

# Integrating Optimization of Berth Allocation and Quay Crane Assignment Decision Under Uncertain Situation

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## Abstract

It is very important to jointly determine effective berth and quay crane allocation plan in order to make full use of the limited berth and quay crane resource and improve container terminal transportation efficiency. The uncertainty of ship's arrival time and the quantity of loading and unloading, however, will disrupt the execution of the berth and quay crane allocation plan and increase the cost. In order to minimize the total cost of time and position deviation, the optimal model of berth and quay crane allocation is proposed. The combination of SWO algorithm and PGA algorithm is presented to solve the optimization model, because SWO algorithm can effectively reduce the dimension of the solution and can avoid the PGA algorithm to converge to the local optimal solution. The correctness of the model and the validity of the algorithm are verified by numerical analysis.

## Keywords

Berth and Quay Crane allocation plan; SWO algorithm; Hybrid Heuristic algorithm.

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## 1. Introduction

There is a trend of large-scale container ships. Large-scale container ships must berth in deep water berths. The deep water berths of a container terminal are a limited and scarce resource. When the number of large container ships that need to be berth increases, it is required to formulate the best the berth plan is to reduce the waiting time of container ships; quay cranes are large-scale equipment deployed at the front of the terminal to load and unload containers for container ships. The reasonable configuration and scheduling of quay cranes can reduce the berthing time of container ships. Therefore, terminal operators should formulate efficient and feasible container ship berthing plans (abbreviated as berth plans) and quay crane deployment plans, and make full use of berths and quay cranes to improve the collection and distribution efficiency of container terminals. In actual operations, terminal operators formulate berth plans and quay crane deployment plans based on the container ship's arrival time learned in advance, minimizing the cost of loading and unloading and the shortest container ship's time in port. Berth plan refers to the allocation of berths and determination of berth time for container ships under the dual constraints of the shoreline space of the terminal and the arrival time of the container ships. The quay crane plan is to effectively allocate the quay crane after determining the berth and berthing time. The berth and quay crane joint plan optimization is to coordinate the berth and quay crane plan at the same time to achieve the overall optimization, thereby improving the port's logistics efficiency.

Most terminals simply rely on the timing of the expected arrival time of container ships to plan berths, and add some simple constraints on the basis of "first in, first out" to provide berths and quay crane allocation, facing the uncertainty of the arrival time of container ships There is no good treatment

strategy, which greatly reduces the efficiency of terminal berth allocation. When the arrival time of container ships is uncertain, a more systematic and reasonable berth quay crane joint plan optimization method is needed.

Park and Kim (2003) [1] first studied the joint planning of berths and quay cranes. The assignment tasks of quay cranes in the model are independent of each other, and the assignment tasks can be changed during the loading and unloading process of container ships. The quay crane productivity is directly proportional to the number of quay cranes serving container ships. These have been corrected and modified by Cordeau (2005) [2] and Hansen (2006) [3]. Because of the mutual restriction and influence between quay cranes, the marginal productivity of quay cranes will decrease. If the docking position of container ships deviates from the ideal position, the productivity of quay cranes will also decrease. Oguz et al. (2004) [4] studied the problem of static continuous berth quay crane allocation and regarded this problem as a parallel machine scheduling problem with the goal of minimizing the time to complete the loading and unloading tasks. This method is different from the method of Park and Kim (2003) [1]. It considers that the quay crane assignment task changes with time, and introduces an interference coefficient to solve the problem of the decline of the quay crane's marginal productivity.

Meisel and Bierwirth (2006) [5] discussed a dynamic berth allocation problem, which was regarded as a multi-resource limited project scheduling problem. The model was determined based on the principle of priority, including the berthing time of each ship and Berthing location. Xu (2011) [6] considered the dynamic continuous berth allocation problem under uncertain conditions and operating time. The optimization goal is the balance of service level delay and buffer time, and it is solved by simulated annealing and branch and bound method. Fang Peng (2011) [7] established a mathematical model through the study of the berth-quayside crane coordinated scheduling problem and solved it with genetic algorithm. Through the analysis of the regularity of incoming ships at the container terminal, the arrival of container ships at the container terminal in the next few years is predicted, and the number of quay cranes in the future terminal quay cranes should be configured when the berth and quay crane are coordinated. Lu (2012) [8] established a dual-objective model considering cost and model robustness, and used the SWO algorithm to solve large-scale cases. Meisel (2013) [9] first proposed a three-stage model for berth and quay crane allocation. The first stage calculates the loading and unloading efficiency of different numbers of quay cranes for each ship, and the second stage is based on the previous stage the calculated loading and unloading efficiency allocates berths, and the third stage determines the scheduling plan of each quay crane. The article uses SWO's heuristic algorithm to solve the berth allocation problem, which verifies the efficiency and correctness of the model. Gui Xiaoya (2013) [10] established an initial dispatch plan model under certain conditions and a basic dispatch plan model under uncertain conditions. The former aims at minimizing the total time of ships in port, and the latter further introduces buffer time. Decision variables, and put forward an agent objective function containing the expected total departure time and total buffer time to optimize the quality robustness and solution robustness of the basic scheduling plan. Finally, numerical simulation experiments prove the effectiveness and superiority of the algorithm, as well as the robustness of the basic scheduling plan obtained, and design parameter experiments to analyze the degree of uncertainty, the upper limit of the buffer time, and the threshold parameters in the heuristic adjustment strategy. Impact on scheduling decisions.

Lin Jiahong (2014) [11] In view of the shortcomings of the existing research, under the continuous berth allocation strategy, the berth service range of the quay crane is considered, and the berth and the quay crane are coordinated to make up for the lack of coordination in the existing research. Scheduling, or considering the lack of discrete berths, makes the scheduling system more realistic; for the proposed mixed integer programming model, a heuristic algorithm that integrates basic genetic algorithms, genetic adjustment algorithms and local optimization algorithms is also designed to solve the problem. Liang Chengji and Wu Yu (2015) [12] aimed at the joint scheduling problem of berths and quay cranes with random ship arrival time and loading and unloading operation time. They comprehensively considered the penalty time caused by ships deviating from the preferred berth

under continuous berths, and added a delay time method. Absorb the impact of uncertain factors. In order to reflect the robustness of the dispatch plan, the delay time is added to the objective function, and a mixed integer program is established that aims at minimizing the total time of the ship in port, the penalty time for deviation from the preferred berth, customer satisfaction and the sum of delay time. For the model, an improved genetic algorithm combining self-altering genetic algorithm and heuristic berthing is proposed to solve the model. Hua Jianhui (2015) [13] lists the dual objective function that minimizes the ship's stay time in the port, the berth operating cost and the quay crane movement cost when the ship performs loading and unloading services. Secondly, the article summarizes the related knowledge of genetic algorithm, introduces the principle and process of genetic algorithm, and discusses the specific method of multi-objective optimization problem using genetic algorithm.

Liu Huilian and Cao Jinxin (2016) [14] studied the joint robust scheduling problem of berth allocation and quay crane allocation in uncertain environments, and established the robust correspondence of the model on the basis of traditional berth and quay crane allocation models. The formula is transformed into a robust optimization model, and a branch and bound algorithm is designed according to the characteristics of the model. The experimental results show that the robustness of the solution can be controlled by adjusting the protection level parameters. Weng Lingtao (2016) [15] abstractly mapped the berth resources, quay crane resources and time resources involved in the ship scheduling problem into a three-dimensional spatial layout problem, and designed a scheduling scheme model based on the layout problem, and based on the large-scale wharf For ports with more berths, longer coastlines, and relatively more quay crane equipment, a priority-based scheduling model considering preferred locations is designed. Qiao Longliang (2016) [16] studied the online model of berth and quay crane joint dispatch for container terminals under unforeseen information and further predicted the online model of joint berth and quay crane dispatch for subsequent  $k \geq 2$  requests. Fan Zhiqiang (2016) [17] studied the continuous berth allocation problem under the dynamic arrival of ships, and constructed a new mixed integer nonlinear programming model by setting new variables and reprogramming time series and space series constraints. It reduces the number of nonlinear constraints and improves the solution efficiency of the branch and bound algorithm.

Zheng Hongxing et al. (2017)[18] considered the impact of tides and the reality of dynamic dispatch in quay crane operations, and set the goal of minimizing the sum of the quay crane operation costs and demurrage costs of all arriving ships during the planning period, and establish a hybrid Integer programming model, and then designed a genetic algorithm embedded with heuristic rules to solve it. Finally, the results of the calculation example give the specific quay crane corresponding to each ship at the exact time and the dynamic operation time window of each quay crane, and the effectiveness of the integrated scheme is verified by comparing with the separately optimized scheme. Ren Jie (2017) [19] studied the discrete berth and quay crane allocation problem, and the continuous berth and quay crane allocation problem, using CPLEX and genetic algorithm to solve the problem. The results show that the proposed genetic algorithm can The optimal solution of the built model is obtained, which provides an effective algorithm for solving large-scale calculation examples. Yang Jie (2017) [20] studied the optimal scheduling of berth quay cranes under discrete and continuous berth layouts, and then studied the optimal scheduling of berths and quay cranes under interference environments, and proposed a berth based on interference management theory. The quay crane optimal scheduling strategy. Xu Wandong et al. (2019) [21] In view of the resource and cost issues in the dispatch of container terminal berths and quay cranes, considering the impact of uncertain factors, the method of adding buffer time is adopted to establish berth and quay crane joint dispatch with the goal of the lowest compound cost. Optimize the model and design an improved genetic algorithm to solve the model.

Most of the existing researches have been carried out in a certain environment, and the research on the uncertainty of the ideal berth and the uncertainty of the arrival time is not deep enough. Therefore, this article mainly analyzes the berth and quay crane plan under the uncertain environment.

## 2. Integrating optimization model of berth allocation and quay crane assignment decision

### 2.1 Description of compliance

The symbols required for modeling and their meanings are described in Table 1.

**Table 1.** Basic symbols and descriptions

symbol	descriptions	symbol	descriptions	symbol	descriptions
N	Number of container ships during the plan period	$m_i$	Cargo handling capacity of container ship i	$EST_i$	The earliest arrival time of container ship i
V	Collection of container ships	$b_i^0$	Ideal berth for container ship i	$EFT_i$	Estimated departure time of container ship i
Q	The number of quay cranes that can be used at the same time	$r_{min_i}$	The minimum number of quay cranes allowed to work simultaneously for container ship i	$LFT_i$	Container ship i penalize departure time
T	Planning cycle (h)	$r_{max_i}$	The maximum number of quay cranes allowed to work simultaneously for container ship i	$c_1^i$	Container ship i accelerated cost factor
L	Berth length	$R_i$	Container ship i can use bridge crane range	$c_2^i$	Container ship i delay cost coefficient
$l_i$	Length of container ship i (including safety distance)	$ETA_i$	Estimated arrival time of container ship i	$c_3^i$	Container ship i delay penalty cost

This model mainly uses container ship berthing time, berthing location and the number of cranes in each time period as decision variables. The used decision variables and their meanings are shown in Table 2.

**Table 2.** Decision variables and their meanings.

variables	meanings	variables	meanings	variables	meanings
$b_i$	Berthing position of container ship i	$r_{it}$	Integer variable, representing the number of cranes allocated by container ship i at time t	$y_{ij}$	0-1 variable, if the berthing time of container ship i is 1 in front of ship j, otherwise it is 0
$s_i$	Berthing time of container ship i	$r_{itq}$	0-1 variable, if the number of cranes allocated by container ship i at time t is q, it is 1, otherwise it is 0	$u_i$	0-1 variable, if the departure time of container ship i is greater than the penalty departure time, it is recorded as 1, otherwise it is 0
$e_i$	Departure time of container ship i	$z_{ij}$	0-1 variable, if the docking time of container ship i is 1 in front of ship j, otherwise it is 0		

### 2.2 Optimization model

This section presents the integrating optimization model of berth allocation and quay crane assignment decision under uncertain situation.

$$\text{Min } \sum_{i \in V_1} (c_1^1 * f(x_i, \varepsilon_i) + c_1^2 * \Delta EFT_i + c_1^3 * u_i + c_1^4 * \sum_{t \in T} \sum_{q \in R_i} q * r_{itq}) \quad (1)$$

$$\sum_{q \in R_i} q * r_{itq} = r_{it}, \forall i \in V, \forall t \in T \quad (2)$$

$$\sum_{t \in T} \sum_{q \in R_i} r_{it} \geq (1 + \beta * (\Delta b_i^+ + \Delta b_i^-) + \gamma * \sum_{t \in T} (\Delta r_{it}^+ + \Delta r_{it}^-)) * m_i, \forall i \in V \quad (3)$$

$$\sum_{i \in V} r_{it} \leq Q, \forall t \in T \quad (4)$$

$$\sum_{q \in R_i \cup \{0\}} r_{itq} = 1, \forall i \in V, \forall t \in T \quad (5)$$

$$\sum_{t \in T} \sum_{q \in R_i} r_{itq} = e_i - s_i, \forall i \in V \quad (6)$$

$$(t + 1) \sum_{q \in R_i} r_{itq} \leq e_i, \forall i \in V, \forall t \in T \quad (7)$$

$$t * \sum_{q \in R_i} r_{itq} + H * (1 - \sum_{q \in R_i} r_{itq}) \geq s_i, \forall i \in V, \forall t \in T \quad (8)$$

$$f(x_i, \varepsilon_i) = \max\{x_i + \varepsilon_i - ETA_i, 0\}, \forall i \in V_1 \quad (9)$$

$$\Delta ETA_i \geq ETA_i - s_i, \forall i \in V_2 \quad (10)$$

$$\Delta r_{it}^+ - \Delta r_{it}^- \geq r_{it} - r_{it-1}, \forall i \in V, \forall t \in T \quad (11)$$

$$\Delta ETA_i^+ \geq ETA_i - s_i, \forall i \in V \quad (12)$$

$$\Delta EFT_i^- \geq e_i - EFT_i, \forall i \in V \quad (13)$$

$$M * u_i \geq e_i - LFT_i, \forall i \in V \quad (14)$$

$$b_j + M * (1 - y_{ij}) \geq b_i + l_i, \forall i, j \in V, i \neq j \quad (15)$$

$$s_j + M * (1 - z_{ij}) \geq e_i, \forall i, j \in V, i \neq j \quad (16)$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1, \forall i, j \in V, i \neq j \quad (17)$$

$$s_i \geq EST_i, \forall i \in V \quad (18)$$

$$\Delta ETA_i, \Delta EFT_i \geq 0, \forall i \in V \quad (19)$$

$$r_{itq}, u_i, y_{ij}, z_{ij} \in \{0, 1\}, \forall i, j \in V \forall t \in T \forall q \in R_i \quad (20)$$

$$e_i, b_i \in N^+, \forall i \in V \quad (21)$$

$$\Delta b_i^+, \Delta b_i^- \in N^+, \forall i \in V \quad (22)$$

$$r_{it} \in N^+ \cup \{0\}, \forall i \in V, \forall t \in T \quad (23)$$

$$r_{i0} = 0, \forall i \in V \quad (24)$$

The time cost in the objective function (1) is mainly divided into three parts. The first part is the time acceleration cost. Container ships can arrive at the terminal earlier than the expected arrival time, but they need to pay a certain cost. This is the time acceleration cost, but this time It cannot be advanced indefinitely and cannot exceed the earliest berthing time agreed upon by the container ship. The second part is the cost of delay time, which refers to the cost incurred by the delay of the actual departure time of the container ship compared to the expected departure time. The third part is the penalty cost, that is, the actual departure time of the container ship is later than the agreed penalty departure time, which will incur additional time penalty cost, which has nothing to do with the length of time. Where  $V_1$  is the set of ships whose arrival time is uncertain,  $V_2$  is the set of ships whose arrival time is determined, and  $\varepsilon_i$  is a random variable.  $f(x_i, \varepsilon_i)$  is the accelerated entry time of the container ship  $i$ , and  $E[f(x_i, \varepsilon_i)]$  is the expected value of the acceleration time of the container ship  $i$ .

The goal of objective function optimization is to minimize the time cost. Therefore, we use constraint (3) to transform the berth deviation cost and the cost of increasing or decreasing the number of quay cranes into time cost. For the berth deviation part, suppose a ship  $m=10$ , that is, there is a loading and unloading task of 10 time units. If the actual berth position of the ship deviates from the ideal berth by 10 units and  $\beta$  is 0.1, then the actual cost of  $10 * (1 + 0.1 * 10)$ , which is 20 time units. This setting can basically simulate the situation of longer loading and unloading time due to deviation from the ideal berth. Although the actual situation may be more complicated, it basically meets the actual needs. This constraint also considers the loss caused by the change of the number of cranes, that is, if there

are 2 cranes working at time  $t$  and 3 cranes working at time  $t+1$ , then the corresponding work efficiency loss is  $\gamma^*(3-2)=0.2$  (where  $\gamma=0.2$ ), constraint (3) can calculate the actual working time of the crane. (4)-(8) are time constraints, which ensure the continuous workability of the bridge crane and ensure that its working time is only after the container ship berths and before it leaves the berth. Constraints (15)-(17) jointly ensure non-overlap between container ships. Constraints (9)-(13) enable the objective function to calculate the berth deviation cost, bridge crane difference cost, acceleration time cost, delayed departure cost, and penalty time cost. (18)-(24) stipulate the basic decision variables range.

### 3. Model validation

This section verifies the rationality and feasibility of the model, using the optimization solution software ILOG CPLEX to solve the model, and the random part contained in the model is set as the mean value of the random variable.

#### 3.1 Model parameter settings

In addition to the basic constraints of the model, the solution also needs to pay attention to the actual workload of the crane (ie the actual cargo loading and unloading volume). This value is the extra caused by the planned loading and unloading of the container ship, the deviation of the berth, and the change of the number of cranes. Bridge crane workload composition. Berth deviation means that when a ship's actual berthing position deviates from the expected berth position, it takes more time to complete the loading and unloading task. In the constraint, we convert this part of the newly added loading and unloading time into a berth deviation coefficient. Additional crane workload; for the same reason, changing the number of bridge cranes will increase the loss of loading and unloading time, so additional crane workload will be generated.

The relevant parameters of the model include variable parameters such as container ship arrival, shoreline length, cycle length, penalty coefficient, expected arrival and departure time, earliest arrival time, penalty departure time, cargo handling volume, etc., to get the berthing position of the container ship in the cycle, The specific parameters of arrival time, departure time, and the number of cranes used are as follows, the cycle time ( $T$ ) is 20, the berth length ( $L$ ) is 260 meters, and the total number of quay cranes ( $Q$ ) is 5. , The berth deviation coefficient ( $\beta$ ) is 0.01, the loss coefficient ( $\gamma$ ) of the number of quay cranes is 0.2, and the values of other parameters are shown in Table 3.

**Table 3.** Parameter settings of the calculation example

Container Ship	Ideal berthing position( $b_i^0$ )	Container ship length( $l_i$ )	Earliest arrival time( $EST_i$ )	Estimated arrival time( $ETA_i$ )	estimated departure time( $EFT_i$ )
0	1	6	1	1	4
1	7	4	1	3	5
2	11	5	5	9	11
3	3	3	11	13	15
4	16	7	14	15	17
5	20	4	14	16	19
6	18	6	1	2	5
Container Ship	Penalize time away( $LFT_i$ )	Accelerated cost factor( $c_1^1$ )	Delay cost factor( $c_2^1$ )	Delay penalty cost( $c_3^1$ )	Estimated loading and unloading volume( $m_i$ )
0	5	1	1	3	8
1	7	1	1	3	5
2	11	1	1	3	3
3	16	2	2	6	6
4	19	1	1	3	2
5	20	2	2	6	6
6	6	2	2	6	3

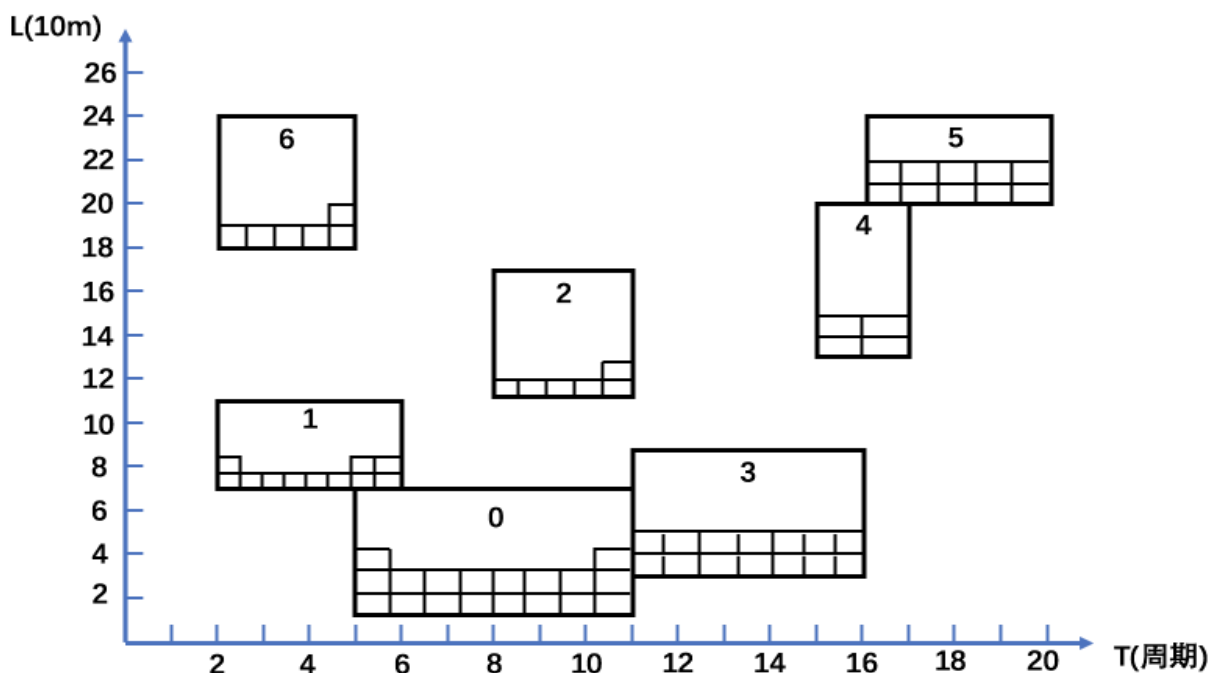
In one cycle, a total of 7 container ships are expected to operate at the port. The parameters that need to be input are in line with the actual situation of the container ship's entry and exit plan, including the container ship's name, planned berth, planned entry and exit time, and a quantitative method to represent the sensitivity of container ships to time. Because of the high time requirements, the corresponding three time coefficients will be larger for container ships with tighter voyage times, on the contrary, it is smaller. The estimated arrival time of all container ships takes the mean of the random arrival time.

### 3.2 Result analysis of calculation example

**Table 4.** Run results of the calculation example

Container Ship	Berthing time( $s_i$ )	Departure time ( $e_i$ )	Actual berthing location ( $b_i$ )	Actual loading and unloading volume
0	5	11	1	18
1	2	6	7	11
2	8	11	11	6
3	11	16	3	14
4	15	17	13	4
5	16	20	20	8
6	2	5	18	6

In this calculation example, the container ship's arrival and departure time is the mean of a random variable that obeys a uniform distribution, which is used to verify whether the model's operating results under deterministic conditions can meet the actual situation. As shown in Table 4 and Figure 1, the operating results are in line with expectations, thus verifying the correctness and feasibility of the model. In the same time period, there is no overlap between the berths of container ships, and the total number of quay cranes used does not exceed five. The actual loading and unloading volume has increased due to the deviation of berths and the increase or decrease in the number of quay cranes, and the arrival time of container ships is no earlier than earliest arrival time.



**Figure 1.** Plans of berth allocation and quay crane assignment

The results of the model solution show that the optimal actual arrival and departure time of container ship No. 0 deviates greatly from the plan, and even time penalty costs are incurred. Because at time 1-6 of the cycle, except for the container ship, container ship No. 1 and container ship No. 6 have also berthed one after another, but the total number of quay cranes that can be used at the same time is limited, which cannot meet the loading and unloading tasks of three container ships at the same time. Therefore, after comparing the costs, the model derives the optimal solution under the goal of the least cost, the arrival and departure times of other container ships are more in line with the planned time, the actual berth is basically the ideal berth position. However, at 16 o'clock, both the No. 4 and No. 5 ships have loading and unloading tasks at the berth 20-23m, so the berthing position of the container ship 4 is slightly changed; and the substantial increase in the actual loading and unloading volume is mainly due to the change in the number of loading and unloading bridges. To better fit the actual situation of the terminal, the size of the  $\gamma$  parameter can be modified appropriately.

#### 4. Heuristic algorithm solution based on SWO and PGA

Since the berth and quay crane joint plan is an NP-Hard problem, when the model scale becomes larger, it cannot be solved directly by the optimization software Cplex. In order to improve the accuracy and feasibility of the algorithm, this paper adopts a hybrid heuristic algorithm combining the Small World Optimization (SWO) algorithm and the parallel genetic algorithm (PGA). Using the SWO algorithm can effectively reduce the dimensionality of the solution, while avoiding the PGA algorithm from converging to the local optimal solution.

Clements (1997)<sup>[22]</sup> uses the SWO algorithm to consider the importance of each element in the feasible solution. The SWO algorithm requires a priority table containing all elements. Combining this priority table, you can find a feasible solution in the solution space; Then according to the above feasible solution, the importance of each element can be recalculated to generate a new priority table. Therefore, using the SWO algorithm, you can switch back and forth between the priority table and the solution domain, that is, you can influence the solution in the solution space according to the priority table, and then use the feasible solution to adjust the priority table. This two-way adjustment can greatly accelerate the convergence rate of the solution, and obtain some optimal solutions that are not easily obtained by other methods. The SWO algorithm was introduced by Meisel (2009)<sup>[23]</sup> to solve the problem of container ship berth planning, mainly because this algorithm can well adjust the solution globally according to the importance of various factors, and can effectively expand the solution space. And designed a set of heuristic algorithms combined with the SWO algorithm, in its model by comparing with Park (2003)<sup>[1]</sup>, has proved the effectiveness of the algorithm.

This section uses the small-scale case in Table 5 to illustrate the algorithm we designed. In this case, the total length of the berth is 140m. For the convenience of calculation, each 10m is set as a unit. The entire quay crane planning cycle is 10 hours. There are three ships in total. The first ship is 30m in length, and the ideal berth is 70m. It takes 6 crane working hours to complete the loading and unloading tasks. The fastest arrival time is the second hour, and the estimated berth time is the third Hours, the estimated departure time is the 5th hour, and the latest departure time is the 8th hour. The minimum and maximum number of bridge cranes allowed to work at the same time is 1 and 3, and the corresponding target penalty coefficient is 1,1,2. The corresponding data of the second and third ships are shown in Table 5.

**Table 5.** Small-scale case description

I	L	$b_0$	M	ETA	EST	EFT	LFT	Rmin	Rmax	$c_1$	$c_2$	$c_3$
0	3	7	6	3	2	5	8	1	3	1	1	2
1	4	1	5	6	2	9	10	1	2	2	2	4
2	5	6	8	4	1	6	7	1	3	3	3	6



### 4.1 The parallel genetic algorithm (PGA)

#### 4.1.1 The composition of genes

For genetic algorithms, the genes that make up its solution are one of the most important parts of the entire algorithm. In this article, the composition of genes is divided into four parts, as shown in Figure 2. The first part combines the SWO algorithm to represent the priority of the container ship. 012 indicates that the first ship has the highest priority, followed by the second ship, and finally the third ship. The priority of a container ship will affect the quality of its corresponding solution. A container ship with a lower priority will be allocated a corresponding berth and quay crane only when the container ship with a higher priority finds a corresponding solution. The second part of the gene corresponds to the berthing time of the container ship, the third part is the berth of the container ship, and the last part is the largest number of allocable bridge cranes for the container ship.



Figure 2. The composition of genes

The above-mentioned gene sequence is translated into the following container ship plan:

Berths are allocated from the first ship, and the first ship is berthed at position 2 from time 3. Considering the cost of berth deviation, the actual amount of cargo to be loaded and unloaded is  $m_0 = (1 + (7 - 2) * \beta) * m_0 = 9$ . Then in order to complete the loading and unloading task, considering that the maximum number of cranes allocated is 2, then at least 5 hours ( $0.9^2 * 5 > 9$ ) are required. In this way, the allocation of berths and cranes for the first ship is completed. Then start the allocation of the second container ship. The second ship is moored at position 5 from time 5. After considering the deviation cost, the amount of loading and unloading tasks is  $m_1 = (1 + (5 - 1) * \beta) * m_1 = 7$ , combined with the limitation of the maximum number of cranes of 2, we need 4 hours to complete ( $0.9^2 * 4 > 7$ ). Finally, the third ship is allocated. The third ship is docked at position 9 at time point 3. The amount of tasks to be completed is  $m_2 = (1 + (9 - 6) * \beta) * m_2 = 10.4$ , The maximum number of cranes for ship 3 is 3, but considering that the total number of cranes that can be allocated is 5, there is only 1 crane that can be allocated to ship 2 at the time point 5, 6, and 7, so it takes 6 hours in total to complete ( $0.9^3 * 3 + 1 * 3$ ). Then calculate the corresponding total target value according to the allocation of container ship berths and cranes.

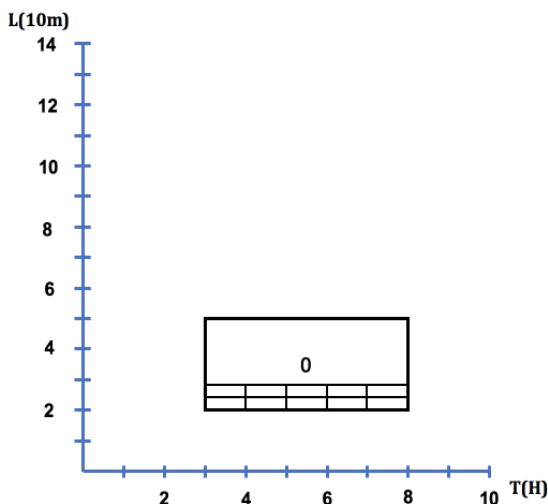


Figure 3. Gene translation 1

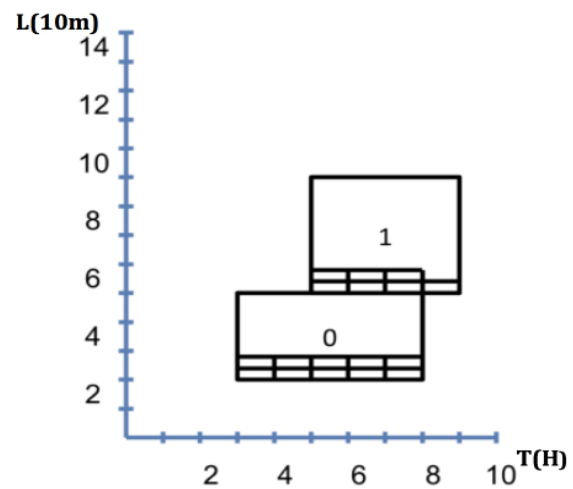


Figure 4. Gene translation 2

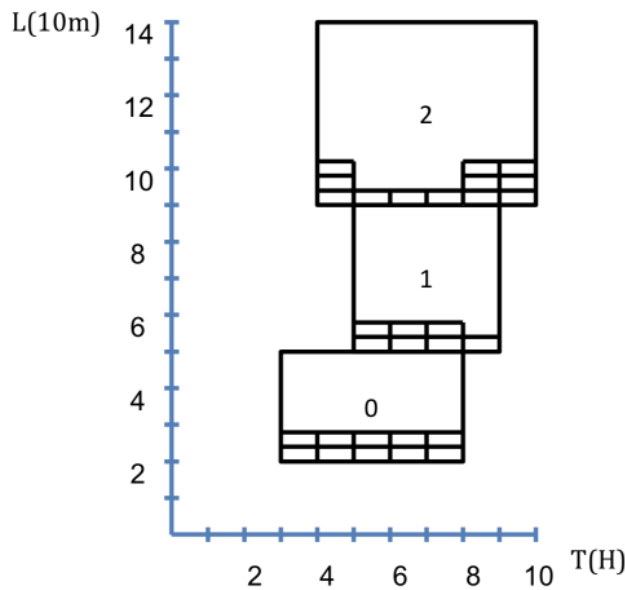


Figure 5. Gene translation 3

4.1.2 Generate initial solution

- (1) Start generating a new gene, the first N genes are 0~N, set t=1.
- (2) If t is less than or equal to N, go to step (3), otherwise jump out of the loop.
- (3) Set k=0; assign the t-th ship to the optimal berth, estimate the arrival time and randomly assign a feasible number of bridge cranes, calculate its departure time, and enter step (4).
- (4) If there is a conflict with the previous container ship, go to step (5), otherwise skip to step (7).
- (5) The estimated time of entry and the number of feasible bridge cranes remain unchanged. Find the berth with the best feasible distance and the smallest berth deviation. If it actually exists, go to step (7). Otherwise, if  $k > 10$ , the gene is invalid, you jump out of the loop; if  $k < 10$ , go to step (6).
- (6) The number of feasible bridge cranes remains unchanged, and a time is randomly generated within the feasible container ship entry time range, which is the entry time of the container ship, the berth is still the optimal berth. Calculate the departure time,  $k++$ , enter Step (4).
- (7)  $t++$ , skip back to step (2).

4.1.3 Gene crossover

This article considers that the first part of genes represents priority, so single point crossover is used. Take the following gene as an example:



Figure 6. Genes before crossover

These are two parent genes. Two child genes need to be generated. First, a random number  $a$  need to be generated between 1 and N. Then the data of the previous ship  $a$  remains unchanged, and exchange the values of the next ship to generate two subtype genes.

As shown in Figure 6, the random number  $a$  is 1, so each gene retains the data of the first ship and exchanges the data of the next two ships.



Figure 7. Genes after crossover ( $a=1$ )

#### 4.1.4 Gene selection

The gene selection of the genetic algorithm is responsible for ensuring that the parent's gene can be inherited to the offspring according to a certain selection rule. Excellent selection operators can effectively retain excellent parent genes, accelerate the convergence speed of the algorithm and prevent premature convergence.

Operators mainly include roulette selection, sort selection and random selection. Among them, random selection is mainly to randomly select two genes from the parent gene, compare the two genes with each other, and select the solution with excellent fitness to enter the offspring. This method not only guarantees the excellence of offspring genes, but also brings diversity to offspring genes. This article uses this method.

Step1: Calculate the objective function value of each gene, and set  $n=1$ .

Step2: Judge whether  $n$  is greater than popsize, if it is greater than popsize, jump out of the loop, otherwise go to step3

Step3: Randomly select two genes, compare their objective function values, select the gene with the smaller function value to retain to the next generation,  $n++$ , transfer to step2.

#### 4.1.5 Genetic variation

Choose uniform variation, where the time variation range of container ship I is determined by EST, the variation range of arrival position is  $[0, L-1]$ , and the maximum number of working bridge cranes is  $[rmin, rmax]$ .

Step1: Initialize the data, set  $n=1, m=1$ .

Step2: Determine whether  $n$  is greater than popsize, if yes, jump to Step3, otherwise jump out of the loop.

Step3: Determine whether  $m$  is greater than  $N$ , if yes, jump to Step4, otherwise reset  $m=1, n++$ , jump to Step2.

Step4: Generate a 0~1 random number  $a$ ,  $m++$ , if  $a < P_m$ , go to Step5, otherwise go to Step3.

Step5: Generate a random number  $p$  of 0~1 to mutate according to the following formula,  $m++$ , and then jump to Step3.

$$\begin{cases} k + (k_{max} - k) * \lambda^{(1 - \frac{g}{g_{max}})^*b} & p > 0.5 \\ k - (k - k_{min}) * \lambda^{(1 - \frac{g}{g_{max}})^*b} & p \leq 0.5 \end{cases}$$

Where  $k$  is the gene value that needs to be mutated,  $k_{max}$  is the upper limit of  $k$ ,  $k_{min}$  is the lower limit of  $k$ ,  $g$  is the current gene algebra,  $g_{max}$  is the maximum algebra,  $\lambda \in [0,1]$  and  $b$  are both fixed, responsible for limiting the rate of mutation. In this article, set  $b$  to 0.5.

4.1.6 Gene repair

For the following genes, it is found that the first ship and the second ship will overlap, although this does not meet the constraint conditions, there may be the above-mentioned cross-mutation or appear directly in the initial solution, which is an unavoidable need Use a certain algorithm to transform it into a feasible solution.

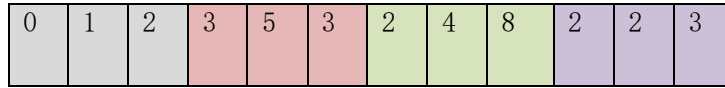


Figure 8. Gene repair

Adjust according to the following algorithm to ensure the feasibility of its solution.

Step1: Initialize the data, set  $n=1$ , jump to step2.

Step2: Judge whether  $n$  is greater than  $N$ , if it is greater, end the loop, otherwise set  $t=1$ , jump to step3.

Step3: Judge whether  $t$  is equal to  $n$ , if yes,  $n++$ , jump to step2, otherwise jump to step4.

Step4: Determine whether the  $n$ th ship and the  $t$ th ship overlap, if they overlap, go to step5, otherwise  $t++$ , jump to step3.

Step5: Generate a random number  $p$  of  $0\sim 1$ , if  $p$  is greater than  $0.5$ , adjust the arrival time, otherwise adjust the arrival berth, return to step3.

In order to prevent the program from falling into an infinite loop after finding all the feasible positions of the container ship  $n$  and still having overlapping parts with other container ships, it is necessary to set  $K$  as the upper limit of the number of times to find a feasible position. If it exceeds  $K$ , jump out of the loop and mark this gene cannot be retained in the next generation of genes.

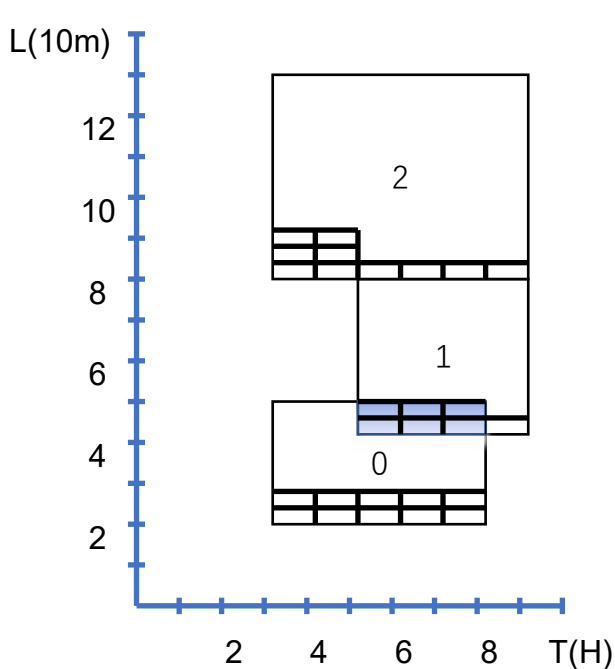


Figure 9. Gene repair 1

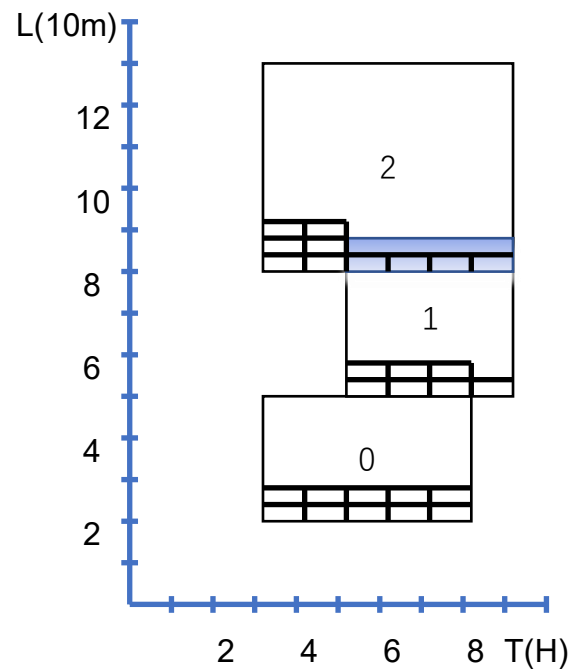


Figure 10. Gene repair 2

Take this gene as an example to show the gene repair algorithm. First, it finds a positional conflict between the second ship and the first ship, then randomly moves its berth position up by one position. The first and second ships no longer collide in position and time, but the third and second ships collide again. Then randomly move its berthing time to the right by one position, get gene repair 3, and find

that there is still conflict with Ship 2. Then move its berthing position up by one unit to get gene repair 4, which is a feasible solution, and the algorithm ends.

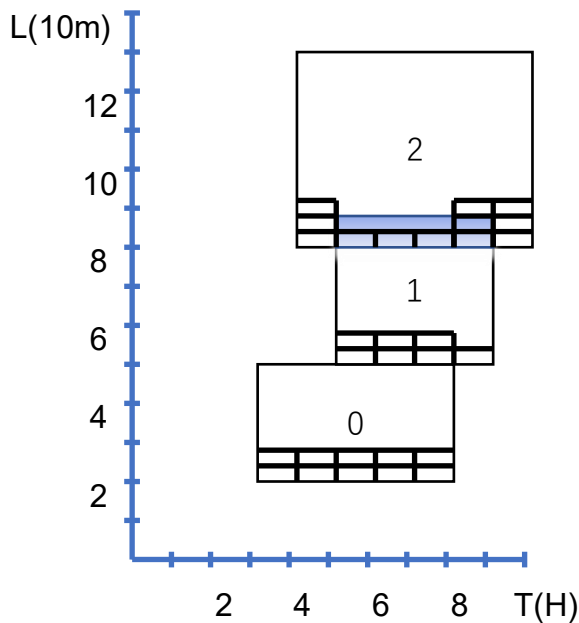


Figure 11. Gene repair 3

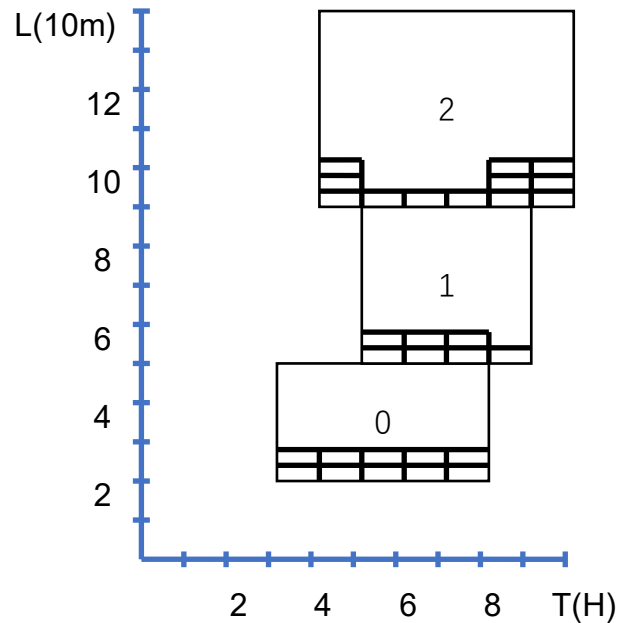


Figure 12. Gene repair 4

#### 4.2 Hybrid heuristic algorithm combined with SWO algorithm

For berth allocation, the quality of each factor's solution is mainly reflected in the last corresponding time cost and bridge crane operation cost of each container ship, but because the acceleration time cost corresponding to container ships with uncertain arrival times cannot be calculated independently, so this cost cannot be included. The priority table is calculated by calculating the time cost of each ship according to the objective function and then sorting from largest to smallest. The container ships at the top of the list can be considered to be very important to the quality of the final solution. We should allocate berths and crane resources to them first to reduce their time costs, then formulate new priorities based on the new plan, and keep looping. In this article, the number of cycles is set to 10.

Combining SWO algorithm and PGA algorithm, the overall algorithm is as follows:

Step1: Initialize the data, remember  $g=1, n=1$ .

Step2: Judge whether  $n$  is less than  $MaxN$ , if yes, jump out of the loop, otherwise jump to Step3.

Step3: Determine if  $g$  is greater than  $MaxGen$ , if it is  $n++$ , jump out of the loop, otherwise go to Step4.

Step4: Determine whether the berthing time corresponding to each ship has the expected target value, if there is, adjust to Step6, otherwise go to Step5.

Step5: Generate the expected target value corresponding to the berthing time through random simulation, and add this mapping to the total mapping, go to Step6.

Step6: Modify the gene according to the target function to formulate the priority table and transfer to Step7.

Step7: Cross, select and mutate genes,  $g++$ , jump to Step3.

### 5. Case analysis

Combining experience and historical data, the relevant data of the three ship types (Feeder, Medium, and Jumbo) generated in this paper are shown in Table 6.

**Table 6.** Relevant data of the three ship types

Ship class	length (10m)	Loading and unloading	Minimum crane	maximum crane	Accelerated cost ( $c_1$ )	Delay cost ( $c_2$ )	Delay penalty cost ( $c_3$ )	Uncertainty factor
Feeder	U[8,21]	U[5,15]	1	2	1	1	3	0.4
Medium	U[21,30]	U[15,50]	2	4	2	2	6	0.2
Jumbo	U[30,40]	U[50,65]	4	6	3	3	9	0

U [a, b] represents the random distribution on the interval [a, b] and the result is rounded for convenience. The length and loading and unloading data of container ships are from ISL (2003). Taking into account the randomness of the algorithm, a total of 20 ships were generated with ten cases in each group. In each case, the initial data was generated according to the ship class of 60% Feeder, 30% Medium and 10% Jumbo. Set the planning period as one week (168H), and the earliest arrival time of the container ship can be up to 10% earlier, which is  $[0.9*ETA]$ . The estimated departure time of each ship is the shortest loading and unloading time plus the berthing time, the penalty departure time is 1.5 times the shortest loading and unloading time plus the berthing time, other fixed data include that the shoreline length of the wharf is 1000m ( $L=100$ ), the maximum number of cranes is 10, and the cost of using bridge cranes  $c_i^4$  is 0.1. Based on the actual situation,  $\alpha$  is set to 0.9, the berth deviation coefficient  $\beta$  is 0.01, which means that each unit of deviation from the berth will incur 1% additional loading and unloading costs, which is also very consistent with the reality.

This article uses a case with a scale of 20 ships. Table 7. shows the detailed information of each ship's arrival at the port. N (9,1) is a normal distribution, so the arrival time of 5 ships is random. Ship types 1, 2, and 3 respectively represent three ship classes: Feeder, Medium and Jumbo.

**Table 7.** Specific data of 20 container ships in the case

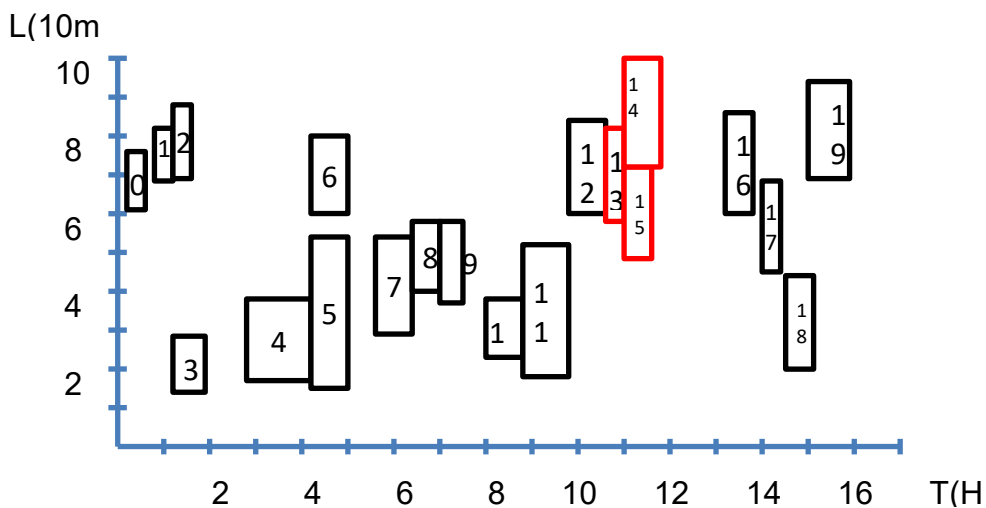
Container Ship	length (10m)	Ideal berth (10m)	Earliest arrival time (H)	Estimated time of arrival (H)	estimated departure time (H)	Latest departure time (H)	Loading and unloading	Ship type
0	15	60	3	3	7	8	6	1
1	8	75	8	9	13	15	7	1
2	19	69	8	N (9,1)	13	14	6	1
3	10	2	12	13	20	23	13	1
4	21	17	26	29	44	51	50	2
5	39	14	37	N (41,2)	53	58	57	3
6	20	59	39	43	51	54	14	1
7	25	31	52	57	66	69	28	2
8	18	42	58	64	69	70	8	1
9	21	39	62	N (69,2)	74	75	15	2
10	15	21	78	86	89	90	5	1
11	34	16	80	88	99	104	53	3
12	14	66	87	96	103	106	12	1
13	10	73	92	N (102,2)	107	109	9	1
14	19	74	98	108	115	118	13	1
15	15	79	99	N (110,3)	119	122	15	1
16	26	55	117	130	136	139	20	2
17	19	43	124	137	143	145	10	1
18	24	23	127	141	147	150	20	2
19	25	73	135	150	160	165	34	2

Next, use the designed algorithm to solve the problem in two cases. The first is under deterministic conditions. For container ships with random arrival time, the average value is taken as the arrival time. The second is to solve the model under uncertain conditions.

**Table 8.** Calculation results

	GA+SWO	GA+SWO (s)	GA+SWO (2)	GA+SWO (2) (s)
10 (1)	--	4.7	69.57	7.7
10 (2)	61.5	10.2	59.97	10.2
10 (3)	53.0	5.4	56.79	8.2
10 (4)	28.2	3.2	30.17	5.2
10 (5)	53.5	6.4	66.95	9.7
10 (6)	58.5	3.7	59.69	6.1
10 (7)	39.5	5.7	73.57	8.0
10 (8)	33.2	3.3	41.36	5.7
10 (9)	62.1	5.1	57.65	7.6
10 (10)	26.1	1.9	29.64	4.9
20 (1)	65.5	25.2	76.54	29.6
20 (2)	117.2	28.5	120.68	34.2
20 (3)	97.5	26.1	100.33	30.8
20 (4)	122.8	27.6	129.56	31.9
20 (5)	104.5	32.8	110.21	37.3
20 (6)	102.3	29.9	107.65	34.7
20 (7)	137.1	28.4	141.25	34.7
20 (8)	69.0	25.1	71.36	28.6
20 (9)	82.6	23.6	86.54	27.6
20 (10)	90.1	26.7	102.25	31.5

The second and third columns of Table 8 are the target value and solution time obtained by using the algorithm to solve the model in a certain environment, and the fourth and fifth columns are the target value and solution time obtained by using the algorithm to solve the model in an uncertain environment. Comparing the solution results, it can be seen that the calculation result under the uncertain environment is slightly larger than that under the certain condition. Because in an uncertain environment, even if these container ships arrive at the estimated time of arrival, they will still incur a certain time cost, which will make the objective function larger. In terms of algorithm solving, because it takes a certain amount of time in random simulation, the solution time is longer than in a deterministic environment, but from the above table can be seen that the increased solution time is basically within an acceptable range. Therefore, it is proved that the algorithm designed in this paper can effectively solve the integrating planning optimization problem.



**Figure 12.** Berth allocation plan for 20 ships

The main difference between the random and deterministic models of container ship arrival time lies in the berth allocation of 13,14,15 (Figure 12 is the berth allocation plan in a random environment).

## 6. Case analysis

In the environment of uncertain container ship arrival time, this paper constructs an integrating optimization model for solving the minimum expected cost of the berth allocation and quay crane assignment decision. First, under the deterministic scenario where the arrival time is the mean value of the random variable, a feasible solution is obtained through Cplex software, which verifies the effectiveness of the model. Subsequently, combined with the parallel genetic algorithm of the SWO algorithm, an algorithm for solving the optimization model was constructed, and the effectiveness of the algorithm was verified through multiple calculation examples. Finally, we obtained a feasible solution for the optimization model.

## References

- [1] Park. Y M, Kim. K H. A scheduling method for berth and quay cranes. *OR Spectrum* 25 (2003), 1–23.
- [2] Cordeau. J F, Laporte. G, Legato. P, Moccia. L. Models and Tabu Search heuristics for the berth-allocation problem. *Transportation Science*. 2005, 39 (4),526–538.
- [3] Hansen. P, Oguz. C, Mladenovic. N, in press. Variable neighborhood search for minimum cost berth allocation. *European Journal of Operational Research*.
- [4] Oguz. C, Błazewicz. J, Cheng. T C E, Machowiak. M. Berth allocation as a moldable task scheduling problem. In: *Proceedings of Ninth International Workshop on Project Management and Scheduling*, 2004, pp. 201–205.
- [5] Meisel. F, Bierwirth. C. Integration of berth allocation and crane assignment to improve the resource utilization at a seaport container terminal. In: Haasis, H.-D., Kopfer, H., Schonberger, J. (Eds.), *Operations Research Proceedings 2005*. Springer, Berlin, 2006, pp. 105–110.
- [6] Xu. Y, Chen. Q, Quan. X. Robust berth scheduling with uncertain vessel delay and handling time. *Ann Oper Res* DOI 10.1007/s10479-010-0820-0, 2011.
- [7] Fang. P. Research on the quay crane configuration based on the coordinated scheduling of berths and quay cranes[D]. Dalian Maritime University, 2011.
- [8] Zhen. L, Chang. D F. A bi-objective model for robust berth allocation scheduling[J]. *Computers & Industrial Engineering*, 2012,63(10):262-273.
- [9] Frank M, Christian B. A framework for integrated berth allocation and crane operations planning in seaport container terminals. 2013,47(2):131-147.
- [10] Gui X. Modeling and optimization of integrated dispatching plan for container terminal berths and quay cranes under uncertainty[D]. Shanghai Jiaotong University, 2013.
- [11] Lin J. Research on coordinated scheduling of berths and quay cranes in uncertain environments [D]. Tsinghua University, 2014.
- [12] Liang C, Wu Y. Joint dispatch of container terminal berths and quay cranes under uncertain environment [J]. *Computer Engineering and Applications*, 2015, 53(07): 212-219
- [13] Hua J. Joint dispatch of container terminal berths and quay cranes [D]. Tianjin University of Technology, 2015.
- [14] Liu. H, Cao J X. Uncertain container terminal berths and quay crane allocation problem[J]. *Journal of Inner Mongolia University (Natural Science Edition)*, 2016, 47(03): 312-320.
- [15] Weng. L T. Research on port berth allocation and quay crane dispatching[D]. Hebei University of Technology, 2016.
- [16] Qiao. L L. Research on the Joint Online Dispatch of Container Terminal Berths and Quay Cranes[D]. Donghua University, 2016.
- [17] Fan. Z Q. Research on continuous berth allocation: model optimization and calculation analysis [J]. *Industrial Engineering and Management*, 2016, 21(03): 81-87.



- [18] Zheng. H X, Yin. H, Cao. H L, Shi Y. Integrated scheduling of discrete berths and quay cranes considering the influence of tides[J]. Operations Research and Management, 2017, 26(06): 167-175.
- [19] Ren J. Research on the allocation of berths and quay cranes in container terminals based on genetic algorithm [D]. Inner Mongolia University, 2017.
- [20] Yang J. Research on Optimal Scheduling Model and Strategy of Quay Cranes for Container Terminal Berths [D]. Dalian Maritime University, 2017.
- [21] Xu. W D, Liu. G Y, Wang. C Y, Luo. L Y. Container terminal berth-quay crane joint scheduling optimization method based on compound cost[J]. Journal of Ningbo University (Science and Technology Edition), 2019, 32(06): 87-91.
- [22] David P. C, James M. Crawford David E. Joslin. Heuristic optimization: A hybrid ai/or approach. In Proceedings of the workshop on Industrial Constrained-directed Scheduling, 1997.
- [23] Frank. M. Seaside Operations Planning in Container Terminals, Physica-Verlag, Berlin, 2009.