

# Analysis and Design of Fuzzy Controller for a Class of Nonlinear Time-varying System

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## Abstract

This paper introduced a self-study and self-organization fuzzy controller with parameter feedback for a kind of nonlinear time varying system in industry. At first, a mathematical model has been established to help analysis of the controller. Then the structure of controller and its components were explained in sequence. In the system, basic fuzzy controller was designed with a simple but utilitarian principle that has been practiced in many circumstances. Reviser of fuzzy regulation was controlled by parameters that come from the object controlled. It can straightly change the parameters set for fuzzy regulation by changing its output value. Genetic algorithm can simplify the designing and enhance the performance of this controller. It is useful to analyze the system with a method based function evaluation when the traditional method of state space is hard to work. The results of simulation showed that this controller was effective.

## Keywords

Nonlinear, Time-varying, Feedback, Fuzzy, Genetic Algorithm.

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## 1. Introduction

Recently, the theory and application of fuzzy control are developing very quickly, thus the fuzzy controller are being used more and more widely in the industrial process for its non-linearly approaching feature and relatively strong robustness. However, it is noted from the practical application and the research of the theory that fuzzy control has improved its flexibility and robustness at the expense of its accuracy and normalization. The main reasons are as follows. The structured analysis and the systematized design method are not perfect enough. Though it is no need to acquire the accurate mathematical model of the system, the structure of fuzzy control and all kinds of compound fuzzy control depends on the characteristic of the specific object to some extent. Since the structure of the controller varies greatly according to the object, there will be great difficulty in the process of designing and analyzing. In spite of the difficult in acquiring the accurate mathematics model of the industrial object, the model can be described partly more or less [1]. It is also an important problem that how we can make full use of the object's information. The paper puts forward a kind of self-organization fuzzy controller with parameter feedback for a kind of common nonlinear time varying system in industry.

## 2. The Description of Problem

There are all kinds of controlled objects, for example the steam temperature in the process of tobacco redrying and PH value of the soft water neutralization system, which can be described as time varying system with the parameters controlled by some uncertain factors. They can be described by the state equations:

$$\begin{cases} \dot{x} = f(x, u, s) \\ Es(t) = m \end{cases} \quad (1)$$

Where  $f(\cdot)$  is a linear or nonlinear function,  $Es(t)$  means finding the mathematical expectation of the random process  $s(t)$  which is a stationary process under many circumstances, namely  $m$  is a constant.

It is difficult to get ideal result only by self-organization fuzzy controller or fuzzy adaptive controller [2]. The reasons are as follows:

(1) While amending the control rules on line, the real-time character of the system must become bad .The reason is that when the parameters of the object vary frequently, the lagging regulation of the control rule may cause the control out of step.

(2) While amending the control rules off line ,the controller ,once put into use normally ,is equal to a basic fuzzy controller and has no self-organization to say nothing of insuring the control performance.

From above analysis, it can be clearly known that we can acquire the major factors that cause the parameter of the object to vary greatly or even to obtain the definite relation between them by some means though unable to describe the characteristic of the object accurately. If we can draw out of the major factor to construct suitable mapping relations with the rule sets of the fuzzy controller and use it as auxiliary variables to control the throwback among different control rule sets then the dynamic rules can be adjusted, as well as the experience and knowledge implied in controlled object can be used efficiently.

### 3. The Structure of Controller

The controller of the system is composed of the basic fuzzy controller, fuzzy rule reviser and the unit seeking for the excellence, as shown in the Figure 1.

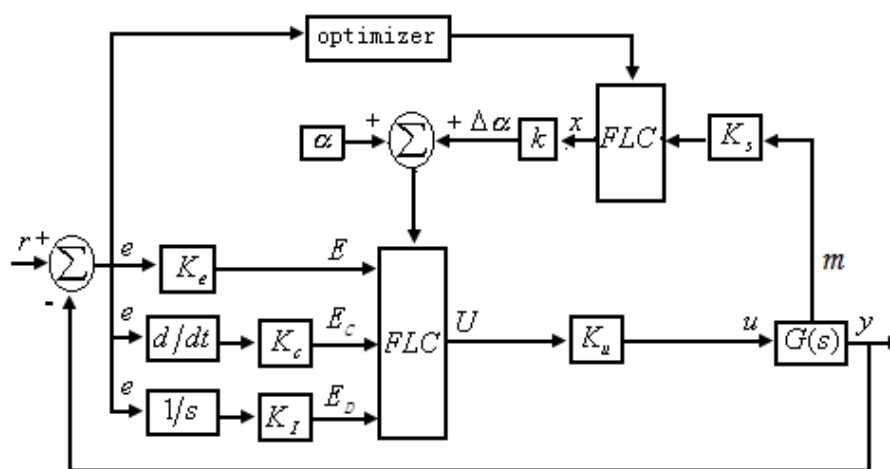


Figure 1. The Structure of Controller

#### 3.1 Basic Fuzzy Controller

The basic fuzzy controller adopts the structure of three inputs and one output, which has higher dimension than the two-dimensional fuzzy controller widely used at present. Increasing the dimension of fuzzy controller can improve the control accuracy, but too high dimension will bring the control rules complex, control algorithm difficult to achieve and other shortcomings [3]. In consideration of the above contradictions, the basic fuzzy controller in this paper makes fuzzy decision based on the formula method, and the output is analytical type. It does not need to rely on the control rule table, and directly calculates the control output  $U$  according to the error  $E$ , the error change rate  $EC$ , and the error integral  $Ed$ . It can be described as the following analytic equations:

$$U = \frac{(\alpha + \beta)}{2} E + \frac{(1 - \alpha)}{2} E_c + \frac{(1 - \beta)}{2} E_d \tag{2}$$

According to experience, the value range of the formula is [0.3,0.7], the value range is [0.2,0.8], on the premise that the control accuracy meets the expectation, it can be taken as = 0.5, then the adjustment of the fuzzy control rules is only affected by, so that the design of the control rules of the basic fuzzy controller is easy to realize.

By doing experiments the scale coefficient can be fixed according to the following principles [4]:

$$K_e \geq \frac{1}{2\delta} \quad (3)$$

Where  $\delta$  is the maximum steady-state deviation of the system.  $K_c = (1.5 \sim 2.5)K_e$ ,  $K_d = (0.5 \sim 1.5)K_e$ ,  $K_u = (2 \sim 3)K_I$ .

### 3.2 Reviser of Fuzzy Regulation with Parameter Feedback

By introducing the control parameters and constructing mapping relation with fuzzy control laws by setting fuzzy rules, the reviser is equivalent to the fuzzy network with feedforward compensation essentially. It can regulate the outputs of the controller dynamically. There are some other characteristics of the controller:

- (1) It is not the external factors but the input variables, which are the self- parameters of the controller.
- (2) By correcting regulations of the fuzzy controller, the accommodation of the controller can be enlarged.
- (3) By introducing the algorithm seeking for the excellence, the design can be simplified and the effect of the controller can be improved.

Here, the fuzzy controller with parameter feedback will be analyzed qualitatively combining with the time-varying process expressed in (1). Firstly, let the output equation be  $y = Cx$ , combined with the equation (1), then  $y = g(u, m)$  can be derived; Secondly, assume the initial condition be  $m_0 = k, y = y_0, u = u_0$  then  $y_0 = g(u_0, k)$ . Thirdly, let the next value of sample time  $m$  be  $m_1$ , then  $\Delta m = m_1 - m_0$  and  $y = y_0 + \Delta y = g(u_0 + \Delta u, k + \Delta m)$ .

Generally the variation of  $\Delta s$  in a sample period is very small, so the variation of deviation  $e$  and deviation variance  $ec$  caused by  $\Delta y$  will not be large enough to change the quantization interval of oneself, that is,  $E$  and  $EC$  can be taken as fixed. By

$u = K_u \beta_i E_i + K_u (1 - \beta_i) C_i + K_I \beta_i E_i + K_I (1 - \beta_i) C_i + K_I \sum_{k=1}^{i-1} u_k$ , it can be known if  $\beta_i$  is

invariable then  $\Delta u = 0$ , namely  $y = y_0 + \Delta y = g(u_0, m + \Delta s)$ .

Recently, the systematic research of fuzzy controller shows that fuzzy controller FLC can approximate to any nonlinear functions, that is, it can be taken as a universal nonlinear approaching device [5]. By means of regulating the accuracy of fuzzy quantization and fuzzy reasoning rules, a mapping  $H$  can always be found i.e.  $\Delta m = H(\Delta t)$ , where  $\Delta m$  can be given by

$$\Delta y = g(u_0 + \Delta u, m + \Delta s) - y_0 = 0 \quad (4)$$

where  $\Delta u = (K_u \Delta \alpha + K_I \Delta \alpha) (E_i - C_i)$ . So it is feasible to apply the reviser of fuzzy regulation with parameter feedback to this kind of object. In this paper, outputs of the fuzzy reviser, the finally outputs, will be normalized, multiplied by scale coefficient and added to the setting value.

The mapping  $H$  can be acquired automatically, according to the optimum-seeking algorithm based on the genetic algorithm (GA). After the searching has been finished,  $H$  can be fixed. While applying online, the basic regulations of the fuzzy controller can be revised quickly only by looking up to a table simply, thus the ability of real-time and self-adaptive can also be ensured.

### 3.3 Optimum-seeking Algorithm Based on GA

Usually, the ideal mapping relation cannot be constructed only by knowledge and experience, for the relation between the input and output parameter is complex and uncertain, so it is necessary to seek

for the optimum by the genetic algorithm[6]. In order to reduce the length of the code, cut down the computational quantity and increase the optimum seeking rate of converge, a fuzzy logic unit with single input and single output will be used as the core part of the control regulation reviser.

To be convenient for the analysis, assume the quantization power group of the inputs to be  $\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$ . The total number of the fuzzy outputs is seven, for each output is corresponding to a quantization value. Let them be  $x_1, x_2, \dots, x_{11}$  separately then the argument vector of the genetic algorithm's object function can be  $\theta = (x_1, x_2, \dots, x_9, x_{10}, x_{11})$ . The process of seeking optimum means to find the best combination of the eleven variables:  $\{x_{1i}, x_{2i}, x_{3i}, \dots, x_{9i}, x_{10i}, x_{11i}\}$  and make the mapping  $H : s \rightarrow x$  meets the needs of the equation(4).

Because the length of the object vector optimized is so long that there will be a combinational exploded problem. Assume the power group of each input or output to be  $\{-n, -(n-1), \dots, -1, 0, 1, \dots, n-1, n\}$  then the total number of the probable number will be  $2n + 1$ . If we adopt the complete encoding method then the whole scale of the code will be  $(2n + 1)^{11}$  and the value of  $n$  which decides the precision of the outputs should be selected carefully. This paper adopts part-encoding method, which judge the quantization value of output according to experience and common sense and then make combinational coding in different intervals[7]. For example, let  $n$  be 7, make a list of every possible output, exclusive of the impossible or nonsensical value. As shown in the Table 1, where "1" means possible, "0" means impossible, and  $x_i$  represents the output corresponding to the  $i$  th input. It can be concluded from the table that the length of the code shortens to 14 bits from 21 bits while the whole scale shortens to  $4 \times 3^5$  from  $7^7$ . Obviously, the amount of calculation has been decreased greatly.

**Table 1.** The Value of Output

Output variable \ Quantizedvalue	-3	-2	-1	0	1	2	3
x1	0	0	0	0	0	1	1
x2	0	0	0	0	1	1	1
x3	0	0	0	1	1	1	0
x4	0	0	1	1	1	0	0
x5	0	1	1	1	0	0	0
x6	1	1	1	0	0	0	0
x7	1	1	0	0	0	0	0

The object function of GA is the ITAE, which has been used widely and can be described as  $J = \int_0^\infty t | e(t) | dt = \min$ . According to  $J$ , the sufficiency of each part can be evaluated and the crossover operator can be selected.

#### 4. The System Analysis Based on Performance Valuation

Recently, the stability analysis, based on the fuzzy language state model, of most fuzzy controllers will describe the conclusion of each fuzzy rule with a state equation. The fuzzy function can be expressed as  $x(k + 1) = \sum_{i=1}^m \alpha_i (A_i x(k) + b_i u(k))$ , where  $A_i$  is the system matrix and  $b_i$  the input matrix. By applying Lyapunov stability theory in the above model, some sufficient condition of system stability can be deduced [7].

Now, the method is coming into shape and gradually reaching its maturity but there still be some difficulty in applying the method in the practical fuzzy control system, for example, the complex computing, hard condition and so on. In this paper, the fuzzy controller with parameter feedback, adopting the analytical method based on function evaluating, which has been introduced in the above, will be expressed qualitatively. To select an assessment index set: {simplicity, robustness, the difficulty of parameter regulating, stability } and describe it with symbol set {A, B, C, D}. Let the comment set be {excellence, good, middle, bad}, the corresponding symbol set be {  $u_1, u_2, u_3, u_4$  }, every power group of comment be [0,100] and the membership function be  $u_i(x) = e^{-\frac{(x-a_i)^2}{b_i^2}}$ , where  $i = 1,2,3,4$  and the values of  $a_i$  and  $b_i$  are shown in the Table 2.

Table 2. The value of  $a_i$  and  $b_i$

i	ai	bi
1	80	10
2	60	20
2	40	30
4	0	50

To acquire comment vector of the variable in the index set, there are two available ways. Provided the quantization index, a mapping between quantization value and the comment can be established directly and the membership grade can be found according to the membership function; otherwise evaluate each comment remark according to the expert result.

As space is limited, each comment is evaluated directly according to index, so that the comment vector can be acquired.

$$\begin{aligned}
 R_A &= (0.1, 0.4, 0.4, 0.2) \\
 R_B &= (0.2, 0.5, 0.3, 0.1) \\
 R_C &= (0, 0.4, 0.5, 0.2) \\
 R_D &= (0.4, 0.3, 0.2, 0.2)
 \end{aligned}
 , \text{ evaluating matrix } R = \begin{bmatrix} 0.1 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.3 & 0.1 \\ 0 & 0.4 & 0.5 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.2 \end{bmatrix}$$

According to the recognition degree of the four indexes in evaluating the performance of the controller, the weight vector can be  $A = (0.1, 0.2, 0.2, 0.4)$ . The synthesis evaluating result  $S = A * R = (0.21, 0.34, 0.28, 0.16)$  can be derived from weighted average model  $M(\bullet, \oplus)$ . It can be known from the result that the performance of the controller can be “good”, when the weight vector A is selected.

### 5. The Simulation

As the model of the object shown in function(1), a nonlinearity and time-varying simulation object can be selected:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.3x_1 - (0.5 + 0.1m)x_2^3 + 0.2u(t - 0.3m) \\ y = x_1 \end{cases} \tag{5}$$

Where m is the random variable between [0,1]. The controller designed in this paper is used to control the object shown in formula (4), and the objective function of genetic algorithm is to select

IAE integral,  $J(IAE) = \int_0^{\infty} |e(t)| dt$ . The parameters of the basic fuzzy controller are shown as below:

$$K_e = 5.5, K_c = 11, K_u = 15, K_d = 6, K_I = 8, \beta = 0.5, k = -0.02$$

The parameter optimization process and system unit step response curve are shown in Figure 2 and Figure 3 respectively. In Figure 2, abscissa is evolutionary algebra and ordinate is logarithm of objective function value. In Figure 3, curve 1, curve 2 and curve 3 respectively represent the response of the system when the evolution algebra is 10, 30 and 50. When the evolution algebra is 50, the unit step response curve of the system has the characteristics of no overshoot and oscillation, small steady-state error, short rise time and so on. It is verified that the fuzzy controller designed in this paper can realize the control of a class of nonlinear time-varying plant.

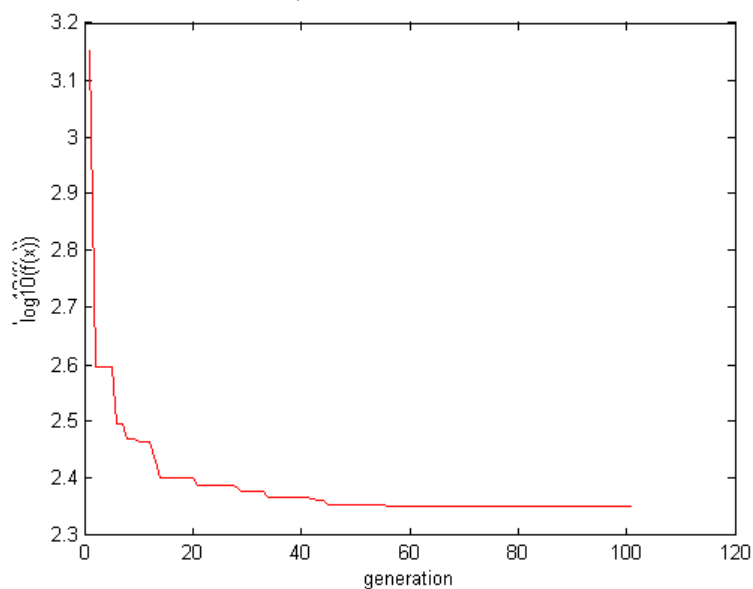


Fig.2 curve of objective function value in optimization process

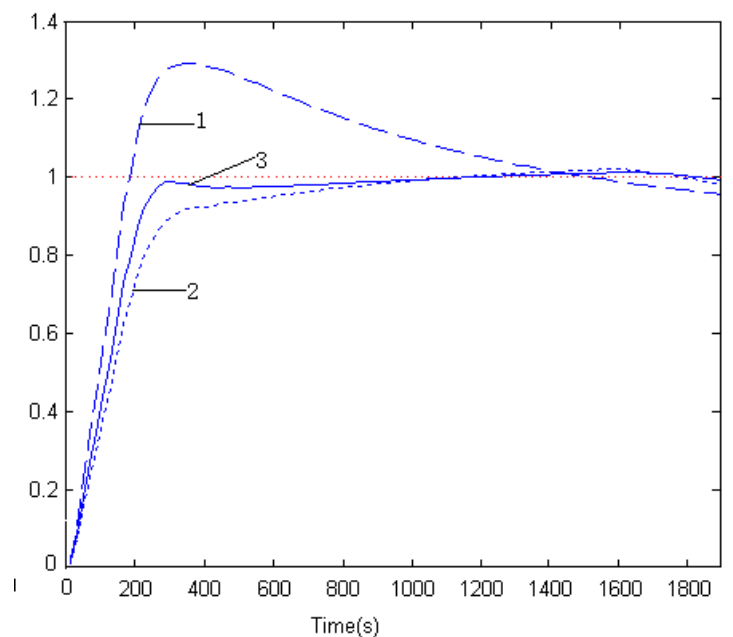


Fig.3 Unit step response curve of optimization process system

## 6. Conclusion

This paper puts forward a kind of self-organization fuzzy controller with parameter feedback for a kind of common nonlinear time varying system in industry and analyzes the feasibility of the fuzzy regulation reviser qualitatively as well as tries to analyze the system with function evaluation method, according to the theory of fuzzy system and the trend of the practice. During the process of designing a controller, we should follow the principle of simple, convenient and economy basically and on the premise of meeting the request of control, make full use of the mature experience of design in fuzzy controller, make the design and analysis method programmed and adopt new disposal skill to solve the specific problem. From the analysis in this article we can know that this kind of controller, possessing both the expertise of intelligence control and the skill of fuzzy control, has utilized the accurate knowledge learnt from the analysis of the object adequately and has drawn out the critical factors and use them as auxiliary measurement of to enhance the effect of control so that the serviceable range of fuzzy control can be expanded.

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