

# Optimal Design of Digital Filter Using Genetic Algorithm

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## Abstract

Aiming at the need of universal high-precision digital filter design, a method of designing finite impulse response digital filter with genetic algorithm is proposed. Use genetic algorithm to optimize the filter coefficients. Based on the maximal error minimization criterion, an optimized fitness function calculation method is given, and the implementation steps of genetic algorithm optimization filter are designed. The simulation design is carried out by programming, and the optimized low-pass filter is used to process the mixed signal. The experimental results show that the digital filter designed based on the genetic algorithm can meet the design requirements.

## Keywords

Genetic Algorithm; Digital Filter; Coefficient Optimization; Fitness Function.

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## 1. Introduction

Filters are an essential part of modern signal processing. Classic filters have the characteristics of complex design, huge structure, and large number of components. With the development of computer technology, digital filters have high precision, flexibility, and are easy to integrate on a large scale. In general, the digital filter is a linear time-invariant discrete-time system, which is implemented by a finite-precision algorithm.

Modern digital filter design can be seen as an approximation to the ideal performance of an ideal filter. Many optimization algorithms have been used in the filter design process. In the early days, the calculation of FIR digital filter design with global optimization algorithm was too large [1], and it was not easy to find the optimal solution with local neighborhood search technology [2]. Since then, more evolutionary algorithms have been used in the design of FIR filters, including simulated annealing algorithm [3], particle swarm algorithm [4], evolutionary programming method [5], immune algorithm [6], ant colony algorithm [7], Neural Network Algorithm [8], etc.

In this paper, the design of FIR digital filter is realized by genetic algorithm. The genetic algorithm is combined with the design requirements of the digital filter parameters, the fitness function is constructed using the maximum error minimization criterion, and the digital filter parameters are designed through the genetic algorithm to obtain an optimized digital filter. A low-pass filter is designed using the method in the article, and the performance of the filter is tested by filtering the mixed signal, which proves that the proposed method can meet the design needs.

## 2. Mathematical Model

### 2.1 Optimal model of FIR digital filter

The ideal frequency response of the FIR filter is

$$H_d(e^{j\omega}) = \begin{cases} 1 & \omega \leq \omega_p \\ 0 & \omega_s \leq \omega \leq \pi \end{cases} \quad (1)$$

Among them,  $\omega_p$  is the passband frequency and  $\omega_s$  is the stopband frequency. Use the frequency response  $H(e^{j\omega})$  of the FIR digital filter of  $N$  to approximate  $H_d(e^{j\omega})$ . If the impulse response  $h(n)$  of the linear phase FIR digital filter is even symmetric and  $N$  is odd, then

$$H(e^{j\omega}) = e^{-j(N-1)\omega/2} H_g(e^{j\omega}) \quad (2)$$

Where

$$H_g(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos(\omega n) \quad (3)$$

$a(n)$  is the filter coefficient, and its relationship with  $h(n)$  is

$$a(n) = \begin{cases} h\left(\frac{N-1}{2}\right) & n = 0 \\ 2h\left(\frac{N-1}{2} - n\right) & n = 1, 2, \dots, \frac{N-1}{2} \end{cases} \quad (4)$$

Define the weighting function as  $W(e^{j\omega})$ , then the error weighting function is

$$E(e^{j\omega}) = W(e^{j\omega}) \left[ H_g(e^{j\omega}) - H_d(e^{j\omega}) \right] \quad (5)$$

Substituting equation (3) into equation (5), it is

$$E(e^{j\omega}) = W(e^{j\omega}) \left[ \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos(\omega n) - H_d(e^{j\omega}) \right] \quad (6)$$

In this way, the problem of uniformly approximating  $H_d(e^{j\omega})$  with  $H_g(e^{j\omega})$  is expressed as: seeking coefficients  $a(n)$ ,  $n=0, 1, \dots, \frac{N-1}{2}$  to minimize the maximum value of the weighted error function  $E(e^{j\omega})$ . After obtaining  $a(n)$  in this way, the amplitude-frequency response can be obtained according to equation (3), and then the impulse response sequence  $h(n)$  of the FIR filter can be obtained according to equation (4).

## 2.2 The Implementation of Genetic Algorithm for Digital Filter Design

When the genetic algorithm is used for filter design, set the number of groups  $N_p$  to 100, a set of filter coefficients as an individual, and use equation (6) as the goal to find the optimal value. The implementation steps are as follows:

- (1) Initialize the population and randomly generate 100 individuals as the initial population. Take the initial population as  $S$ , and the initial population as the filter coefficients. In each generation of genetic algorithm, the individual count value is transformed to obtain a child.
- (2) Calculate the fitness of each offspring. The greater the adaptability, the closer the filter design is to the ideal filter, and the more likely the individual is to be retained by the race.

- (3) Selection operation, keeping individuals with higher fitness for inheritance, while individuals with low fitness cannot be inherited by offspring.
- (4) Crossover operation, use probability  $p_c$  to select individuals to be paired. If the number of selected individuals to be paired is odd, the last individual will be automatically discarded to ensure that the number of intersecting individuals is always an even number, and the two adjacent individuals Randomly generate the intersection point, and implement the intersection operation at this point.
- (5) Mutation operation, use probability  $p_m$  to perform mutation operation, and replace the value of some specific gene positions in the individual code string with other values.
- (6) If the number of cycles reaches the termination evolutionary algebra, then the algorithm ends, otherwise, skip to step (2).

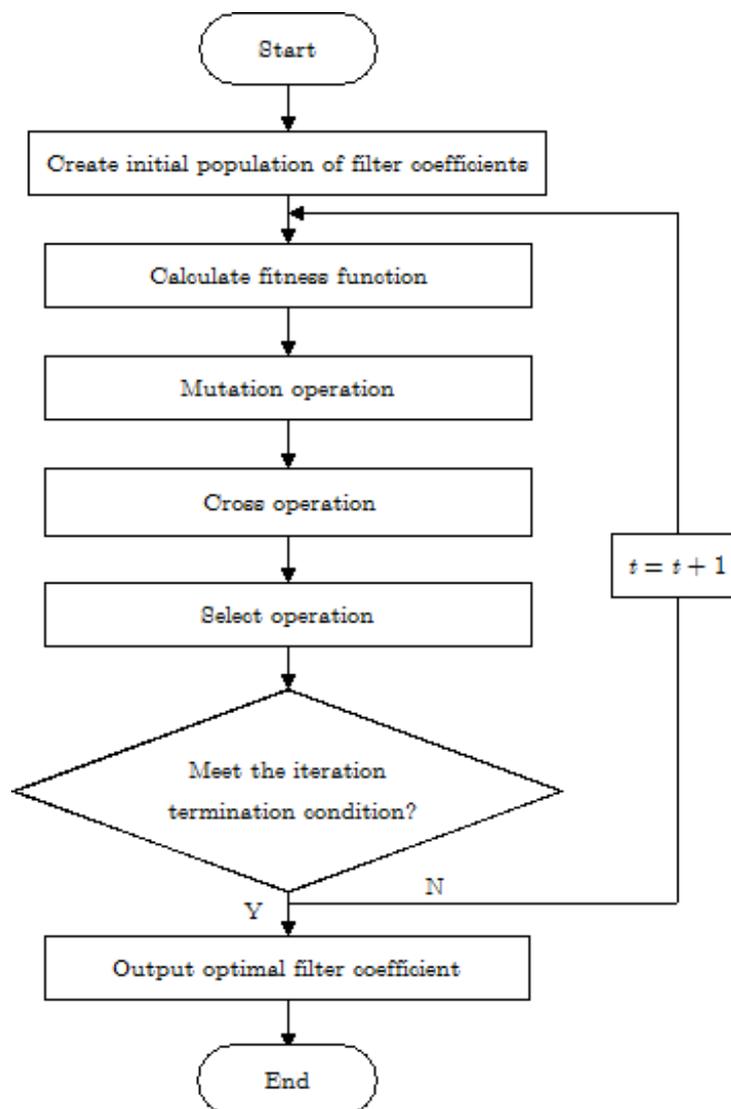


Figure 1. Digital filter design flow chart.

### 3. Calculation and Analysis

Construct a sine wave original signal, the amplitude of the signal is 1, the frequency is 10Hz. In addition, construct an interference signal that is also a sine wave, with an amplitude of 0.3 and a frequency of 20 Hz. Create a Gaussian noise signal with a noise amplitude of 0.2 and a variance of 0.01. The three signals are synthesized, the time-domain waveform of the synthesized signal is shown in Figure 2, and the amplitude spectrum of the synthesized signal is shown in Figure 3. It can be easily

seen from Figure 3 that the frequency distribution of the original signal, interference signal and noise signal.

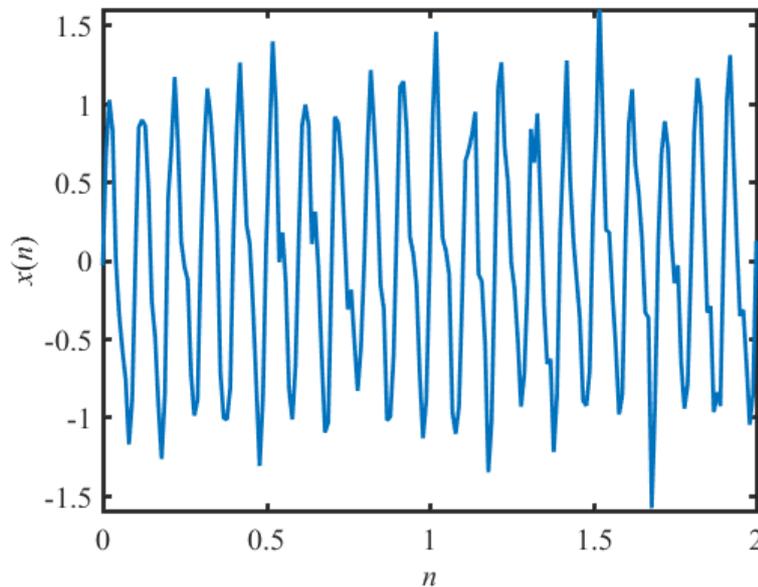


Figure 2. Time signal with interference and noise.

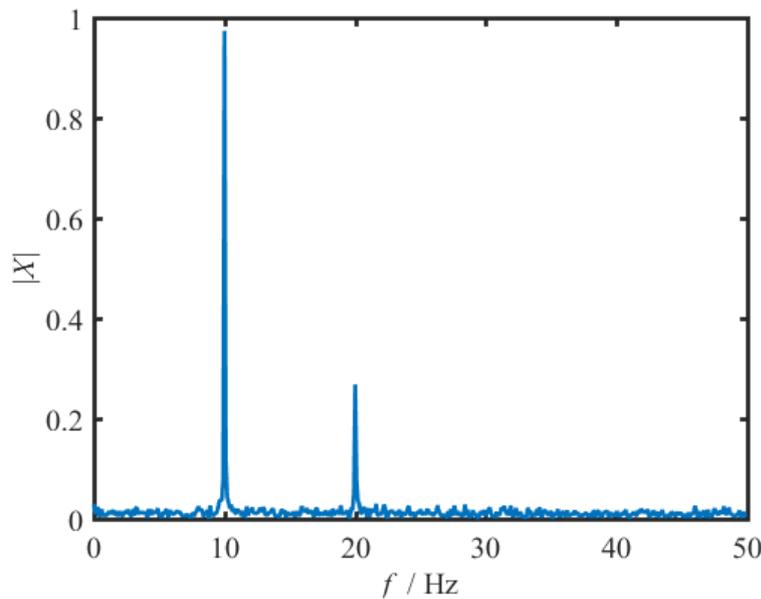


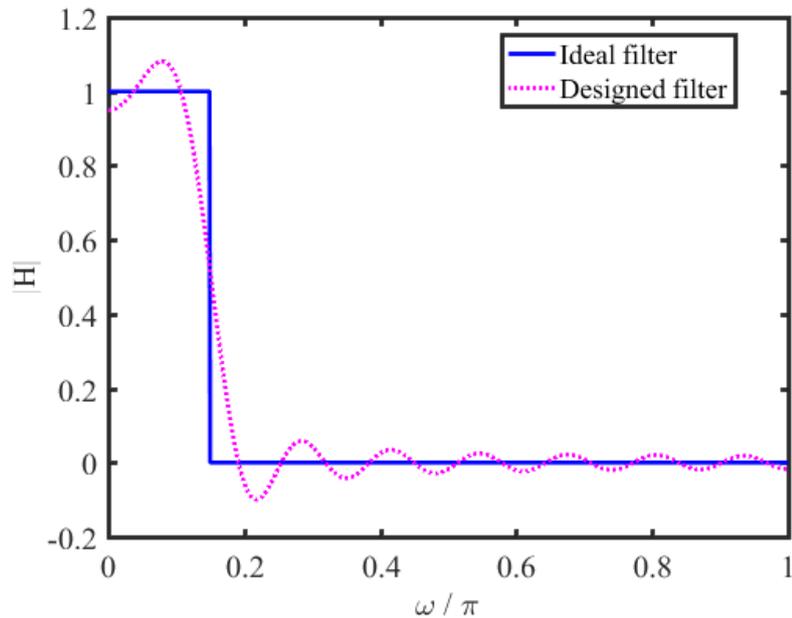
Figure 3. Amplitude spectrum of mixed signal.

Design a low-pass filter with an order of 30, the technical indicators are as follows:

$$H_d(e^{j\omega}) = \begin{cases} 1 & \omega \leq 0.15\pi \\ 0 & 0.15\pi \leq \omega \leq \pi \end{cases} \quad (7)$$

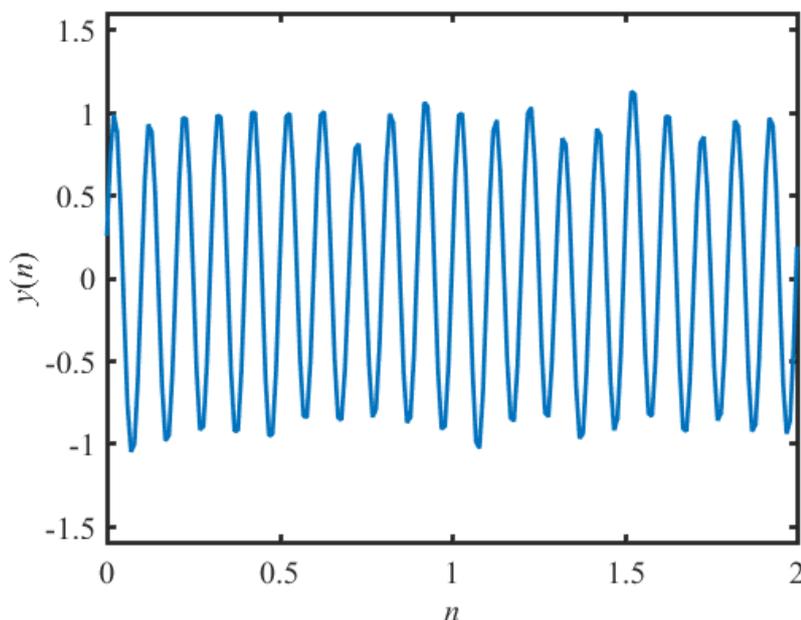
In the design process, the frequency is uniformly sampled, and the number of sampling points is 50. For the optimization of genetic algorithm, set population size  $N_p = 100$ , crossover probability  $p_c = 0.25$ , and mutation probability  $p_m = 0.01$ . The frequency code length is 20 bits. Maximum genetic algebra  $G = 100$ . Using the maximum error minimization as the optimization criterion, the comparison between the optimized low-pass filter and the ideal filter is shown in Figure 4. It can be

seen that better filter coefficients can be obtained through genetic algorithm, and the characteristics of the designed filter are basically consistent with the expected filter.

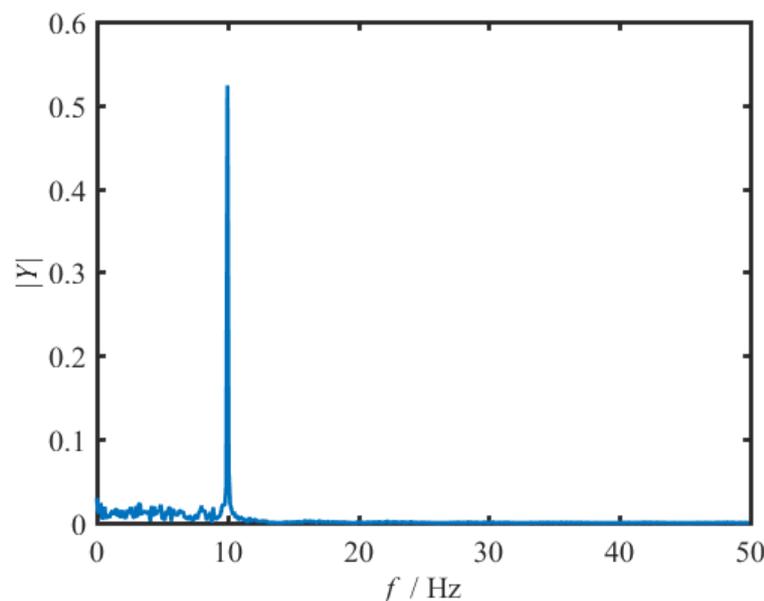


**Figure 4.** Design result of low-pass digital filter based on mean square error minimization design.

The synthesized signal including the original signal, interference signal and noise signal is passed through the designed low-pass filter. The time-domain waveform of the filtered signal is shown in Figure 5, and the amplitude spectrum of the filtered signal is shown in Figure 6. It can be seen that the designed filter can effectively filter out high-frequency interference signals and high-frequency noise. Although the filtered signal still contains low-frequency noise with limited amplitude, the characteristics of the filtered signal are closer to the original uninterrupted signal. It can be seen that the filter designed by genetic algorithm can meet the design requirements.



**Figure 5.** Filtered time domain signal.



**Figure 6.** The amplitude spectrum of the filtered signal.

#### 4. Conclusion

In this paper, genetic algorithm is used to optimize the design of FIR digital filter, and the mathematical optimization model of FIR digital filter, the design rule of minimizing the maximum error, and the design steps of genetic algorithm to optimize the design of FIR filter are studied. Given the design index of the low-pass filter, the simulation design is completed by the proposed method, and the noise-added mixed signal is filtered. The experimental results show that through the genetic algorithm optimization, the designed filter can effectively control the error from the ideal filter and meet the application needs.

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