

Dynamic Analysis of Vehicle-Bridge Coupling System Based on Bond Graph Method

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Abstract

Taking the main beam of the bridge as the research object, the bond graph model of the vehicle-bridge coupling system is established by the bonding graph method, and the system state equation is derived. The state space method is used to solve the problem. The variation law of the frequency of each phase of the coupled system with the relative position of the vehicle on the bridge and the variation of the fundamental frequency of the system with the vehicle speed and acceleration are studied. Research indicates: The bond graph method can quickly establish a mathematical model and can provide a good solution for complex system modeling; The frequency of each phase of the coupled system is not only related to the position of the trolley on the beam, but also related to the quality of the trolley. Under the condition that the beam is constant, the larger the mass of the trolley, the smaller the coupling frequency; The greater the initial velocity of the car, the greater the mid-span displacement of the beam, and the greater the acceleration of the car during the acceleration process, the greater the mid-span displacement of the beam.

Keywords

Bond graph, axle coupling, trans-middle displacement.

1. Introduction

Field bridge cranes are the main equipment for port cargo handling. With the development of high-speed, large-scale and heavy-duty port cranes, the coupled vibration problem of the main girder structure of the bridge under moving loads is becoming more and more significant. Therefore, its research results are of great significance for crane design and vibration control.

Considering the coupled vibration of the crane's main beam structure and the lifting trolley, it is often simplified as a moving load-beam coupling system. At present, a lot of researches have been done on mobile load-beam coupling systems at home and abroad. Fryba et al. [1] listed many modeling and analysis methods for moving load-beam coupling systems, and studied the main influencing factors of beam dynamic deflection; Wang Hualin et al. [2] studied the moving mass in uniform speed, acceleration or deceleration. The vibration characteristics of the lower simply supported beam. Peng et al. [3] studied the influence of the acceleration and deceleration of moving mass on the mid-span deflection of the bridge when the moving mass model was simply supported. Zhao Xiaohua et al. [4] studied the simple support of moving mass load by mass-spring model. The vibration characteristics of the box beam and the influence of the moving speed on the beam vibration; Xia He et al. [5] constructed a moving sprung mass model, analyzed the influence of the sprung mass acceleration and deceleration on the mid-transverse deflection of the bridge and conducted resonance research.

In this paper, the main beam of the bridge crane is taken as the research object, and the main beam of the bridge is simplified into a simple supported beam, which is based on the bonding diagram method.

Based on the force balance and displacement continuous conditions between subsystems, the coupled system bond graph model is established, the system state equation is derived, and the state equation is numerically solved by the iterative solution of state space. The vibration response of the main beam of the field bridge is studied, and the factors affecting the vibration response of the main beam of the field bridge are analyzed. The variation law of the frequency of each phase of the coupled system with the relative position of the vehicle on the bridge and the variation of the fundamental frequency of the system with the vehicle speed and acceleration are studied.

2. Establishment of Bond Graph Model for Vehicle-Bridge Coupling System

2.1 Model establishment

The field bridge is often simplified to a simply supported beam. When the mass of the moving load cannot be neglected compared with the mass of the beam, the gravity of the moving load and the inertial force generated by the moving load during the vibration must be considered. It is assumed that the simply supported beam is homogeneous in equal section, the damping is neglected, the moving mass is m , and the moving speed is V . If the moving mass does not leave the beam during the motion, the displacement is consistent with the deflection of the beam. Therefore, for the case of concentrated mass action, the model is simplified as shown in Fig. 1.

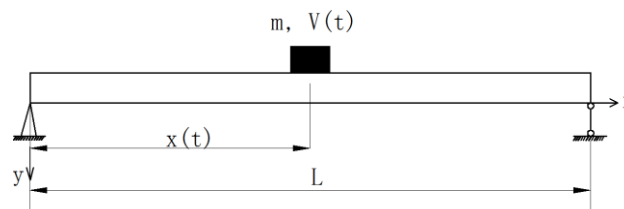


Fig. 1 Mechanical model of the trolley-suspended-simplified beam coupling system

Set the mass of the trolley m ; the elastic modulus of the beam is E , the moment of inertia is I , the mass per unit length is m_b , and the span is L . In this section. For the beam bonding diagram model,

take the finite mode $N=5$. which is $y(x,t) = \sum_{i=1}^5 \phi_i(x)q_i(t)$.

According to the bonding graph modeling step, a bond graph model is established for the coupled system, as shown in Fig 2.

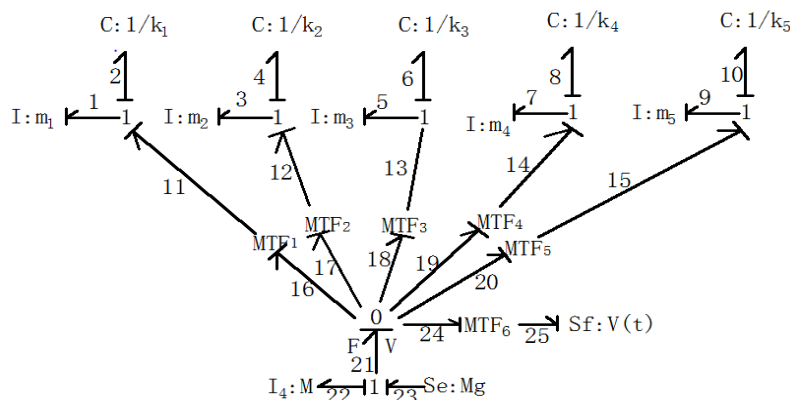


Fig. 2 Axle coupling system bond diagram model

Where m_1, m_2, m_3, m_4, m_5 is the modal quality of simply supported beams, k_1, k_2, k_3, k_4, k_5 is the modal stiffness of a simply supported beam, k_1, k_2, k_3, k_4, k_5 is the modal stiffness of a simply supported beam. Where $MTF_1, MTF_2, MTF_3, MTF_4, MTF_5$ is the position of the trolley on the beam corresponds to the first five-order mode function.

$$\phi_i = \sin\left(\frac{i\pi x_c}{l_b}\right), (i = 1, 2, 3, 4, 5) \quad MTF_6 = \sum_{i=1}^5 \eta_i(t) \frac{d}{dx} \phi_i(x) \quad (1)$$

2.2 Column system state equation

According to the bond graph rule, the generalized momentum of the inertial component with integral causality and the generalized displacement of the capacitive component are taken as the state variables of the system.

Set the system state variable: $X = [p_1, p_2, p_3, p_4, p_5, q_6, q_7, q_8, q_9, q_{10}]^T$;

Set the input vector of the system is: $U = [Mg, v(t)]^T$;

It can be known from the bond diagram causality and potential equations and flow equations:

$$\begin{aligned} F &= Mg - M \dot{f}_{22} \\ V &= f_{22} = f_{16} + f_{17} + f_{18} + f_{19} + f_{20} + v(t) MTF_6 \\ \dot{p}_1 &= -k_1 q_2 + F \times MTF_1 \\ \dot{p}_3 &= -k_2 q_4 + F \times MTF_2 \\ \dot{p}_5 &= -k_3 q_6 + F \times MTF_3 \\ \dot{p}_7 &= -k_4 q_8 + F \times MTF_4 \\ \dot{p}_9 &= -k_5 q_{10} + F \times MTF_5 \end{aligned} \quad (2)$$

Defining model generalized momentum: $p_1 = m_1 \dot{\eta}_1$, $p_2 = m_2 \dot{\eta}_2$, $p_3 = m_3 \dot{\eta}_3$, $p_4 = m_4 \dot{\eta}_4$, $p_5 = m_5 \dot{\eta}_5$; Defining model generalized displacement: $q_6 = \eta_1$, $q_7 = \eta_2$, $q_8 = \eta_3$, $q_9 = \eta_4$, $q_{10} = \eta_5$.

Substituting generalized momentum and generalized displacement into the above equations yields:

$$\begin{aligned} m_1 \ddot{\eta}_1 &= -k_1 \eta_1 + F \times MTF_1 \\ m_2 \ddot{\eta}_2 &= -k_2 \eta_2 + F \times MTF_2 \\ m_3 \ddot{\eta}_3 &= -k_3 \eta_3 + F \times MTF_3 \\ m_4 \ddot{\eta}_4 &= -k_4 \eta_4 + F \times MTF_4 \\ m_5 \ddot{\eta}_5 &= -k_5 \eta_5 + F \times MTF_5 \end{aligned} \quad (3)$$

Introducing state variables $X(t)$:

$$X(t) = \begin{pmatrix} \eta_i(t) \\ \dot{\eta}_i(t) \end{pmatrix} \quad (i=1, 2, 3, 4, 5) \quad (4)$$

The dynamic equation of the system can be converted to:

$$\dot{X} = DX + P \quad (5)$$

In the formula:

$$D = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad P = \begin{pmatrix} 0 \\ M^{-1}F(t) \end{pmatrix}$$

The resulting equation is the state equation of the axle-coupled system, and the state equation describes a dynamic process. According to the state space theory, the solution of the equation is:

$$X = e^{Dt} X(0) + \int_0^t e^{D(t-\tau)} P(\tau) d\tau \tag{6}$$

In the equation, there are 10 dynamic responses, 5 displacement responses, and 5 speed responses. At the same time, the eigenvalues of the matrix are calculated, and the frequencies of the axle coupling system can be obtained.

3. Establishment of Bond Graph Model for Vehicle-Bridge Coupling System

Analyze the influence of acceleration and initial velocity on the mid-span displacement of the beam. The calculation parameters are shown in Table 1.

Tab.1 Calculation parameters

parameter	Numerical value
$E / (\text{N} \cdot \text{m}^2)$	2.15×10^{11}
I / m^4	0.8
$m / (\text{kg} \cdot \text{m}^{-1})$	1.2×10^4
M / kg	1.2×10^4
l_b / m	96

The calculation results are shown in Figures 3 and 4.

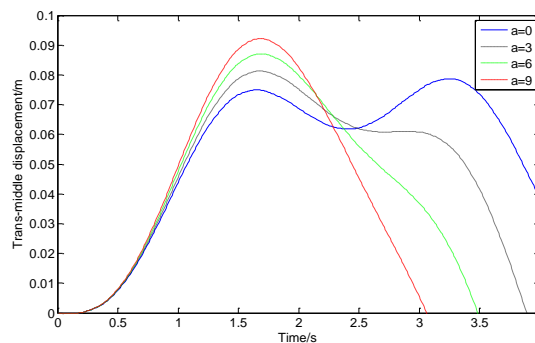


Fig. 3 Effect of acceleration on the mid-span displacement of the beam

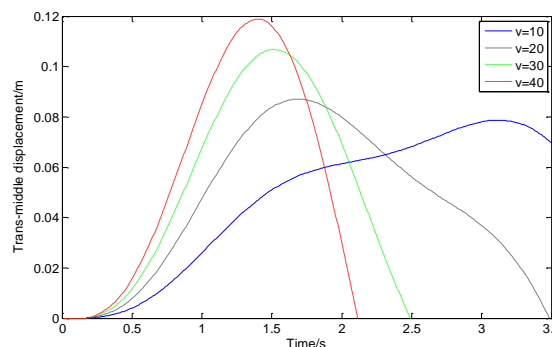


Fig. 4 Effect of initial velocity on the displacement of the beam span

Through the above calculations and the calculation results of the literature [3], the calculation results are basically the same, which verifies the correctness of the model and the feasibility of the bond graph modeling. And the greater the acceleration of the trolley, the greater the maximum vibration response of the beam; the larger the initial velocity of the trolley, the greater the maximum vibration response of the beam.

The state equation is obtained by the bond graph. By calculating the eigenvalues of the matrix of the state equation, the frequency of each phase of the coupled system can be obtained. Take the calculation parameters of the beam in Table 1, calculate the frequencies of the vehicle-bridge coupling system, and analyze the influence of different quality cars on the coupling frequency.

Where $M1 = 6.12 \times 10^4 \text{ kg}$, $M2 = 8.12 \times 10^4 \text{ kg}$.

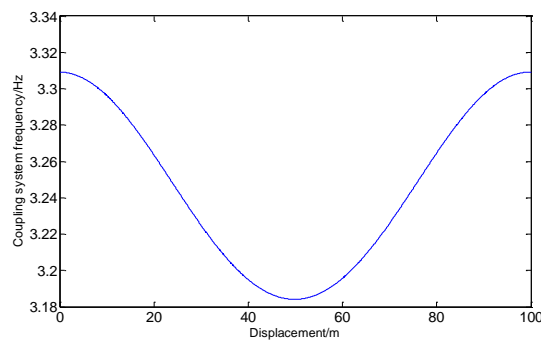


Fig. 5 First order coupling frequency

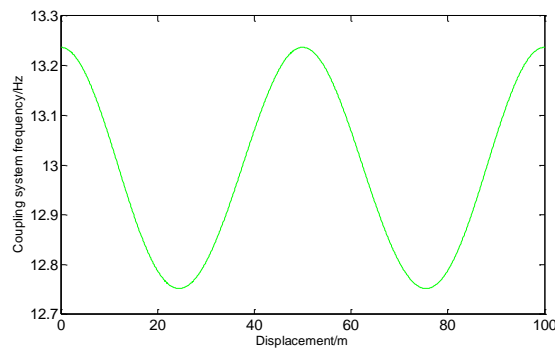


Fig. 6 Second order coupling frequency

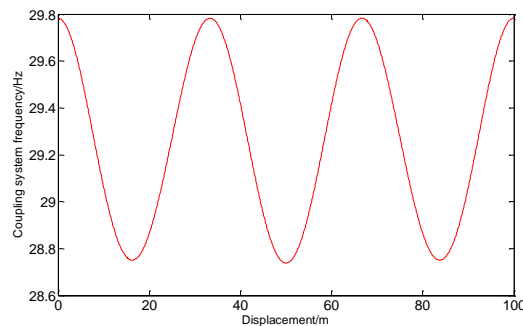


Fig. 7 Third-order coupling frequency

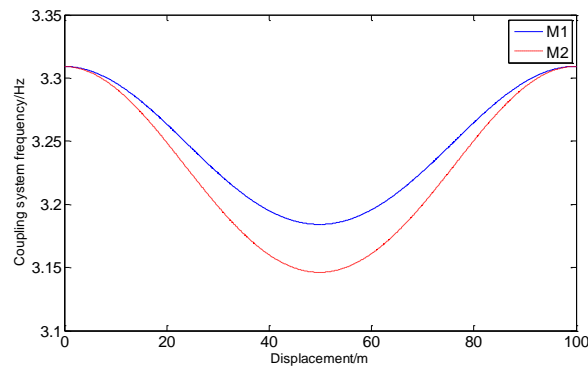


Fig. 8 first-order coupling frequency

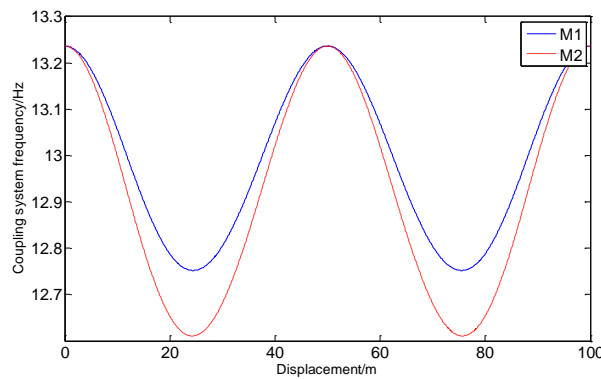


Fig. 9 second-order coupling frequency

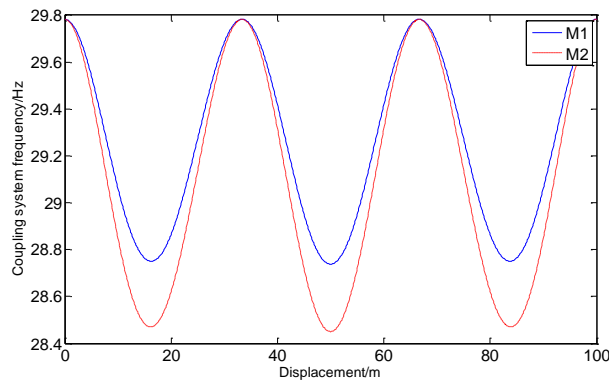


Fig. 10 Third-order coupling frequency

The frequency of the trolley-suspended-simplified beam coupling system is not only related to the position of the trolley on the beam, but the greater the total mass of the trolley and the hoist, the smaller the coupling frequency of each step.

4. Conclusion

Based on the bond graph method, the bond graph model of the trolley-hanging weight-simple supported beam coupling system is established, and the state equation is derived. Through the state space iterative solution of the coupled system, the following conclusions can be drawn:

- (1) The bond graph method can quickly establish a mathematical model and can provide a good solution for complex system modeling.
- (2) The greater the initial velocity of the trolley, the greater the mid-span displacement of the beam, and the greater the acceleration of the trolley during the acceleration process, the greater the mid-span displacement of the beam.

(3) The frequency of each step of the trolley-suspended-simplified beam coupling system is not only related to the position of the trolley on the beam, but also related to the speed and acceleration of the trolley and the total mass of the trolley and the hoist.

It is feasible to apply the bond graph theory for vibration analysis, and it is very simple and practical from modeling to calculation. It is more convenient in mathematical modeling, and can be obtained by simple addition, subtraction, multiplication and division.

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