Research on Adaptive Control Method of Cantilever Beam Vibration Suppression

Yaxiong Gu, Feng Liu, Qing Hu

School of Mechanical and Electrical Engineering, Southwest Petroleum University, Chengdu 610500, China;

1441658770@qq.com

Abstract

A method of adaptive control of vibration suppression by establishing the cantilever Bernoulli-Euler beam model is proposed.Firstly, based on the study of the model reference adaptive theory, the Lyapunov stability model reference adaptive control method is established to suppress the cantilever vibration. The transverse vibration differential equations of the cantilever beam system and the transverse vibration differential equations of the modal coordinates are established and simulated under the ANSYS Workbench software environment to obtain the optimal position of the vibration sensor/brake. According to the differential equation of transverse vibration of the modal coordinates of the system model, the state space equation and transfer function of the system corresponding to the system are obtained, which are mathematically transformed into difference equation and used as the mathematical model of parameter identification. On this basis, band-pass filter and recursive least squares method are combined to identify the natural frequency of sensor signal. Finally, the adaptive controller for vibration suppression of cantilever beam is simulated by using MATLAB software. The results show that the control method can achieve better suppression effect.

Keywords

Vibration Suppression Adaptive Control; cantilever beam; Differential equation of transverse vibration; natural frequency; Model Reference Adaptive Control.

1. Introduction

In the industries of aviation, aerospace, satellite, high-precision electronic equipment and precision medical equipment manufacturing, the use of mechanical arms has become the norm. At the same time, due to the high requirements of control precision in various industries, the elastic vibration problem of mechanical arms will seriously interfere. Its running accuracy, in order to get a wider application of the arm, must solve the elastic vibration problem of the arm, so that the manufacturing precision of the device can be more effectively improved [1].

In [1], the Euler-Bernoulli beam is used as the flexible arm model. The vibration suppression strategy based on active vibration control is studied, and the control parameters can be adjusted according to the change of the end load. Reference [2] uses a flexible mechanical arm to make a combined structure, which uses the frictional effect between the contact surfaces to consume energy to suppress the vibration of the flexible mechanical arm, but has poor controllability to complex systems. In [3], the pedestal vibration is regarded as an external uncertain disturbance, a coupled model is established, and then the singular perturbation technique is used to eliminate the vibration.

There are two methods for solving the problem of elastic vibration in the existing methods, one is passive vibration suppression, and the other is active vibration suppression [4-5]. In order to solve the elastic vibration problem of the flexible manipulator and improve the control precision and

stability of the flexible manipulator, a model reference adaptive control method is proposed. This method simulates the flexible manipulator as a cantilever beam, and studies the control method through the general model of the cantilever beam combined with the characteristics of the arm material, and uses the piezoelectric film sensor and the piezoelectric ceramic actuator as the main control components. The flexible manipulator performs corresponding simulation and experimental research [6].

2. Modeling and analysis of cantilever beam system

The flexible manipulator is a time-varying system with high nonlinearity and strong coupling characteristics. The complexity of the machine determines the mathematical model is difficult to be precise. In this paper, the manipulator is regarded as a cantilever beam, and the forced vibration of the cantilever beam usually satisfies the Euler-Bernoulli model without special circumstances. As a model, a mathematical model of the flexible manipulator can be established. When an external force acts on the cantilever beam in the y direction, the cantilever beam will start to vibrate. When the cantilever beam vibrates, a small section is randomly removed from the beam for lateral vibration force analysis. The vibration analysis of the cantilever beam is shown in Fig. 1.

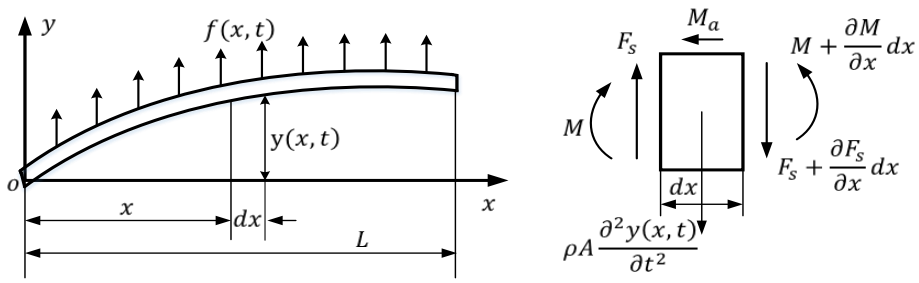


Fig.1 Stress analysis of cantilever beam vibration bending

2.1 Cantilever beam lateral vibration differential equation

According to the D'Alembert principle, the influence of shear deformation and section rotation is neglected. Through the mechanical analysis, the differential equation of the laterally forced vibration of the cantilever beam is:

$$\rho S \frac{\partial^2 y(x,t)}{\partial t^2} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = \frac{\partial^2 M_a}{\partial x^2} \tag{1}$$

The transverse vibration differential equation excitation is equal to zero:

$$\rho S \frac{\partial^2 y(x,t)}{\partial t^2} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = 0 \tag{2}$$

Because the solution is separated in time and space, the method is solved by using the method of separating variables:

$$y(x,t) = (C_1 \cos \beta x + C_2 \sin \beta x + C_3 ch\beta x)(A \sin \omega t + B \cos \omega t) \tag{3}$$

Angular frequency equation:

$$\cos(\beta_n L)\cosh(\beta_n L) = -1 \tag{4}$$

$$\begin{cases} \beta_1 L = 1.875 & i = 1; \\ \beta_2 L = 4.694 & i = 2; \\ \beta_3 L = 7.855 & i = 3; \\ \beta_i \approx (i - 1/2)\pi & i > 3; \end{cases} \tag{5}$$

$$\omega_i = (\beta_i L)^2 \sqrt{\frac{EI}{\rho S L^4}} \qquad i = (1,2,3,4...) \tag{6}$$

The mode function corresponding to the natural angular frequency of the cantilever beam is expressed as:

$$\varphi_i(x) = \cos\beta_i x - \cosh\beta_i x + \eta_i(\sin\beta_i x - \sinh\beta_i x) \tag{7}$$

In the middle $\eta_i = \dfrac{\sinh\beta_i L}{\sin\beta_i L}, \beta^4 = \dfrac{\rho S}{EI}\omega^2$

Differential equation of vibration of cantilever beam under modal coordinates:

$$\ddot{q}_i(t) + 2\xi_i f_i \bar{q}_i(t) + f_i^2 q_i(t) = \int_0^l \varphi_i(x)\frac{\partial^2 M_a}{\partial x^2}dx \tag{8}$$

In the formula:

- the i-th normalized orthogonal mode function; - the i-th order mode coordinate;
- the i-th modal damping ratio; - the natural frequency of the i-th order system.

2.2 Cantilever beam dynamic analysis

In this paper, 65 Mn steel with good vibration characteristics is used as the main body of cantilever beam. The core characteristic parameters are shown in Table 1:

Table 1 Core characteristic parameters of cantilever beam

name	Cantilever beam
Length / width / thickness (m)	0.3/0.02/0.001
Cross-sectional area (m2)	2×10-5
Density (kg/m3)	7180
Moment of inertia (m4)	1.7×10-12
Modulus of elasticity (Pa)	1.98×1011
Poisson's ratio	0.3

When the flexible cantilever arm is in the initial state, a very short step external force is applied thereto, that is, an impulse signal is input to the system to study the system response of the flexible cantilever arm, thereby obtaining the system parameters thereof. After the external force input, the flexible cantilever beam will begin to dampen the amplitude of the damping motion. In the process that the flexible cantilever beam starts to vibrate until the energy is completely depleted and reaches a static state, the low-order mode stiffness will be weaker, and the vibration effect caused by the impulse is more obvious in the low-order mode, while the higher-order mode is opposite. At the same time, when subjected to the same external force impulse, the low-order mode will absorb more energy. Therefore, when analyzing the mathematical model of the flexible cantilever beam, the first three-order modes with greater influence will be taken. The properties can determine most of the performance of the entire flexible cantilever system, and the first three modes occupy a large weight in the system core parameters.

The structural parameters of the cantilever beam of Mn steel are obtained by using MATLAB simulation software. The front three-order natural angular frequency value, the natural frequency value and the vibration mode function of the first three-order modal parameters are obtained, as shown in Table 2. Subsequently, according to the function in Table 1, the simulation function parameters in MATLAB are set, and the steps of the cantilever beam are represented by the stepwise display method, and the first three-order mode shape curve can be obtained, as shown in Fig. 2 below:

Table 2 Natural angular frequency, natural frequency and first-order mode function of cantilever beam structure

First three natural angular frequencies (rad/s)	$\omega_1 = 59.8053$
	$\omega_2 = 374.8206$
	$\omega_3 = 1049.6$

First three natural frequencies (Hz)	$f_1 = \omega_1/(2\pi) = 9.5183$
	$f_2 = \omega_2/(2\pi) = 59.6546$
	$f_3 = \omega_3/(2\pi) = 167.0490$
First-order mode function	$\varphi_1(x) = \cos 3.75x - \cosh 3.75x + 0.734(\sin 3.75x - \sinh 3.75x)$
	$\varphi_2(x) = \cos 9.39x - \cosh 9.39x + 1.109(\sin 9.39x - \sinh 9.39x)$
	$\varphi_3(x) = \cos 15.7x - \cosh 15.7x + 0.99(\sin 15.7x - \sinh 15.7x)$

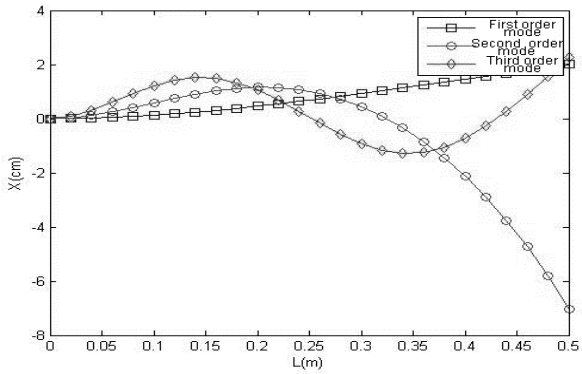


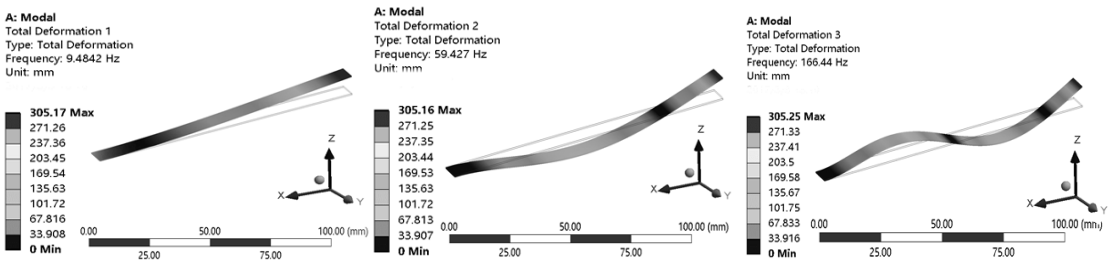
Fig. 2 The first three-order mode shape MATLAB curve display

2.3 ANSYS modal analysis verification

According to the characteristic parameters of the cantilever beam structure material in Table 1, the simulation model is established in ANSYS, and the cantilever beam modal simulation analysis is carried out by running the processing.

After the ANSYS modal analysis, the natural frequency of the cantilever beam is obtained. The calculated values and simulation results are shown in Table 3. After the comparative analysis, the deviation between the theoretical value and the simulated value of the software is within 0.4%. Therefore, the lateral vibration differential equation can be established by a series of measurement parameters of the cantilever beam, and the natural frequency is obtained therefrom with high precision. The first-order and second-order mode curves of the cantilever beam have lower natural frequency values, and the third-order mode curve has a higher natural frequency value.

The ANSYS curve of the first three modes is shown in Fig. 3. The modal shape has the lateral vibration displacement coordinate of the cantilever beam. However, this displacement is not the true lateral displacement of the arm beam. This is mainly because of the magnitude of the displacement. The initial energy obtained by the cantilever beam is related. For the same cantilever beam, the amount of displacement generated under different energy excitations is different, but the distribution of energy and the rate of change of attenuation are basically constant.



First-order mode curve (b) Second-order mode curve (c) third-order mode curve

Fig. 3 shows the first-three order mode shape ANSYS curve display

From the comparative analysis, it can be seen that the theoretical calculation analysis of the first three-order mode shape curve is almost the same as that of the mode shape obtained by ANSYS simulation

analysis. Therefore, it can be considered that the theoretical mode shape curve of the above theoretical analysis has Correct guidance

Table 3: The first three natural frequency values of the cantilever beam modal analysis

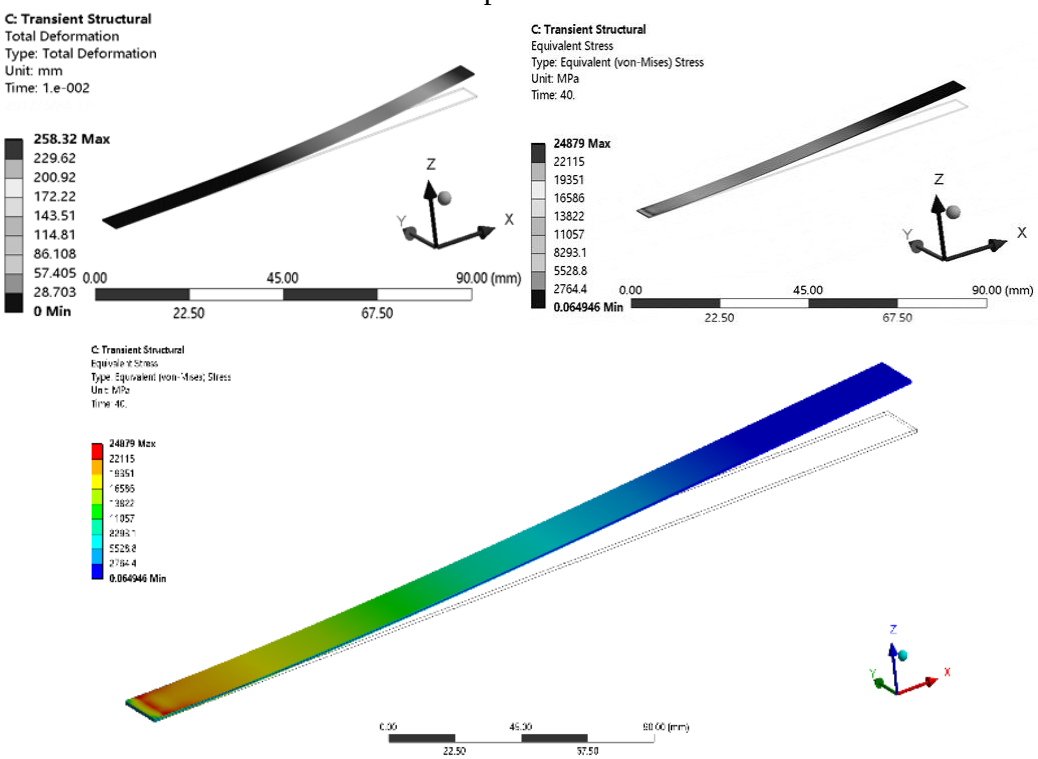
Natural frequency order	Natural frequency calculation value (Hz)	Modal frequency simulation value (Hz)	Error (Hz)
level one	9.5184	9.4842	0.0341
Second order	59.653	59.425	0.2271
Third order	167.06	166.43	0.6100

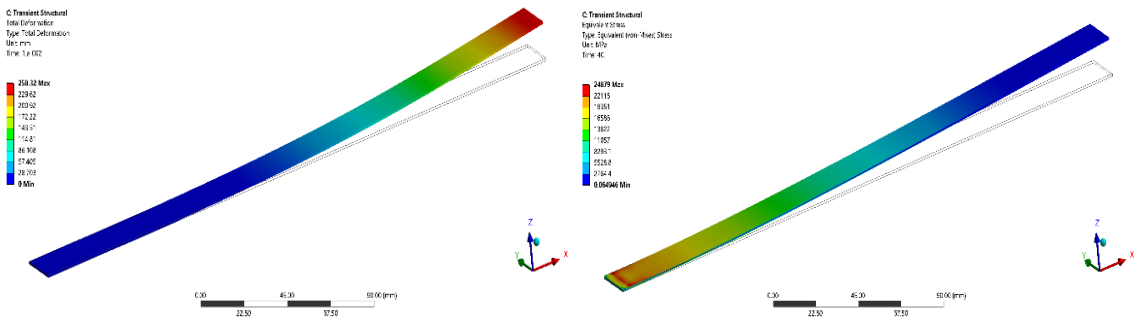
3. Sensor and actuator

The controlled structure in the vibration suppression control system discussed in this paper not only has lateral vibration, but also has bending and longitudinal vibration. In order to take into account the vibrations of these aspects, the PVDF piezoelectric film is used as the sensor. Since the piezoelectric material is small in size and can be integrated with the cantilever beam, piezoelectric materials are used herein as actuators.

In addition to the choice of materials, it is also necessary to consider the installation position. Different installation positions have different signal sensing capabilities for the cantilever beam, and also have a greater impact on the control effect of the entire system [4]. In order to obtain a reasonable installation position, this paper uses ANSYS to perform transient dynamic analysis of the cantilever beam to determine the installation position of the sensor and the actuator.

Therefore, the model of the cantilever beam needs to be solved first, and the optimal installation position of the sensor and the actuator can be determined by solving, simulating and analyzing the corresponding cloud image. Fig. 4 is a simulation diagram of transient simulation deformation and strain analysis obtained by solving the model. It can be analyzed in the figure that after the input of an impulse excitation, the cantilever beam leaves the initial position and starts to vibrate, and it can be seen that the maximum stress concentration position is at its root.

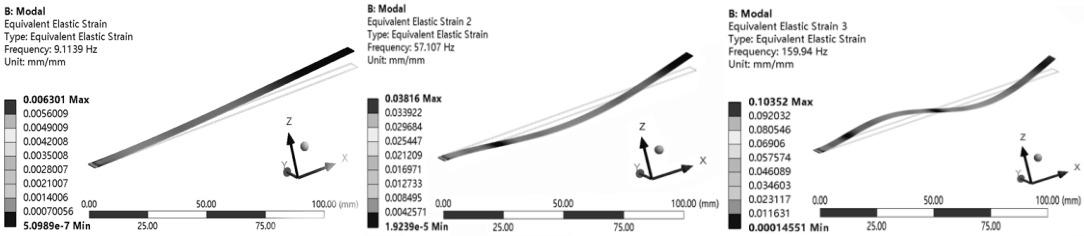




(a) deformation analysis cloud image (b) stress analysis cloud image

Fig. 4: Cantilever beam model transient dynamics simulation cloud image

Fig. 5 is a front-fourth simulation stress cloud analysis diagram of the cantilever beam model for its modal analysis. From the stress maps of the various modes, one can clearly draw a conclusion: the cantilever beam in vibration, the stress is basically concentrated in the root. In the vibration state, the maximum stress is concentrated at the root.



First-order modal stress cloud diagram (b) Second-order modal stress cloud diagram (c) Third-order modal stress cloud diagram

Fig. 5 Stress analysis cloud image of cantilever beam modal simulation

Since the vibration of the cantilever beam is transmitted to the root, a stress opposite to the existing vibration stress can be set at this position to offset the instantaneous stress at this time, and the stability of the cantilever beam can be accelerated.

4. Implementation and simulation of control algorithms

4.1 Vibration suppression control state space equation

Piezoelectric brake equation:

$$M(x,t)=kU(t)[H(x-x_1)-H(x-x_2)] \tag{9}$$

$$\frac{\partial^2}{\partial x^2}M(x,t)=\frac{\partial^2}{\partial x^2}kU(t)[H(x-x_1)-H(x-x_2)] \tag{10}$$

Differential equation of vibration of cantilever beam under modal coordinates:

$$\ddot{q}_i(t)+2\xi_i f_i \bar{q}_i(t)+f_i^2 q_i(t)=\int_0^l \varphi_i(x)\frac{\partial^2 M_a}{\partial x^2}dx \tag{11}$$

Differential equation of cantilever beam with piezoelectric actuator in modal coordinate system:

$$q_i(t)+2\xi_i\omega_i\bar{q}_i(t)+\omega_i^2 q_i(t)=k[\varphi_i'(x_2)-\varphi_i'(x_1)]U \tag{12}$$

The above equation is rewritten as the state space equation of the cantilever beam system, and its expression is:

$$\begin{cases}\bar{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases} \tag{13}$$

In the formula: $A=\begin{pmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_1 \end{pmatrix}$; $B=\begin{bmatrix} 0 \\ k[\varphi_i'(x_2)-\varphi_i'(x_1)] \end{bmatrix}$; $C=[1 \quad 0]$

Write the state space equation as a transfer function form, which can be represented by multiple second-order systems:

$$G(s)=\sum_{i=1}^{n} G_i(s)=\sum_{i=1}^{n}\frac{k[\varphi_i'(x_2)-\varphi_i'(x_1)]}{s^2+2\xi_i f_i s+f_i^2} \tag{14}$$

4.2 Model reference adaptive controller design

In the analysis, it is found that the low-order absorption has the most energy and occupies the largest weight. Therefore, in order to simplify the control system, the mode larger than the first-order mode is temporarily ignored.

The transfer function of the cantilever beam control system is:

$$G(s)=\frac{\lambda_1}{s^2+2\xi_1 f_1 s+f_1^2} \tag{15}$$

Order to $\lambda_1=k[\varphi_i'(x_2)-\varphi_i'(x_1)]$, bring the known amount into the above formula:

$$G(s)=\frac{50.8}{s^2+0.15s+90.598} \tag{16}$$

The reference model is an important part of the adaptive control system:

$$G_m(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \tag{17}$$

The overshoot is less than 5%, the settling time is less than 10 s, and the stability error is less than 5%. The reference transfer function is:

$$G_m(s)=\frac{\omega_m^2}{s^2+1.8s+1.6} \tag{18}$$

The conditions under which the system can be stabilized are:

$$\lim_{x\to\infty}\varphi(t)=\lim_{x\to\infty}\varepsilon(t)=0 \tag{19}$$

4.3 Control system simulation

The simulation diagram of the control system is established, and the Simulink simulation block diagram of the control system designed by MATLAB Simulink simulation software is established.

Apply an impulse to the cantilever beam and the system will attenuate the damped vibration. After adding the Lyapulov model reference adaptive control, the control effect is obvious, and the system quickly reaches a steady state. In the simulation environment, the control algorithm achieves vibration suppression under impulse conditions, as shown in Fig. 6.

In the case where a sinusoidal vibration signal is applied to the cantilever beam, the amplitude will become larger and larger until amplitude oscillation occurs. After adding the model reference adaptive control, the amplitude of the system is significantly reduced. In the simulation environment, the control algorithm achieves vibration suppression under the action of a sinusoidal signal, as shown in Fig. 7.

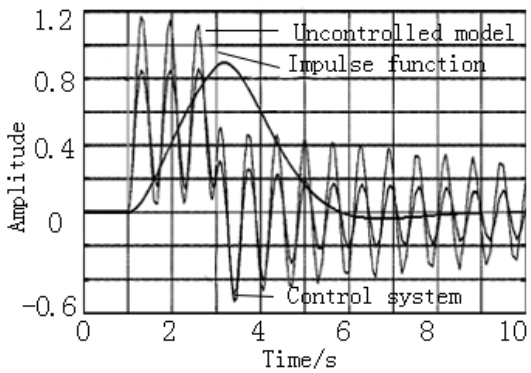


Fig.6 Response curve of the cantilever beam subjected to impulse

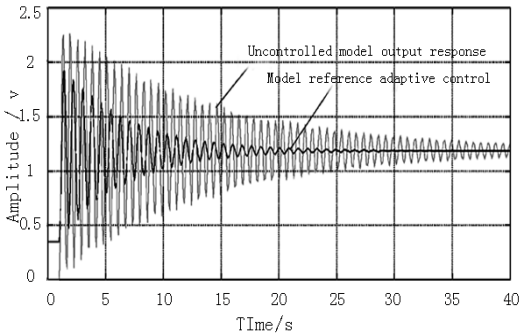


Fig.7 Response curve after applying sinusoidal vibration signal to the cantilever beam

5. Experiment and analysis

The construction of the cantilever beam vibration control experimental platform is carried out, and the effect of the model reference adaptive control algorithm in the vibration suppression control is verified. Debug the various modules of the vibration suppression control experimental platform, apply transient excitation to the cantilever beam, and let it enter the vibration state. In the uncontrolled state, the time required for the cantilever beam to start vibrating to a stationary state is 32.5 s. The amplitude of 0 to 10 s is large and the attenuation is large. Most of the energy of 10 s to 22 s has been consumed, so that the attenuation is reduced,as shown in Fig. 8.

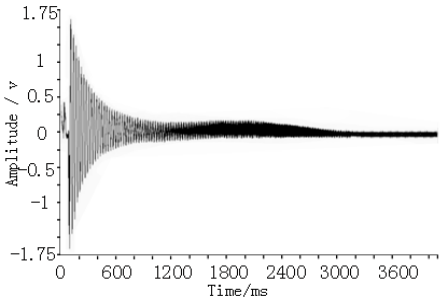


Fig. 8 Vibration curve of the cantilever beam after being excited under the uncontrolled state

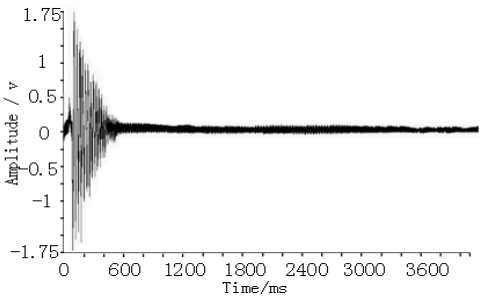


Figure 9 model reference adaptive control effect curve

The adaptive control algorithm is based on the structural stability of Lyapunov stability. Combined with the simulation analysis model of the cantilever beam, the control method for suppressing the vibration of the cantilever beam is designed to make the amplitude fast decay time become 3.6 s. Fast decay in the uncontrolled state is fast. After 5.8 s, the cantilever beam amplitude tends to be stable. At this time, the maximum output voltage of the cantilever beam is 0.08V, and the steady-state error is 4.8%, as shown in Fig. 9.

The cantilever beam vibration suppression method should meet the following conditions: overshoot <5%, settling time <10s, steady state error <4%. However, in the actual experimental test, it was found that only the stabilization time was fully met, while the other two did not reach the standard, especially the stability effect was worse than expected. According to the analysis of the specific situation, the main factors causing this result are as follows: First, in the study of the cantilever beam model, only its own role is considered, and the influence of the pedestal on the cantilever beam parameters is not considered, and the pedestal If the energy is not completely absorbed, the energy will be reflected to the cantilever beam in a certain proportion, resulting in a mismatch of the suppression parameters; the second is that the resolution of the sensor itself is insufficient, or the signal conversion delay of the sensor causes the control amount to appear Phase deviation.

6. Conclusion

In this paper, the hazard of the elastic vibration of the cantilever beam is analyzed, and the flexible cantilever beam is taken as the research object. The D'Alembert principle is used to study the lateral vibration differential equation, and then its natural frequency and mode shape function are obtained. The dynamic performance of the cantilever beam is analyzed, and the first three-order mode shape curve of the cantilever beam is obtained in MATLAB. . In order to verify the feasibility of the theory, the ANSYS software is used to simulate the cantilever beam. The obtained results are less than 1% from the calculated values above. The subsequent experiments can be carried out according to this model.

The first three modes of the model are extracted to identify the natural frequency of the system vibration signal, and the natural frequency of the system is obtained. The vibration law and the deformation cloud map are used to analyze the vibration law and the distribution of the different modes. At the root of the cantilever beam. Then the brake equations and the vibration differential equations of the cantilever beam are obtained by the Labrador variation to obtain the transfer function of the system, and the adaptive control method of the system is obtained by combining the Lyapunov stability principle. Finally, the simulation is carried out in MATLAB, and the impulse signal is input to the system. The system parameters such as system response speed, steady speed and steady-state error are greatly improved when there is no control algorithm.

According to the theoretical analysis and simulation results, the cantilever beam experimental control platform is built. The experiment shows that when the amplitude is attenuated to 10%, the no control method takes 10.1s, and the adaptive control algorithm only takes 3.6s. System stabilization time, no control method takes 32.5s, and it takes only 5.8s to join the adaptive control algorithm. Moreover, after adding the adaptive control algorithm, the steady-state error of the system is 4.8%, less than 5%, which satisfies the system stability requirements.

References

[1] Yu Yang. Research on adaptive vibration control of cantilever beam under load based on PZT[D]. Harbin Institute of Technology, 2014.

[2] Chen Bin. Research on Vibration Suppression Performance of Combined Flexible Manipulator[J]. Machine Tool & Hydraulics, 2012, 40(11): 83-84.

[3] Zhu Chang. Research on nonlinear robust control of vibrating-based flexible manipulator based on singular perturbation method[D]. Wuhan University of Science and Technology, 2018.

[4] Dong Yonggui, Chai Xu. Active vibration control of piezoelectric laminated curved shell structure [D]. Dalian University of Technology, 2014

[5] Xiaoshun Zhang,Dingguo Zhang,Sijia Chen,Jiazhen Hong. Modal characteristics of a rotating flexible beam with a concentrated mass based on the absolute nodal coordinate formulation[J]. Nonlinear Dynamics,2017,88(1).
[6] Yang Hongbin. Experimental study on active vibration control of flexible cantilever beam[D]. Lanzhou University of Technology, 2012.