The Correspondence between Graph and Its Adjacent Matrix

Xiaoqing Chi

College of Mathematics and Systems Science, Shandong University of Science and Technology, Shandong 266590, China

Abstract

A graph is completely determined by its adjacency and relevance, indicating that a graph can be represented by a matrix. Adjacency matrices and directed graphs are two different expressions of the same system structure. This paper studies the relationship between a graph and its adjacency matrix. There is a one-to-one correspondence between the matrix and the graph. The directed graph is determined, and the adjacency matrix is uniquely determined. Conversely, if the adjacency matrix is determined, the directed graph is uniquely determined.

Keywords

Directed graph; adjacency matrix; correspondence.

1. Introduction

Graph theory is a widely used branch of mathematics[1]. His pioneers should be attributed to the Swiss mathematician Euler. The matrix of the graph shows that a graph can be represented by a matrix, which gives an algebraic structure, so that algebraic techniques can be used to solve the graph problem, and it is beneficial to perform operations on a computer. In the 1970s, the matroid theory of graphs appeared. Its development plays an important role in promoting the study of graphs[2]. A graph is completely determined by its adjacency and relevance. This kind of information can be represented by a matrix. Commonly used are adjacency matrices. The adjacency matrix is a common storage structure of the graph. It has the characteristics of simple and intuitive description. In the graph operation, many algorithms are usually processed by using the adjacency matrix as the storage structure. The discussion of the adjacency matrix will undoubtedly give Graph theory and the study of many problems in real life bring convenience[3]. The adjacency matrix can also define the Laplacian matrix of the graph, the signless Laplacian matrix of the graph, and the reachable matrix of the directed graph. These matrices are good tools for studying graphs and have a very wide range of applications in practice[4].

This paper introduces the algorithm of the adjacency matrix of the graph and studies the correspondence between the graph and its adjacency matrix. In terms of structure, the second section reviews some basic concepts of the graph; the third section gives the definition of the adjacency matrix of the graph; the fourth section gives an example to prove the one-to-one correspondence between the graph and its adjacency matrix.

2. Diagram concept

The graph is a binary group $G = \langle V, E \rangle$, where V is the set of vertices and E is the set of edges. Each element in E can correspond to a binary unordered element pair of V. If e ∈ E, e corresponds to the two vertices u and v, then e is denoted as uv or (uv), while the vertices u and v are adjacent, also called u and e are interrelated. Edges associated with the same vertex are said to be contiguous. The two edges with the same endpoint are called rings. Two edges associated with the same pair of vertices and more than two edges are called heavy edges. A graph that does not contain heavy edges and rings is called a simple graph. The sequence formed by the edges formed by the vertices and adjacent vertex sequence pairs is called the path of the graph[5].

This paper mainly discusses the adjacency matrix of the graph.

3. Graph and adjacency matrix

Definition 1 Let G $= \langle V, E\rangle$ be a simple graph with n vertices, $V = \{v_1, v_2, \cdots, v_n\}$, then the n-order matrix (a_{ij}) is called the adjacency matrix of G. among them,

$$a_{ij}\begin{cases}1 & There\ is\ a\ relationship\ between\ v_i\ and\ v_j \\ 0 & There\ is\ no\ relationship\ or\ the\ same\ between\ v\ v_i\ and\ v_j\end{cases}$$

Let G be a directed graph as follows:

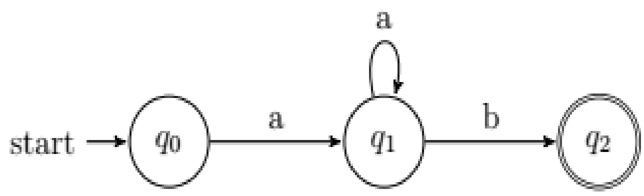

Fig. 1 Figure G

The adjacency matrix of Fig. 1 is:

$$A = \begin{bmatrix}0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0\end{bmatrix}$$

Vertex 1 has only a direct relationship with vertex 2, so the first row and second column elements are 1 and the rest are 0; the same vertex 2 has a direct relationship with itself and vertex three, so the second row and the third column element are 1 remaining. It is 0; vertex has no vertices directly related to it, so the elements are all 0, so that the above matrix A can be obtained. The adjacency matrix of the graph can be used to characterize the number of chains with a length k between any two points in the graph.

The adjacency matrix and the directed graph are two different expressions of the same system structure. In the case of graph isomorphism, there is a one-to-one correspondence between the matrix and the graph, and the directed graph is determined, and the adjacency matrix is uniquely determined. Conversely, if the adjacency matrix is determined, the directed graph is uniquely determined. If the element of the j column is all 0 in the adjacency matrix, the vertex corresponding to this column can be determined as the input point of the graph. As in the adjacency matrix A of Fig.1 above, the elements of the first column are all 0, and the vertex q_0 is the input point of the graph. In the adjacency matrix, if the elements of the i row are all 0, the corresponding vertex of the row can be determined as the output point of the graph. As in the adjacency matrix A of Fig.1 above, the elements of the third row are all zero, that is, the vertex q_2 can be determined as the output point of the graph.

4. Example description correspondence between graph and adjacency matrix

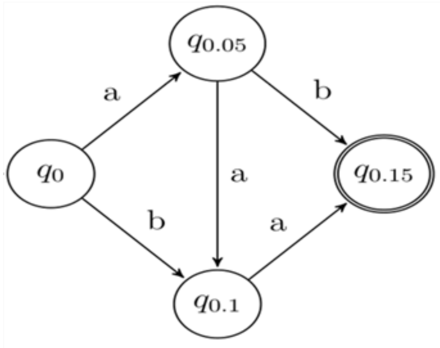

Fig. 2 state diagram

Where a indicates that the amount of the input banknote is five yuan, and b indicates that the amount of the input banknote is ten yuan.

The adjacency matrix of M_c is

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Obtain:

$$a_{11} = a_{11} \times a_{11} + a_{12} \times a_{21} + a_{13} \times a_{31} + a_{14} \times a_{41} = 0,$$
$$a_{12} = a_{11} \times a_{21} + a_{12} \times a_{22} + a_{13} \times a_{32} + a_{14} \times a_{42} = 0,$$
$$a_{13} = a_{11} \times a_{13} + a_{12} \times a_{23} + a_{13} \times a_{33} + a_{14} \times a_{43} = 1,$$
$$a_{14} = a_{11} \times a_{14} + a_{12} \times a_{24} + a_{13} \times a_{34} + a_{14} \times a_{44} = 2.$$

Analyze the a_{13} expression in the first line of A^2:

$$a_{11} = 0\text{，so } a_{11} \times a_{13} = 0;$$
$$a_{22} = 0\text{，so } a_{12} \times a_{22} = 0;$$
$$a_{13} = 1\text{，} a_{33} = 1\text{，so } a_{13} \times a_{33} = 1;$$
$$a_{14} = 0\text{，} a_{43} = 0\text{，so } a_{14} \times a_{43} = 0.$$

The elements in matrix A represent the number of paths that can be reached in one step between states. For example, one step from state q_0 can reach $q_{0.05}$ and $q_{0.1}$, and the number is all 1; one step can be reached from state $q_{0.05}$. $q_{0.1}$ and $q_{0.15}$, the number is also 1; from the state $q_{0.1}$ one step can reach $q_{0.15}$, the number is 1.

The elements in the matrix A^2 represent the number of paths that can be reached in two steps between states. For example, two steps from the state can reach q_0.1 and $q_{0.15}$, the numbers are 1 and 2 respectively; the slave state is $q_{0.05}$. Two steps can be reached to $q_{0.15}$, the number is 1, and the combination matrix A can be seen that q_0 to $q_{0.1}$ pass $q_{0.05}$, ie $q_0 \to q_{0.05} \to q_{0.1}$, from q_0 to$q_{0.15}$ respectively through $q_{0.05}$ And $q_{0.1}$, ie $q_0 \to q_{0.05} \to q_{0.15}$ and $q_0 \to q_{0.1} \to q_{0.15}$, from $q_{0.05}$ to $q_{0.15}$ via $q_{0.1}$, ie $q_{0.05} \to q_{0.1} \to q_{0.15}$.

The elements in the matrix A^3 represent the number of paths that can be reached in three steps between states. For example, three steps from the state q_0 can reach $q_{0.15}$, the number is, and the combination matrix A and A^2 look at q_0 to $q_{0.15}$ pass $q_{0.15}$ and $q_{0.1}$, ie $q_0 \rightarrow q_{0.05} \rightarrow q_{0.1} \rightarrow q_{0.15}$. Since q_0 is the initial state, $q_{0.15}$ is the termination state, so the path recognizable by the graph M_c is $q_0 \rightarrow q_{0.05} \rightarrow q_{0.15}$, $q_0 \rightarrow q_{0.1} \rightarrow q_{0.15}$ and $q_0 \rightarrow q_{0.05} \rightarrow q_{0.1} \rightarrow q_{0.15}$.

5. Conclusion

The adjacency matrix and the directed graph are two different expressions of the same system structure.The directed graph is determined and the adjacency matrix is uniquely determined. Conversely, if the adjacency matrix is determined, the directed graph is uniquely determined.It can be seen that there is a one-to-one correspondence between the directed graph and its adjacency matrix.

References

[1] Xu Junming. Graph Theory and Its Application[M]. University of Science and Technology of China Press, 2010.

[2] F. Harari. Graph Theory. Shanghai Science and Technology Press, 1976.

[5] Zhu Zhengyu, Zhu Qingsheng. Matrix Model Method for Finite Automata Research[J]. Computer Science, 2001, (04): 46-48.

[6] Qu Yuling. Ruan Suyun. Zhang Liang. Discrete Mathematics[M]. Higher Education Press, 2008.

[7] Hao Rongxia. Graph theory guidance[M]. Beijing Jiaotong University Press, 2014.