{ref} and V_i satisfy the Lyapunov stability condition. According to Eqs. (10) and (31), the first derivative of V_0 can be obtained as:

$$\dot{V}_0 = s_{PI-TSM1}\dot{s}_{PI-TSM1}$$

$$= s_{PI-TSm1}\left(\Omega_{ref}(e_{ref}) + \kappa_{ref}(e_{ref}) + \varpi_1\left(\frac{p}{q\chi_1}\dot{e}_{ref}^{p/q-1}(\ddot{\theta}_d - A(t)\dot{\theta}_i - L_1(t)u_{ref} - d_i)\right)\right)$$

$$= s_{PI-TSM1}\left(-\rho_1\, sgn(s_{PI-TSM1}) - \psi_1 s_{PI-TSM1} - \frac{p}{q\chi_1}\dot{e}_{ref}^{p/q-1}d_i\right)$$

$$\le -\rho_1|s_{PI-TSM1}| - \psi_1(s_{PI-TSM1})^2 - \frac{p}{q\chi_1}\dot{e}_{ref}^{p/q-1}d_i s_{PI-TSM1}$$

$$\le -\rho_1|s_{PI-TSM1}| - \psi_1(s_{PI-TSM1})^2 + \tau_1 s_{PI-TSM1} \tag{32}$$

where $\tau_1 = -\frac{p\rho_1}{q\chi_1}\dot{e}_{ref}^{p/q-1}$.

Appendix1: In order to facilitate the proof, the first-order nonlinear inequality is given as follows:

$$\dot{V}(x) + aV(x) + bV^c(x) \le 0 \tag{33}$$

where $V(x)$ is a positive Lyapunov function, $a,b>0$ and $0<c<1$.

There are two cases for the Lyapunov Eq. (32).

Case1: When $s_{PI-TSM1} > 0$,we can rewrite Eq. (32) into the following form:

$$\dot{V}_0 \le -\left(\psi_1 - \frac{\tau_1}{s}\right)(s_{PI-TSM1})^2 - \rho_1|s_{PI-TSM1}| \tag{34}$$

Hence, if $y_1 - \frac{t_1}{s} > 0$ and $\dot{e}_{ref}^{p/q-1} \ne 0$. According to the inequality proposed in the appendix1, there exits $J_1, J_2 > 0$ to make the following formula

$$\begin{aligned}\dot{V}_0 &\le -\vartheta_1(s_{PI-TSM1})^2 + \vartheta_2|s_{PI-TSM1}| \\ &\le 0\end{aligned} \tag{35}$$

Case 2: When $s_{PI-TSM1} < 0$, we can rewrite Eq. (32) into the following form:

$$\dot{V}_0 \le -\psi_1(s_{PI-TSM1})^2 - \left[\rho_1 - \frac{\tau_1}{s_{PI-TSM1}}\right]|s_{PI-TSM1}| \tag{36}$$

Similarly, if $r_1 - \frac{t_1}{s_{PI-TSM1}} > 0$ and $\dot{e}_{ref}^{p/q-1} \ne 0$, $\dot{V}_0 \le 0$.

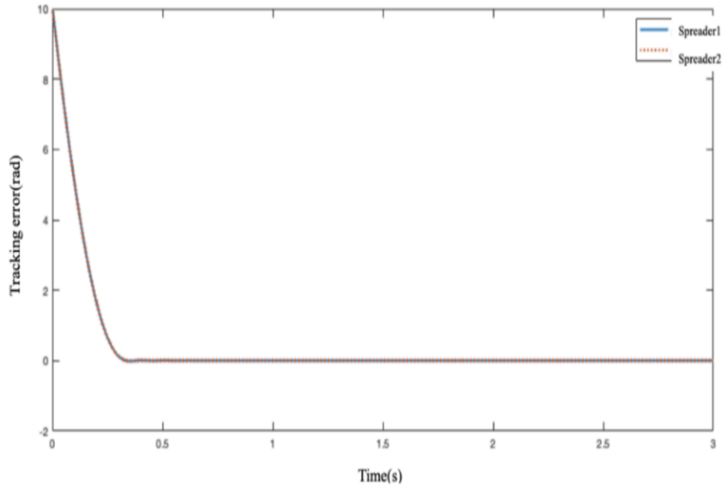
The above proof shows that when the PI non-singular fast terminal sliding mode (Eq. 8) proposed in this paper is applied to the overhead crane system, it is progressively stable under the Lyapunov criterion. Similarly, from Theorem 1, the synchronization error controller based on PI non-singular fast terminal sliding mode theory is also stable.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202005_6(5).0021

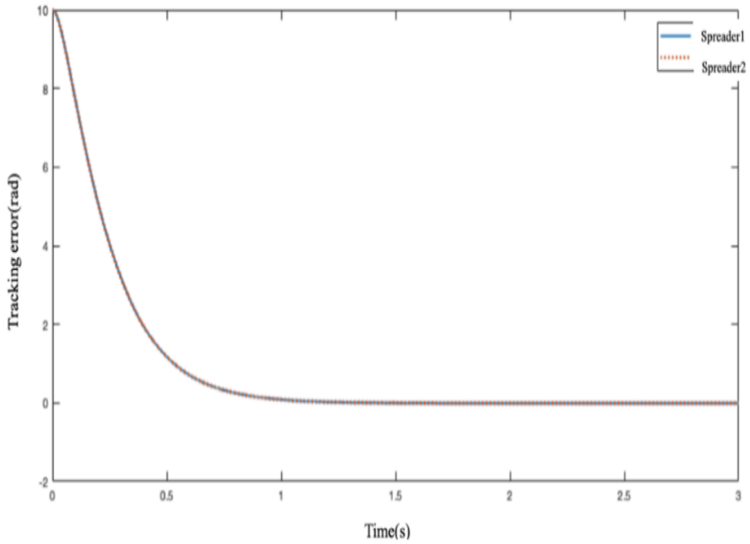
4. Simulation

In order to verify the effectiveness of the PI non-singular fast terminal sliding mode proposed in this paper, MATLAB / SIMULINK software is used for simulation experiments. For different load and disturbance situations, multiple sets of simulations are set for comparison.

Case1: The load of the spreader 1 is set to 3 N/m, the load of the spreader 2 is set to 3 N/m, the desired angle $\theta_d = 10rad$,the external disturbance is 0. Compare the effects of PI nonsingular fast terminal sliding mode control and traditional sliding mode control on the control of the overhead crane system. As can be seen from Figures 2 and 3, the control method proposed in this paper controls the synchronization error of the system within $6.8 * 10^{-4}rad$, and the tracking error converges to 0 at 0.3s. The traditional sliding mode control can only control the synchronization error within $1.8 * 10^{-3}rad$, and the tracking error converges to 0 at 1.2s. This shows that the PI non-singular fast terminal sliding mode control system proposed in this paper has significantly better convergence speed and control effect than the traditional sliding mode control in both synchronization error and tracking error.



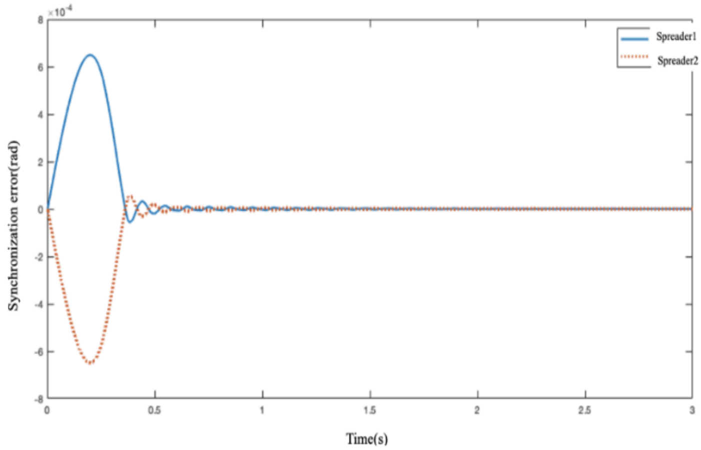
(a)



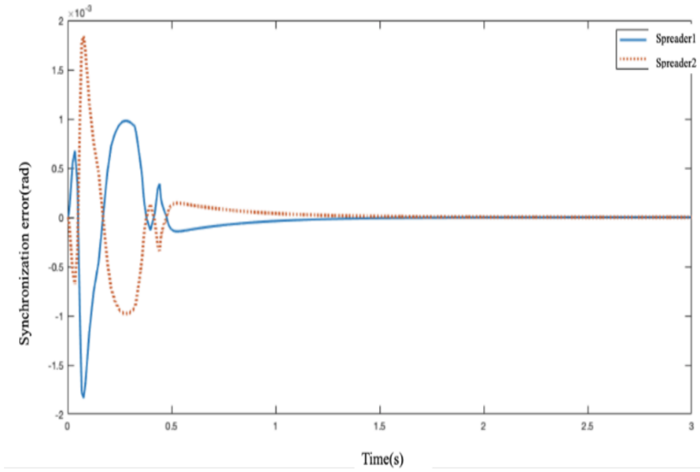
(b)

Figure 2 Tracking error during double spreader operation (a) under PI fast non-singular terminal sliding mode control (b)under traditional sliding mode control

ISSN: 2414-1895
DOI: 10.6919/ICJE.202005_6(5).0021



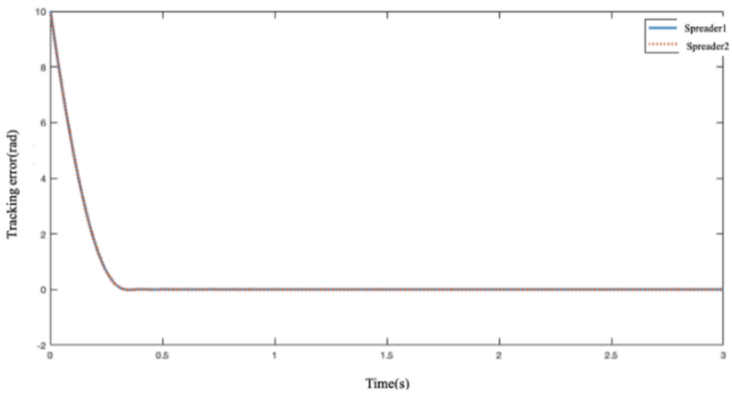
(a)



(b)

Figure 3 Synchronization error during double spreader operation (a) under PI fast non-singular terminal sliding mode control (b) under traditional sliding mode control

Case 2: When the load torques are different, that is, the load of spreader 1 is $3N$, and the load of spreader 2 is $2N$. The desired angle $\theta_d = 10rad$. Compare the effects of PI nonsingular fast terminal sliding mode control and traditional sliding mode control on the control of the overhead crane system. As can be seen from Figures 4 and 5, the control method proposed in this paper can make the synchronization error converge at 1 s, the tracking error converge at 0.3 s, and the variation range of the synchronization error is small. Under the traditional sliding mode control, its synchronization error converges in 2s.



(a)

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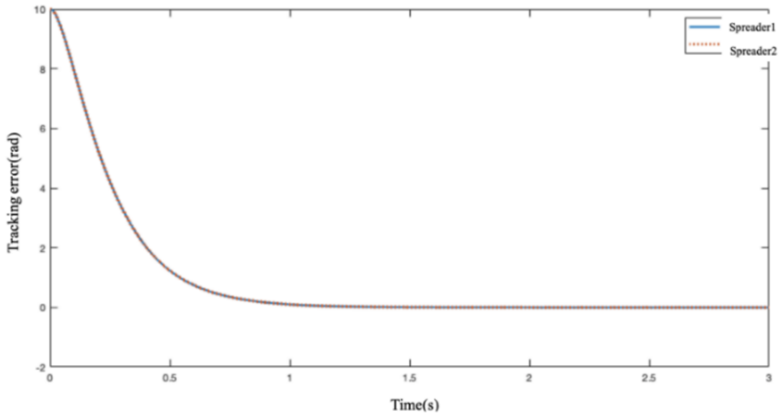
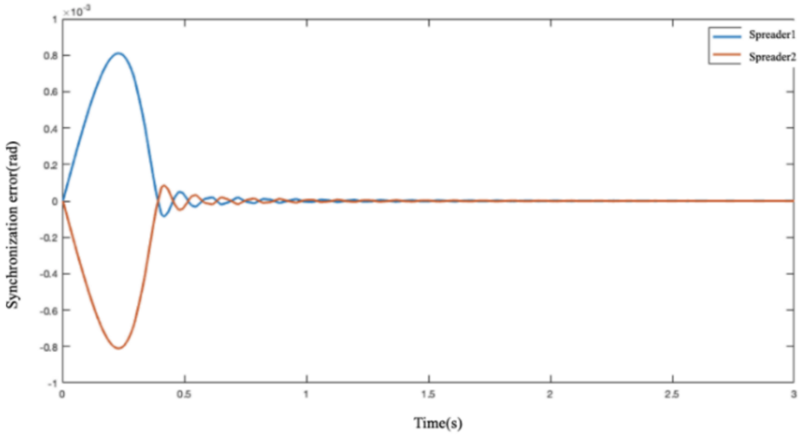
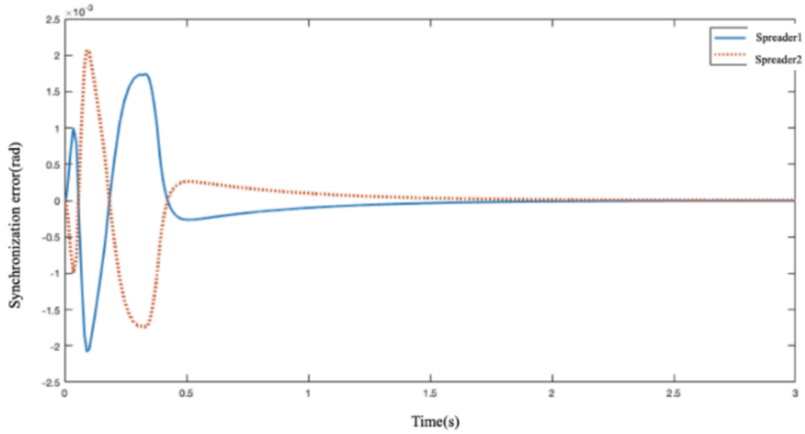


(b)

Figure 2 Tracking error during double spreader operation (a) under PI fast non-singular terminal sliding mode control (b) under traditional sliding mode control



(a)



(b)

Figure 3 Synchronization error during double spreader operation (a) under PI fast non-singular terminal sliding mode control (b) under traditional sliding mode control

5. Conclusion

This paper proposes the PI nonsingular fast terminal sliding mode control method based on the mean deviation coupling strategy for the double-container overhead crane. This method guarantees that the system can converge in a limited time, the system can respond quickly, the system's robustness is enhanced, and the steady-state error is small. It can be seen from the simulation results that compared with the traditional sliding mode control, the control method proposed in this paper can make the synchronization error and tracking error of the spreaders converge to zero faster. When the system is

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subject to external interference or the load of the spreader changes, the control method proposed in this paper shows its characteristics of anti-interference and strong robustness. Lyapunov stability analysis proves the global stability and convergence of the controller.

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