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# Slot Allocation in Container Liner Revenue Management

Zhenghua Huang

College of Transport, Shanghai Maritime University, Shanghai 200000, China.

35888112@qq.com

## Abstract

**In recent years, with the continuous development of international trade, the Marine container transport industry is also developing rapidly. According to statistics, more than 90% of the freight volume of foreign trade is completed by shipping. Due to the fierce competition in the shipping market, container freight rates are falling, fuel costs are rising sharply along with policies such as sulphur restrictions. In addition, the empty container transportation costs caused by trade are huge. In this case, liner shipping enterprises must establish a revenue management system to solve the problem of reasonable allocation of shipping space on container ships and improve the operating revenue of container ships.This paper compares the characteristics of container transport and air transport and proves the feasibility and necessity of the application of container revenue management.The revenue management and put forward a liner shipping space distribution model, by introducing parameters: the level of uncertainty, not feasibility, tolerance, consider two constraints container shipping volume and weight, solve the liner companies under bounded uncertain demand considering empty container transportation of container shipping slot allocation problem, through the specific routes of numerical experiment, using GAMS software to solve the model, it is concluded that the reasonable slot allocation of the results.**

## Keywords

**Liner transport, Revenue management, Slot allocation, Empty container transport.**

## 1. Introduction

RM or yield management, in general, refers to the management of the price and supply of a product or service so as to maximize revenue in a stochastic environment. It aims to use proper mechanisms/ methods to provide the “right product” to the “right customer” at the “right time”. [1]. The research and application of revenue management originated from the aviation industry and has achieved remarkable benefits. Due to the successful application of revenue management in aviation industry, more and more industries and fields have realized the important role of revenue management. There are some differences between container liner shipping industry and air passenger industry is shown in Table 1.

In addition to these differences, there are significant differences in capacity utilization configurations between container liner shipping and air passenger shipping in terms of the combination of required space and loading and unloading ports (length of occupancy), as illustrated by a simple example. As shown in Figure 1, Assuming only one slot is left on the container ship, which can be assigned to a container (say Container A) that would be loaded at port 2, and unloaded at port 5.Assigning this slot to Container A definitely means that when the ship reaches port 3,this space would remain occupied and will not be available. There may be a case where another container (say Container B) can be loaded at port 3, for unloading at port 10.Obviously, Container B will generate more revenue than Container A. Hence we have to consider future opportunity costs. In particular, the multi-port arrival

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demand and configuration of space utilization are essentially uncertain in nature.Like air passenger transport, container liner transportation has the typical characteristics of revenue management application, which are mainly manifested in the following aspects: demand is classified according to market segmentation, perishable, demand volatility is strong, products or services can be sold in advance, fixed cost is high and marginal cost is low [2]. These characteristics make the application of revenue management in container liner transportation industry possible.

**Table 1.** Differences between container liner shipping industry and air passenger industry

| | Air transport | Container liner transport |
|---|---|---|
| Transport path | Passengers pay attention to the time, the specific flight path and whether the transfer has strict requirements | There is no clear requirement for the specific navigation path |
| Carrying object | Passengers, there is a certain division of seats | The type of case (size, kind), the type of goods, and the service to be provided all vary |
| Empty container transportation | Without dispatching | Empty container transportation |
| Different customer sources | Tickets are reserved mainly for individuals | Contract customers and spot customers |

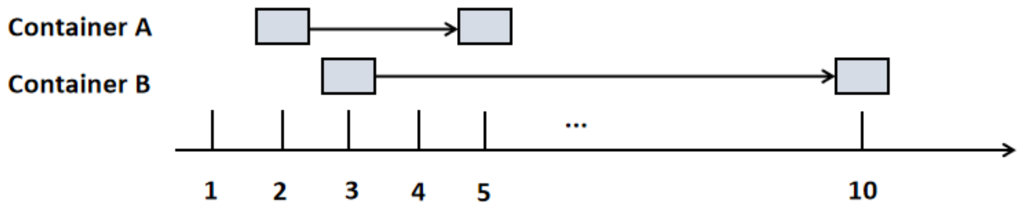


**Figure1.** Network structure in slot allocation of container liner

Liner transport is a mode of fixed port, fixed ship and fixed line. Container ships pass through each port in order according to the established route. The transport accessibility between ports can be expressed by the following adjacent matrix:

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{vmatrix}$$

**Figure. 2** Transport accessibility between ports

Where, 1 means there is an arrival relationship between two ports, and 0 means there is no arrival relationship.

## 2. Related studies

In the literature, liner shipping revenue management is an upcoming research topic, there are few research results in this area.

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In 1994, the first article on liner shipping revenue management was published.Brooks and Button[3] (1994) analyzed the potential of liner transportation revenue management and studied the existing pricing and rate structure.

Xiang-zhi Bu et al. [4] (2008) based on the idea of revenue management of shipping container shipping slot allocation problem is studied, established considering multiple products and for dispatching empty shipping containers segment capacity allocation model, and based on the uncertainty of demand, using the robust optimization method to solve the model, slot allocation of the optimized scheme is obtained by numerical simulation.

Shin-chan TING and Gwo-Hshiung TZENG[5] summarized relevant studies on revenue management in the transportation industry.A conceptual model of liner transport revenue management (LSRM) is proposed, and a liner transport revenue distribution model is established through mathematical programming to maximize the contribution of freight. A liner company in Taiwan is taken as an example to analyze the model. The results show the applicability and superiority of the model.

Sebastian Zurheide and Kathrin Fischer[6] proposed a new bid price (BP) strategy and compared it with the previous proposed reservation price limit strategy.The simulation method is used to evaluate the strategy in different scenarios. Simulation studies reveal that strategies can have very different impacts, such as depending on the quality of the reservation forecast. As the new BP strategy shows very promising results in many different practical situations, it can be highly recommended for liner transport.

Berit Dangaard Brouer et al. [7] explained the importance of empty container transport, and proposed a liner company revenue management model considering empty container repositioning to maximize the profit of goods transported in the network based on the cost and availability of empty containers.A feasible IP solution for LP relaxation is obtained by using delayed column generation algorithm.

Yelin Fu et al. [8] proposed a robust optimization model to solve the problem of container slot allocation with minimum quantity commitment (MQC) under the uncertain demand of international companies exporting to the United States.A new robust optimization method for linear programming (LP) is designed and the right uncertainty is developed by introducing new parameters.A deterministic and processable mixed integer linear programming model is established and a robust solution is obtained which is not affected by demand uncertainty.The robust optimization model proposed in this paper is based on Yelin Fu et al., considering weight limitation and empty container transport, and the reasonable result of slot allocation is obtained.

## 3. Container liner shipping slot allocation model

### 3.1 Problem description

Container liner shipping is a capital intensive industry and liner companies invest a lot of money in transportation facilities. Because of the keen competition in the shipping market, most of freight container unit is on the decline, again due to the trade imbalance has a huge empty container transportation costs, which makes the container transport marginal costs rise further, in this case, the liner companies want to obtain the objective gains a lot of difficulties, especially during the epidemic situation, many shipping companies more losses. Therefore, it is particularly important to apply revenue management to container liner transport. Under the two constraints of volume and load, how to sell container space to the most suitable shippers at the most appropriate price at the most appropriate time is an urgent problem to be solved by liner companies under the concept of revenue management.

### 3.2 Symbol Description

$V$ :Available capacity of container ships

$I$ : Loading port, $i = 1,..., P-1$

$J$ :Port of discharge, $j = 2,..., P$

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$x_{i,j}$ :Number of heavy cases from port $i$ to Port $j$

$y_{i,j}$ :Number of heavy cases from port $i$ to Port $j$

$r_{i,j}$ :Average return per container from port $i$ to Port $j$

$c_{i,j}$ :Average shipping cost per empty container from port $i$ to port $j$

$wz$ :Average weight of heavy cases

$we$ :Average weight of empty case

$W$ :Ship load limit

$U_{i,j}$ :Uncertain booking requirements from port $i$ to port $j$

### 3.3 Model Construction

In this study, we considered that the designated containers would be loaded at one port at a specified time and shipped to another port according to the booking requirements to maximize the revenue.

The total number of containers leaving the port $i$ is $\sum_{j=i+1}^{P}(x_{i,j}+y_{i,j})$, and the number of containers arriving at the port $j$ is $\sum_{i=1}^{j-1}(x_{i,j}+y_{i,j})$.We consider a special port $k$, $k\in\{2,3,...,P-1\}$, and the following equation simulates the occupancy state of container ships in the port:

$$\sum_{i=1}^{k-1}\sum_{j=k}^{P}\left(x_{i,j}+y_{i,j}\right)-\sum_{i=1}^{k-1}\left(x_{i,k}+y_{i,k}\right)+\sum_{j=k+1}^{P}\left(x_{k,j}+y_{k,j}\right) \tag{1}$$

The first term is the container capacity on the ship that has been occupied and that continues to be occupied; the second term is the space previously occupied but released at port $k$. The last term represents the newly loaded containers that are to be transported to forward ports.Container ships have very strict control over capacity and load. In the allocation of slot, not only should the utilization rate of space be improved as much as possible, but also the carrying capacity of the ship should be considered. In a lot of existing literature, however, is not for intermediate port liner liner shipping space take up state and load state description carefully.Taking the ship's volume as an example, when the empty container transportation is not considered, only through the constraint $\sum_{i=1}^{P-1}\sum_{j=i+1}^{P}x_{i,j}\le V$ or constraint $\sum_{i=1}^{P-1}\sum_{j=i+1}^{P}x_{i,j}\le V_i$.The former said maximum total carrying capacity is less than the shipping container shipments, This will lead to a waste of container ship's transport capacity. The latter means that the upper limit of the loading capacity of each port is the maximum carrying capacity, which will lead to the number of containers carried by some ports far exceeding the capacity limit. According to Figure 1, it can be seen that the two constraint modes are incorrect.

Therefore, the container liner shipping slot allocation model can be described as follows:

$$\text{MAX} \sum_{i=1}^{P-1}\sum_{j=i+1}^{P}(r_{i,j}x_{i,j}-c_{i,j}y_{i,j}) \tag{2}$$

$$0\le\sum_{i=1}^{k-1}\sum_{j=k}^{P}\left(x_{i,j}+y_{i,j}\right)-\sum_{i=1}^{k-1}\left(x_{i,k}+y_{i,k}\right)+\sum_{j=k+1}^{P}\left(x_{k,j}+y_{k,j}\right)\le V \tag{3}$$

$$0\le\sum_{i=1}^{k-1}\sum_{j=k}^{P}\left(x_{i,j}\cdot wz+y_{i,j}\cdot we\right)-\sum_{i=1}^{k-1}\left(x_{i,k}\cdot wz+y_{i,k}\cdot we\right)+\sum_{j=k+1}^{P}\left(x_{k,j}\cdot wz+y_{k,j}\cdot we\right)\le W \tag{4}$$

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$$0 \le \sum_{j=2}^{P}(x_{1,j} + y_{1,j}) \le \mathrm{V} \tag{5}$$

$$0 \le \sum_{j=2}^{P}(x_{1,j} \cdot \mathrm{wz} + y_{1,j} \cdot \mathrm{we}) \le W \tag{6}$$

$$\mathrm{x}_{i,j} \le \overline{U}_{i,j} \tag{7}$$

$$\sum_{\mathrm{i}}^{P-1} y_{i,j} \ge NE_j \tag{8}$$

$$\sum_{\mathrm{j}}^{P} y_{i,j} \le \mathrm{SU}_{\mathrm{i}} \tag{9}$$

$$\mathrm{x}_{i,j} \ge 0, 1 \le i < j \le P, k = 2,3,..., P-1 \tag{10}$$

Objective function (2) aims to maximize the gross revenue of ship operation. Constraint condition (3) indicates that the number of containers of container ships in Port $\mathrm{k}$ is greater than or equal to 0 and less than or equal to the maximum load capacity.Constraint condition (4) indicates that the container weight carried by the ship is greater than 0 and less than the net load limit of the ship. Constraint condition (5) Ensure that the sum of load capacity of 0 heavy boxes and empty boxes at the port is greater than or equal to 0 and less than or equal to the maximum load capacity.Constraint condition (6) Ensure that the total weight of heavy boxes and empty boxes in port 0 is greater than 0 and less than or equal to the ship's net load limit.Constraint condition (7) indicates that the amount of space allocated between each port pair is less than a nominal value. Constraint condition (8) indicates that the number of empty containers transported to port $\mathrm{j}$ is greater than or equal to the empty container demand of port $\mathrm{j}$, and constraint condition (9) indicates that the number of containers transported from port $\mathrm{i}$ is less than or equal to the empty container quantity that port i can transfer. The constraint condition (10) is to ensure that the slot allocation quantity is a positive number, $0 \le i < j \le P$ indicating the port order, $\mathrm{k} = 2,3,..., P-1$ indicating the scope of port K.However, the $\overline{\mathrm{U}}_{i,j}$ in constraint (7) is always inherently uncertain, so we use a robust optimization method to solve this right-side uncertainty problem.

### 3.4 Robust optimization for general integer LP with right-hand-side uncertainty

In this section, we study general LP in which right hand sides is uncertain. We are concerned about developing a robust optimization methodology to generate "reliable" solutions to LP, which are immune against data uncertainty.

We consider the following generic LP with $\mathrm{m} \times n$ variables and $\mathrm{m}$ constraints:

$$\begin{cases} \max \mathrm{cx} \\ \mathrm{A}x \le b, x \ge 0 \end{cases} \tag{11}$$

Where A is an $\mathrm{m} \times n$ integer matrix of rank $\mathrm{m}$ and $\mathrm{b} \in \Re^{\mathrm{m}}$. The uncertainty comes from the right-hand side parameter of the inequality constraint, namely $\mathrm{b}_i, \mathrm{i} = 1,2,..., m$.In a robust optimization framework, we are concerned about the feasibility of the following constraints:

$$\sum_{\mathrm{j} \in J} \mathrm{a}_{i,j} x_{i,j} \le b_i \tag{12}$$

The goal of this section is to generate robust solutions to general linear programming problems, in a sense immune to uncertainty. The robust optimization method proposed in this paper was first proposed by Ben-Tal and Nemirovski[9] (2000), and then promoted by Lin et al. [10] (2004).Deal with MILP problem with uncertainty coefficient and inequality constraint on right - hand parameter.

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The method of introducing robustness into the original model in this study is very similar to that of Lin et al. (2004).In this paper, we discuss the allocation of shipping space in the case of bounded uncertainty and obtain a robust solution to the revenue management problem in the case of uncertain market demand.

### 3.5 Bounded uncertainty

The range of uncertainty data is assumed as follows:

$$\left|\tilde{b}_i - \bar{b}_i\right| \le \varepsilon\left|\bar{b}_i\right| \tag{13}$$

Where $\tilde{b}_i$ is the true value, $\bar{b}_i$ is the nominal value, and is defined as the level of uncertainty.

We give the definition of "robust" solution for LP problems with bounded uncertain parameters at the right end:

Definition 3.1 (Lin et al., 2004) When the right-hand-side uncertainty is described in a bounded manner, we call a solution x robust if it satisfies the following conditions:

x is feasible for the nominal problem;

Whatever the true values (say $\tilde{b}_i$ ) of the right-hand-side parameters from the intervals(12), x mustsatisify satisfy the ith inequality constraint with an error of at most $\delta \cdot \max\{1, |\bar{b}_i|\}$,where $\delta$ is interpreted as a given infeasibility level.

More specifically, condition (2) can be expressed as:

$$\forall\left(\left|\tilde{b}_i - \bar{b}_i\right| \le \varepsilon\left|\bar{b}_i\right|\right): \sum_{j\in J} a_{i,j} x_{i,j} \le \tilde{b}_i + \delta \cdot \max\{1, |\bar{b}_i|\} \tag{14}$$

In order to obtain a robust solution, we use the worst-case value of uncertain parameters:

$$\tilde{b}_i \ge \bar{b}_i - \varepsilon\left|\bar{b}_i\right| \tag{15}$$

Substitute (15) into (14).Obviously, X is robust if and only if x is a feasible solution to the following optimization problem:

$$\begin{cases} \max \mathrm{cx} \\ \sum_{j\in J} a_{i,j} x_{i,j} \le \bar{b}_i \\ \sum_{j\in J} a_{i,j} x_{i,j} \le \bar{b}_i - \varepsilon|\bar{b}_i| + \delta \max\{1, |\bar{b}_i|\} \\ x_{i,j} \ge 0, \forall i, j. \end{cases} \tag{16}$$

### 3.6 Robust revenue management model for liner transportation under uncertain demand

Reviewing the model of liner transport revenue management in the previous section, We consider the application of uncertainty level( $\varepsilon$ ) and infeasibility tolerance( $\delta$ ) in this model.Assume that each port has at least one booking requirement, therefore, $\max\{1, |\bar{b}_i|\} = \bar{b}_i$ .

Now the robust optimization model of liner transportation revenue management is transformed into the following contents:

$$\mathrm{MAX} \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} (r_{i,j} x_{i,j} - x_{i,j} y_{i,j}) \tag{17}$$

$$0 \le \sum_{i=1}^{k-1} \sum_{j=k}^{P} (x_{i,j} + y_{i,j}) - \sum_{i=1}^{k-1} (x_{i,k} + y_{i,k}) + \sum_{j=k+1}^{P} (x_{k,j} + y_{k,j}) \le V \tag{18}$$

$$0 \le \sum_{i=1}^{k-1} \sum_{j=k}^{P} (x_{i,j} \cdot wz + y_{i,j} \cdot we) - \sum_{i=1}^{k-1} (x_{i,k} \cdot wz + y_{i,k} \cdot we) + \sum_{j=k+1}^{P} (x_{k,j} \cdot wz + y_{k,j} \cdot we) \le W \tag{19}$$

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$$0 \le \sum_{j=2}^{P} (x_{1,j} + y_{1,j}) \le \mathrm{V} \tag{20}$$

$$0 \le \sum_{j=2}^{P} (x_{1,j} \cdot \mathrm{wz} + y_{1,j} \cdot \mathrm{we}) \le W \tag{21}$$

$$\mathrm{x}_{i,j} \le \overline{U}_{i,j} \tag{22}$$

$$\mathrm{x}_{i,j} \le (1-\varepsilon+\delta)\overline{U}_{i,j} \tag{23}$$

$$\sum_{\mathrm{i}}^{P-1} y_{i,j} \ge NE_j \tag{24}$$

$$\sum_{\mathrm{j}}^{P} y_{i,j} \le \mathrm{SU}_{\mathrm{i}} \tag{25}$$

$$\mathrm{x}_{i,j} \ge 0, 1 \le i < j \le P, k = 2,3,\ldots, P-1 \tag{26}$$

Note that the structural characteristics of the constraint (23) mean that, when the parameters are respectively satisfied $\delta \ge \varepsilon$ , the situation obtained by our robust optimization model is $\mathrm{x}_{i,j} = \overline{\mathrm{U}}_{i,j}$.Therefore, in order to study the performance of our proposed robust optimization model, it is necessary to study the $\varepsilon \le \delta$ case.

# 4. Case Analysis

## 4.1 Data Collection


**Figure 3.** Asia-North Europe Route

This paper takes one of COSCO's Asia-North Europe routes as a case study.Figure 3 shows the route of this service, which consists of 10 legs,the port sequence is shown in Table 2. Port 1 and port 11 are Ningbo ports. In our example, the $\overline{\mathrm{U}}_{i,j}$ in constraint (22) is shown in Table 3, the revenue from heavy container transportation between port pairs is shown in Table 4, and the volume of empty containers available for deployment and demand is shown in Table 5 and Table 6, respectively. See Table 7 for the empty container transportation cost of each port. These data are estimated according to the actual situation of the route. The revenue of heavy containers and the cost of empty containers allocation not only consider the transportation distance, but also consider the market competition. For

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example, the freight rate from Ningbo to Antwerp is more expensive than that from Ningbo to port Kelang. The vessel is a container ship with a cargo capacity of 10036 TEUS, with a deadweight of 121800 tons. Container utilization rate and load utilization rate are two indicators that both the terminal and the shipping company are very concerned about. The stowage personnel need to integrate these two indicators to maximize the interests of the shipping company. In the actual loading process, the loading situation of container ship "full slot but under load" or "full load but not enough slot" often occurs. In this paper, for the convenience of calculation, we only consider the container of 1 TEU, we assume the weight of container with cargo is 13 tons, and the weight of empty container is 2 tons. There are no containers on board before the port of departure, and all containers must be unloaded from the container ship at or before the last port.

**Table 2.** Route port sequence

| Routes | Ports |
|---|---|
| Asia-North Europe Route | Ningbo—Shanghai—Kaohsiung—Yantian—Singapore—Colombo—Antwerp—Hamburg—Rotterdam—Port Kelang—Ningbo |

**Table 3.** Nominal values of demand $\left(\overline{\mathrm{U}}_{i,j}\right)$

| i \ j | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 300 | 540 | 1500 | 1440 | 1380 | 620 | 1230 | 1280 | 140 | |
| 2 | | 240 | 440 | 1230 | 1040 | 1390 | 710 | 530 | 620 | 140 |
| 3 | | | 270 | 500 | 580 | 1580 | 450 | 670 | 440 | 170 |
| 4 | | | | 620 | 780 | 1540 | 1410 | 410 | 370 | 130 |
| 5 | | | | | 610 | 1850 | 380 | 210 | 220 | 230 |
| 6 | | | | | | 420 | 430 | 510 | 250 | 510 |
| 7 | | | | | | | 600 | 350 | 1360 | 720 |
| 8 | | | | | | | | 320 | 580 | 1320 |
| 9 | | | | | | | | | 440 | 520 |
| 10 | | | | | | | | | | 460 |

**Table 4.** Unit weight container transportation revenue

| i \ j | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 110 | 150 | 180 | 230 | 270 | 460 | 500 | 560 | 330 | |
| 2 | | 100 | 140 | 200 | 240 | 430 | 470 | 530 | 300 | 260 |
| 3 | | | 130 | 170 | 240 | 400 | 430 | 510 | 320 | 250 |
| 4 | | | | 150 | 190 | 370 | 400 | 450 | 280 | 260 |
| 5 | | | | | 170 | 320 | 370 | 410 | 180 | 320 |
| 6 | | | | | | 310 | 380 | 400 | 230 | 370 |
| 7 | | | | | | | 130 | 170 | 430 | 500 |
| 8 | | | | | | | | 150 | 400 | 470 |
| 9 | | | | | | | | | 350 | 430 |
| 10 | | | | | | | | | | 280 |

**Table 5.** Number of empty containers available for deployment

| Port | Number of empty containers (TEU) |
|---|---|
| 6 | 300 |
| 7 | 300 |

***Table 6.*** *Demand for empty containers*

| Port | Number of empty containers (TEU) |
|---|---|
| 7 | 200 |
| 8 | 180 |
| 10 | 150 |

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***Table 7.*** *Freight rates of empty containers between ports*

| | | | | | j | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| i | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 100 | 140 | 150 | 180 | 200 | 300 | 320 | 340 | 220 | |
| 1 | | 110 | 130 | 160 | 190 | 280 | 290 | 310 | 200 | 120 |
| 2 | | | 120 | 140 | 170 | 270 | 280 | 290 | 180 | 140 |
| 3 | | | | 120 | 160 | 250 | 270 | 280 | 170 | 180 |
| 4 | | | | | 130 | 250 | 260 | 370 | 130 | 210 |
| 5 | | | | | | 240 | 270 | 300 | 150 | 250 |
| 6 | | | | | | | 150 | 180 | 270 | 290 |
| 7 | | | | | | | | 130 | 260 | 280 |
| 8 | | | | | | | | | 220 | 290 |
| 9 | | | | | | | | | | 200 |

### 4.2 Solution results

In this section, we use the above data to solve the model.When the uncertainty level ($\varepsilon$) and the infeasibility tolerance level ($\delta$) are set to 0, the nominal solution is obtained as follows:

**Table 8.** Robust container slot allocation strategy with bounded uncertainties

| | | | | | j | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| i | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1 |
| 1 | 300 | 540 | 1500 | 1440 | 480 | 395 | 1144 | 1280 | | |
| 2 | | 240 | 440 | 1220 | | | | 530 | | |
| 3 | | | 270 | | | | | 670 | | |
| 4 | | | | 620 | 780 | 400 | | 410 | | |
| 5 | | | | | 610 | 1850 | 380 | 210 | | 230 |
| 6 | | | | | | 420 | 430 | 510 | | 510 |
| 7 | | | | | | | 600 | 350 | 1360 | 720 |
| 8 | | | | | | | | 320 | 580 | 1320 |
| 9 | | | | | | | | | 440 | 520 |
| 10 | | | | | | | | | | 460 |

When $x_{i,j} \le \overline{U}_{i,j}$, the total revenue was 7926700USD, among which the empty container transportation cost was 121100USD, carrying 24479TEU containers in total.

When the uncertainty level ($\varepsilon$) is set to 20% and the infeasibility tolerance level ($\delta$) to 10%, the robust solution is obtained as follows:

**Table 9.** Robust slot allocation scheme with 20% $\varepsilon$ and 10% $\delta$

| | | | | | j | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| i | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1 |
| 1 | 270 | 486 | 1350 | 1296 | 1089 | 558 | 1107 | 1152 | | |
| 2 | | 216 | 396 | 1098 | | | | 477 | | |
| 3 | | | 243 | | | | | 603 | | |
| 4 | | | | 558 | 270 | 161 | 631 | 369 | | |
| 5 | | | | | 549 | 1665 | 342 | 189 | | 207 |
| 6 | | | | | | 378 | 387 | 459 | 225 | 459 |
| 7 | | | | | | | 540 | 315 | 1224 | 648 |
| 8 | | | | | | | | 288 | 522 | 1188 |
| 9 | | | | | | | | | 396 | 468 |
| 10 | | | | | | | | | | 414 |

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When the uncertainty level ( $\varepsilon$ ) is 20% and the infeasibility tolerance level ( $\delta$ ) is 10%, the total revenue is 757,9609USD and the number of carrying containers is 2,3193TEUs.The robust solution ensures that the robust scheduling obtained under a given level of uncertainty and infeasible tolerances is feasible.By comparing the results obtained with the uncertainty level ( $\varepsilon$ ) and the infeasibility tolerance level ( $\delta$ ) both being 0, it is known that the total revenue of container liner transport has decreased by 34,7091USD, and the container transport capacity has decreased by 12,86 TEUS.

The values of uncertainty level and infeasibility tolerance represent managers' risk aversion. More specifically, the greater the uncertainty level, the smaller the infeasibility tolerance, and the higher the risk aversion.Figure 4 summarizes the benefits at different levels of uncertainty. The results show that, under the given infeasible tolerances, the maximum profit that can be achieved decreases with the increase of uncertainty level, which means that the slot allocation strategy is more conservative. On the other hand, at a given level of uncertainty, earnings increase with the increase of infeasibility tolerance, which means that the slot allocation strategy is more aggressive.

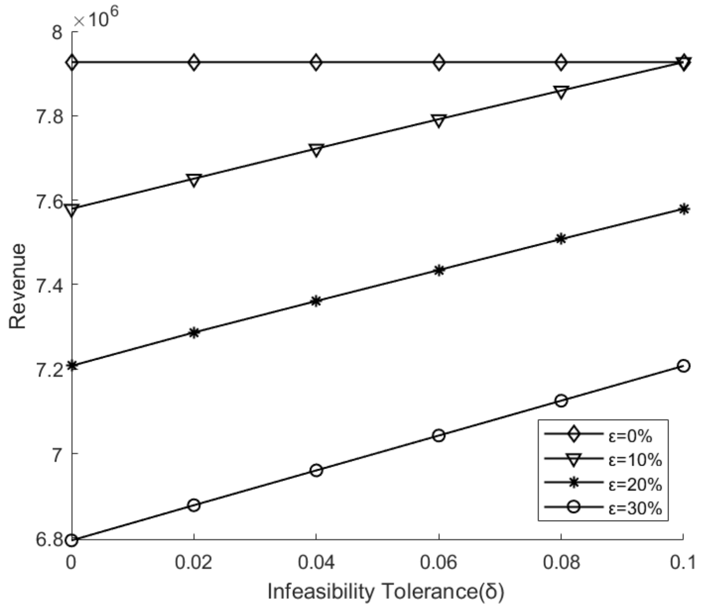


**Figure 4.** Revenue vs. infeasibility tolerance at different uncertainty levels

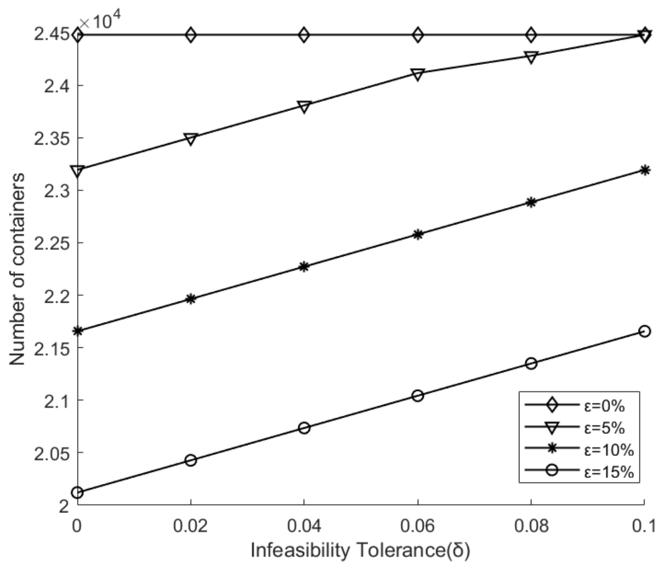


**Figure 5.** Number of containers vs. infeasibility tolerance at different uncertainty levels

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Figure 4 The number of containers carried under different uncertainty levels is summarized. The results show that the number of containers carried is closely related to the revenue.Moreover, under a given infeasible tolerances, the number of containers that can be transported decreases with the increase of the uncertainty level; on the other hand, under a given uncertainty level, the volume of container transport increases with the increase of infeasible tolerances.

According to the slot allocation data calculated by the uncertainty level (20%) and the infeasibility tolerance level (10%), the empty container transfer is shown in Table 10.

**Table 10.** Empty container transport results

| Port i, j | 8 | 9 | 10 |
|---|---|---|---|
| 6 | 100 | 80 | 50 |
| 7 | 100 | 100 | 100 |

According to the empty container freight rate table, empty containers at port 7 have priority to meet the demand of empty containers at the back port, and then empty containers are transferred at port 6.

The loading quantity and occupancy status of container space at each port are shown in Table 11 and Table 12 respectively.

**Table 11.** Quantity of containers loaded at each port

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number | 7308 | 2187 | 846 | 1989 | 2952 | 1908 | 2727 | 1998 | 864 | 414 |

**Table 12.** Occupancy status of shipping slot at each port

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number | 7079 | 9209 | 9369 | 9369 | 9369 | 9369 | 9334 | 9000 | 5680 | 3760 |

It can be seen from Table 12, the port 3 to 6 slot occupancy state is 9369 TEUs, according to the load limit divided by the space limit, the average weight of each container is 12.14 tons, this paper assumes that the average weight of the container loaded with cargo is 13 tons, which indicates that the container ship is “full load but not enough slot”,part of the demand was rejected because it was full load. The number of slot occupied in port 7 has decreased, which is mainly due to the demand of empty containers at the back port.Some empty containers need to be transferred from port 6, so they need to occupy part of the deadweight. Therefore, some of the heavy containers must be rejected. This shows that empty container transportation not only needs empty container transportation cost, but also refuses heavy container reservation under the factors of transportation capacity limitation, which further reduces the revenue.

### 4.3 Summary of this chapter

This chapter is an example analysis part. It selects a route of COSCO Group as the research object, collects data, solves the robust model of class allocation of liner transportation revenue management through GAMS software, obtains the class allocation strategy under a certain parameter level, and then calculates the revenue change under different uncertainty levels and infeasibility tolerance.The effects of parameter uncertainty level and infeasibility tolerance on ship earnings are obtained.According to the allocation strategy under certain parameters, analyze the empty container allocation, ship traffic volume and occupancy status of ship slot,the shipping company's strategy of accepting or rejecting the booking under the condition of empty container transportation demand is obtained.

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## 5. Conclusion and prospect

This paper mainly studies the revenue of liner transportation in revenue management, puts forward the allocation model of container liner shipping space, calculates the revenue by gams software, and finally analyzes the revenue.Under certain uncertainty level and infeasibility tolerance, the slot allocation strategy considering empty container transportation is obtained. The research proves the feasibility of the model, provides the solution ideas and basis for solving the problem of shipping slot allocation, and provides theoretical guidance and support for the development decision-making of liner companies.

The contribution of this paper is to study the network structure of container liner shipping slot allocation, put forward the correct model of container liner shipping slot allocation, analyze the strategy of container liner shipping slot allocation under different parameters and empty container allocation, and obtain the result of slot allocation in line with the concept of revenue management through empirical analysis.Future research can consider market segmentation, overbooking slot allocation strategy and revenue analysis.

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