Heat transfer model of thermal protective clothing based on non-steady-state finite difference equation

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Abstract

In this paper, using the knowledge of unsteady heat conduction, finite difference method and regression theory, the heat transfer model of multi-layer materials for thermal protective clothing is established. The problem is solved by finite difference equation and dichotomy, and the optimal design of garment parameters is completed. Using the known law of temperature changes on the outside of the skin, we found that the heating process of the garment can be divided into three stages. The first and second stages are the unsteady heat transfer process, and the third stage reaches the steady state heat transfer state. The unsteady stage is the difficult point of the problem. Based on the assumption of one-dimensional unsteady heat transfer without internal heat source, we use the finite difference method to establish the heat transfer model.The temperature distribution of the interior of the garment with respect to time and space can be obtained by using the known law of external temperature change of the skin and the outermost temperature of the garment as boundary conditions and initial conditions. The steady-state temperatures obtained by the unsteady model and the flat model are compared to verify the correctness of the model.

Keywords

High temperature special clothing; heat transfer model ;difference method ;non-steady state ; dichotomy.

1. Problem Retelling

When working in a high temperature environment, people need to wear special clothing to avoid burns. Special clothing usually consists of three layers of fabric material, recorded as layer I, II, III, where layer I is in contact with the external high temperature environment, and there is a gap between layer III and skin ,this gap is referred to as the IV layer. Special clothing parameters must meet certain requirements to ensure the safety of the staff.At the same time, we should minimize R&D costs and shorten the R&D cycle.

In order to design a special heat-resistant garment, the dummy whose body temperature is controlled at 37ºC is placed in a high temperature environment of the laboratory to measure the temperature outside the skin of the dummy. According to some parameter values of the special clothing materials, the experiment was carried out on the outside of the skin of the dummy, with an ambient temperature of 75ºC, a thickness of the II layer of 6 mm, a thickness of the IV layer of 5 mm, and a working time of 90 minutes. Establish a mathematical model to calculate the temperature distribution.

2. Assumptions

A_1 Heat transfer is perpendicular to the skin and is considered as a one-dimensional heat transfer model;

A_2 The initial temperature of the special clothing is 37℃;

A_3 The thickness of the air layer does not exceed 6.4mm, ignoring the influence of heat radiation;

3. Symbol Description

Symbol	Description
q	Heat flux density (W/m2)
T_i	Temperature at the layer-to-layer interface (℃)
t	Time (s)
λ_i	Material thermal conductivity (W/(m·℃))
Q_L	Heat flow through a cylindrical wall per unit length (W/m)
r_i 、 r_{i+1}	Inner and outer radii of curvature of the i-th cylinder (m)
ρ_i	Material density (kg/m3)
c_i	Material specific heat (J/(kg·°C))
ϕ_i	Internal heat source power (W/m3)

3. Problem Analysis

We abstract the problem into one-dimensional heat transfer of the multi-layer material without internal heat source, and divide the heating process of the work clothes into three stages. During the first stage, there is no heat passing through the fabric layer of the garment; from the second stage, there is external heat flowing into the skin, that is, all the fabric layers of the garment have been separated from the original temperature; the third stage is the steady-state heat transfer process, in which the internal temperature distribution of the garment is stable, and the heat passing through the sections of the garment no longer changes and the total heat flux equal. In order to avoid solving the complicated piecewise partial differential equations, we use the finite difference method to establish and simplify the heat transfer model based on the assumption of one-dimensional unsteady heat transfer without internal heat source. The temperature distribution of the interior of the garment with respect to time and space can be obtained by using the known law of external temperature change of the skin and the outermost temperature of the garment as boundary conditions and initial conditions.

4. Model establishment and solution

The establishment and solution of the problem-model

Preliminary knowledge

(1) Steady heat transfer

When the temperature distribution inside the heat-conducting object and the heat passing through the object no longer change with time, the process is steady heat transfer [1]. As mentioned in the previous analysis, the heat transfer process is divided into three phases, the first two phases being unsteady heat transfer.It is steady heat transfer when the temperature outside the skin is constant. Steady-state heat transfer is closely related to the shape of the heat-conducting object.

Due to the small curvature of the thermal protective suit, isotropic and uniform heat transfer, it can be regarded as a flat model treatment, and at the same time compare with the cylinder model.

Solve the equation for the n-layer flat model[2]:

$$q=\frac{T_1-T_{n+1}}{\sum_{i=1}^{n}\frac{b_i}{\lambda_i}} \tag{5-1}$$

$$T_{i+1}=T_1-q\sum_{j=1}^{i}\frac{b_j}{\lambda_j} \tag{5-2}$$

Where T_i is the temperature at the interface between the layers and the unit is °C; b_i is the thickness of each layer of material, the unit is m; λ is Thermal conductivity in W/(m·°C).

For the n-layer cylinder model, the equation is[2]:

$$Q_L=\frac{T_1-T_{n+1}}{\sum_{i=1}^{n}\frac{1}{2\pi\lambda_i}\ln\frac{r_{i+1}}{r_i}} \tag{5-3}$$

$$T_{i+1}=T_1-Q_L\sum_{j=1}^{i}\frac{1}{2\pi\lambda_j}\ln\frac{r_{j+1}}{r_j} \tag{5-4}$$

Where r_i and r_{i+1} are the inner and outer radius of curvature of the cylinder of the i-th layer, the unit is m.

(2) Unsteady heat transfer equation

For an unsteady heat transfer process, the governing equation in a Cartesian coordinate system can be expressed as [2]:

$$\rho c\frac{\partial T}{\partial t}=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\Phi \tag{5-5}$$

Where Φ is the internal heat source power of the object, the unit is W/m3; ρ is the density of the material, the unit is kg/m3; c is the specific heat capacity of the material, and the unit is J/(kg·°C). Since the outside is heated uniformly to the dummy of the thermal protective suit, and there is no heat source in the thermal protective suit, the process is regarded as one-dimensional unsteady heat transfer without internal heat source. Then the formula can be simplified to:

$$\rho c\frac{\partial T}{\partial t}=\lambda\frac{\partial^2 T}{\partial x^2} \tag{5-6}$$

The above formula is the core equation of the problem.

(3) finite difference concept

It is mentioned in the problem analysis that if MATLAB is used to solve the partial differential equations directly, four equations, four initial conditions and eight boundary conditions will be involved, which is difficult to implement. If the differential is replaced by finite difference and the derivative is replaced by a finite difference quotient, then the partial differential equation problem can be transformed into an algebraic equation problem to obtain an approximate solution. In this paper, the explicit difference method is used.

(4)Model establishment

Unsteady model

The one-dimensional unsteady heat-free partial heat differential equation is given by equation (5-6). During the non-steady heat transfer process, the internal temperature of the object has obvious spatial and temporal distribution. In the process of finite difference discrete variables, the time and space within the target range must be divided into small intervals and form a grid, as shown in Figure 5-2. The temperature of any grid point (m, i) is expressed as , m is the space interval number, and i is the time interval number.

From the assumption A2, the initial temperature of the garment is set to 37 ℃, and the change of the surface temperature in the one-dimensional unsteady heat conduction under the first type of boundary conditions is known[3],we can infer that the outermost surface of the garment will instantly rise to 75 ℃. Knowing the above initial conditions, the value of the previous time point can be used to determine the value of the latter time point, so the derivative of time is forward difference method, and the derivative of space is selected by center difference method. Next, the differential differential equation is explicitly differentiated.

The result of the equation (5-6) forward difference is:

$$\frac{\partial T}{\partial t} \approx \frac{T_m^{(i+1)} - T_m^{(i)}}{\Delta t} \tag{5-7}$$

The result of the second partial derivative center difference is:

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{m+1}^{(i)} - 2T_m^{(i)} + T_{m-1}^{(i)}}{\Delta x^2} \tag{5-8}$$

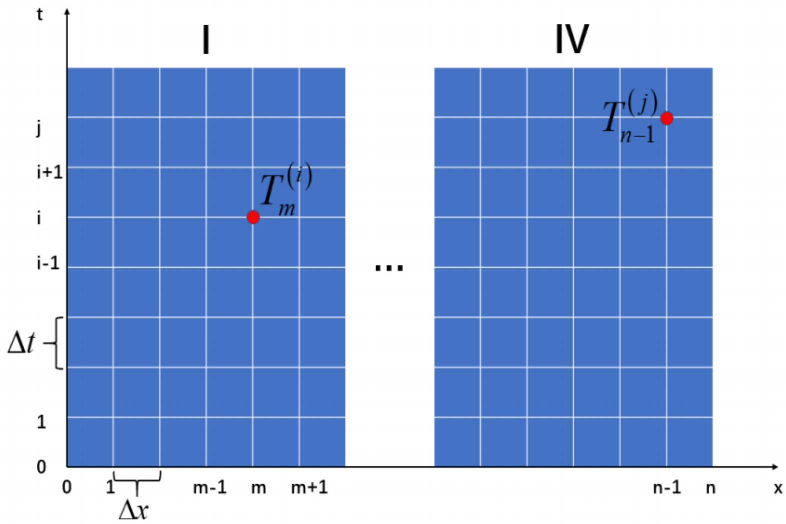


Figure 5-3

Bringing the above difference result into equation (5-6), you can get:

$$\rho c \left[T_m^{(i+1)} - T_m^{(i)}\right] = \lambda \frac{\Delta t}{\Delta x^2}\left[T_{m+1}^{(i)} - 2T_m^{(i)} + T_{m-1}^{(i)}\right] \tag{5-9}$$

Substituting r to obtain a mathematical model of the temporal and spatial distribution of temperature inside the garment:

$$T_m^{(i+1)} = rT_{m+1}^{(i)} + (1-2r)\,T_m^{(i)} + rT_{m-1}^{(i)} \tag{5-10}$$

It can be seen from the above formula that the temperature distribution at any time in the garment can be obtained from the temperature distribution of the previous time interval. The condition that the solution of the explicit equation exists and converges is

$$0 < r \le 0.5 \tag{5-11}$$

The above formula thermodynamically shows that the temperature value of any grid point cannot be a factor that reduces the temperature of the grid point behind. The coefficients on the right side of equation (5-10) must be positive. In addition, similar to the calculation of the outer skin temperature, the same problem exists in the temperature calculation at the boundary of each layer of material. In order to give the boundary condition equation, we introduce the Fourier theorem :

Where T_j and T_j^- are the temperature of the left side of the interface and the interface, the unit is ℃; Δx is the value determined according to the convergence condition. It is known from the conservation of energy that the energy at the inflow interface and the outflow interface are equal, thereby establishing boundary conditions:

$$T_m^{(i+1)} = rT_{m+1}^{(i)} + (1-2r)\,T_m^{(i)} + rT_{m-1}^{(i)} \tag{5-14}$$

Finished up:

$$T_j = \frac{\lambda_1 \Delta x_{j+1} T_j^- + \lambda_{j+1} \Delta x_j T_j^+}{\lambda_1 \Delta x_{j+1} + \lambda_{j+1} \Delta x_j}, j = 2, 3, 4 \tag{5-15}$$

Where T_j^+ is the temperature on the right side of the interface, the unit is ℃. In addition, another boundary condition is established by the fact that the rightmost temperature of the N layer is equal to the leftmost temperature of the N+1 layer:

$$T_N\,(b_N, t) = T_{N+1}\,(0, t)\,, N = 1, 2, 3 \tag{5-16}$$

In summary, the model of Problem One can be summarized as the following mathematical model:

$$\begin{cases} T_m^{(i+1)} = rT_{m-1}^{(i)} + (1-2r)\,T_m^{(i)} + rT_{m+1}^{(i)} \\ T_j = \frac{\lambda_1 \Delta x_{j+1} T_j^- + \lambda_{j+1} \Delta x_j T_j^+}{\lambda_1 \Delta x_{j+1} + \lambda_{j+1} \Delta x_j}, j = 2, 3, 4 \\ T_N\,(b_N, t) = T_{N+1}\,(0, t)\,, N = 1, 2, 3 \end{cases} \tag{5-17}$$

Steady state model

The surface shape of the dummy is more complicated, but most of the areas have smaller curvature and are isotropic, which can be approximated as a 4-layer flat model. To verify its reliability, we used the calculation results of the cylinder model to compare it. If the two are close to each other, the accuracy of the model can be proved, and it can be used as a basis for verifying whether the temperature distribution is correct when the unsteady model is calculated to the steady state process. In the one-dimensional steady-state heat transfer process, the temperature inside the object does not change with time. Therefore, the partial derivative of the left side of equation (5-6) is zero, then the general solution is:

$$T = C_1 x + C_2$$

The above formula shows that the linear temperature distribution in each layer can be obtained by solving the one-dimensional steady-state heat transfer process by simply determining the boundary temperature of each layer, the cylinder model can be analyzed by a similar method to obtain a logarithmic distribution of temperature in the wall of each layer of the cylinder.

(5)Model solving

Steady-state process

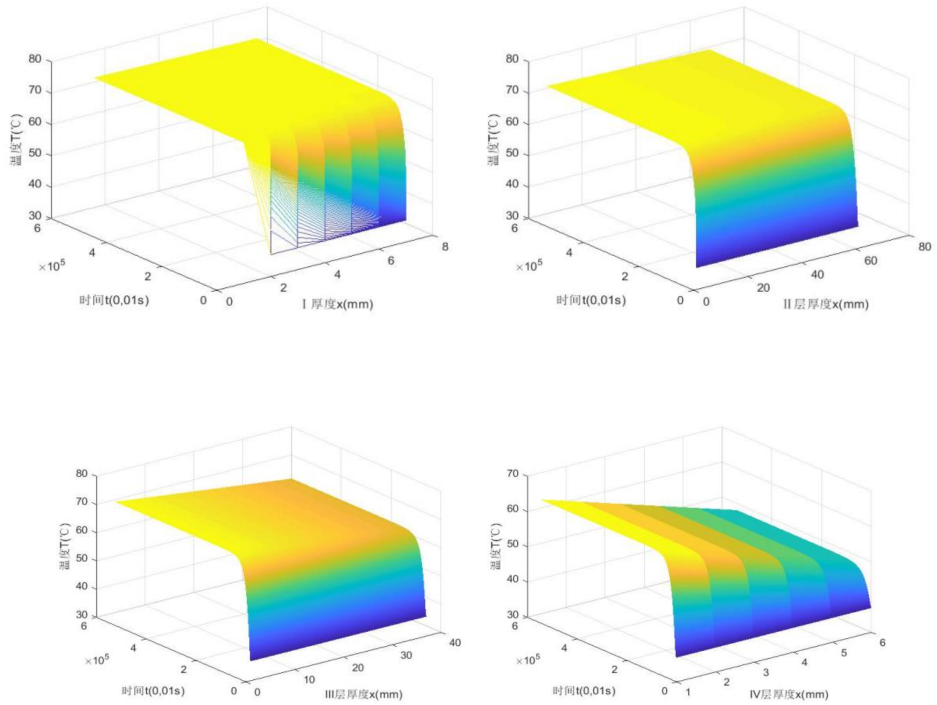
For the cylinder model, we need to assume the initial value of the parameter r. We use the average of the male and female shoulders with a height of 175 cm and a weight of 65 kg as the diameter of the cylinder, and round up to r=15 mm. The temperature of each interface of the flat and cylindrical models was obtained by using the MATLAB program as follows:

Table 5-1

Temperature/℃	T_1	T_2	T_3	T_4	T_5
Flat model	75.00	74.30	72.75	65.12	48.08
Cylindrical model	75.00	74.35	73.53	67.50	48.08

Unsteady process

For the time and space interval, in order to ensure that the temperature distribution of each layer converges and the time interval is consistent, the first three layers Δx are selected to be 0.1 mm, the air layer is selected to be 0.5 mm, and the time interval Δt is unified to 0.01 s. The program output gets the temperature distribution data at intervals of 0.01s, but in order to make the table data more intuitive, we set the interval 1s. Substituting the determined and known parameters into equations (5-10), the following four graphs can be obtained using the MATLAB program, which represent the temperature distribution of each layer.



The characteristics of layer-by-layer heating and delayed temperature rise can be clearly seen from the figure, which is consistent with the assumptions. In particular, we extracted the temperature of each layer interface at steady state from the excel table, as shown in Table 5-2.

Table 5-2

Temperature/℃	T_1	T_2	T_3	T_4	T_5
Difference model	75.00	74.30	72.75	65.12	48.08

Comparing this table with Table 5-1, it can be seen that the steady-state temperature obtained by the unsteady model is almost the same as the plate model (retaining four significant figures), and the correctness of the model can be seen.

References

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