Strain softening function of composite solid propellant suitable for loading and unloading

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Abstract

Based on the uniaxial tensile test data of composite solid propellant, the method of calculating strain softening function is discussed in this paper. A more refined strain softening function model under unloading condition is given. The predicted results of the model are in good agreement with the experimental data.

Keywords

Strain softening function, , loading and unloading.

1. Introduction

The mechanical behavior of solid propellant is related to temperature and strain rate. Temperature and strain rate can influence the mechanical characteristic parameters of material, such as tensile modulus, yield stress, yield strain, maximum stress and fracture stress[1,2,3,4].

Swanson constitutive model could described the phenomenological constitutive relationship about the correlativity between the temperature and the strain rate of the solid propellant, it has been widely used in the solid motor engineering projections. The difficulty is that the strain softening function and change strain rate correction function in Swanson constitutive model are difficult to measure, which limits its application to some extent [5,6].

In this paper, the determination and calculation methods of strain softening function and strain rate correction function in Swanson constitutive model are discussed, and a more refined strain softening function model adapting to unloading conditions is given. The rationality of the strain softening function model is proved by comparing a set of loading and unloading test data.

2. The arithmetic of Strain softening function and change strain rate correction function

Swanson constitutive model can be written as the following equation

$$S'_{ij} = g(E)\int_0^t 2G_{rel}(t-\tau)\phi(\tau)\frac{\partial E'_{ij}}{\partial \tau}d\tau \tag{1}$$

The usual constant crosshead speed uniaxial tensile tests of solid propellant is fitted to the Eq.(1) so that the Cauchy stress is given by

$$\sigma_{11} = \frac{g(\varepsilon)(2\lambda_1^2+1/\lambda_1)}{3}\int_0^t G_{rel}(t-\tau)\phi(\tau)\frac{d}{d\tau}(\lambda_1^2-1/\lambda_1)d\tau \tag{2}$$

λ_1 is principal stretch ration in the Eq.(2). Taking the rate of change of ($\lambda_1^2 - 1/\lambda_1$) as being approximately constant and taking $\phi(\tau)$ as unity.

By Eq.(2)

$$\frac{\sigma_{11}}{g(\varepsilon)(2\lambda_1^2+1/\lambda_1)/3}=3\dot{\lambda}_1\int_0^t G_{rel}(t-\tau)d\tau \tag{3}$$

The strain softening function $g(\varepsilon)$ is determined for uniaxial tensile tests of solid propellants. The strain softening function can be expressed as a function of the second invariant $\sqrt{IIE'}$ of the strain deviation .

In order to calculate the strain softening function, the function ϕ is normalized to 1. This is possible at constant or zero strain rates, but the effect of function ϕ must be considered in the case of variable strain rates.

The function ϕ will depend on the difference between the current value of the components of the heredity integral and the value that would obtain if the current strain level had been reached by a strain rate and temperature constant at their current values. Thus defining

$$f_c(t)=\overline{R}_E\int_0^{\psi}G_{rel}(\psi-\tau)d\tau \tag{4}$$

Where $\psi=\overline{E}'/\overline{R}_E$, $\overline{R}_E$ is the equivalent constant strain rate.

Define

$$\phi(\tau)=1+\gamma[f_c(\tau)-f(\tau)] \tag{5}$$

Where $\gamma(0)=0$, the shear relaxation modulus is expressed by Prony series, convolution on t_n time is.

$$f(t_n)=\int_0^{t_n}G_{rel}(t_n-\tau)\frac{\partial E'}{\partial\tau}\phi d\tau=\sum_{i=1}^{m}I_{n,i} \tag{6}$$

Here

$$I_{n,i}=\int_0^{t_n}G_i\exp[-\alpha_i(t_n-\tau)]\frac{\partial E'}{\partial\tau}\phi_i(\tau)d\tau \tag{7}$$

Recursion can be written as

$$I_{n,i}=I_{n-1,i}\exp(-\alpha_i\Delta t_n)+p_{n,i}\phi_{n,i} \tag{8}$$

Define

$$p_{n,i}=\Delta E_n{}'G_i\frac{[1-\exp(-\alpha_i\Delta t_n)]}{\alpha_i} \tag{9}$$

$$\phi_{n,i}=1+\frac{(IC_{n-1,i}-I_{n-1,i})\exp(-\alpha_i\Delta t_n)\theta}{p_{n,i}} \tag{10}$$

Where θ is a constant change from 0 to 1 and $IC_{n,i}$ is a constant strain rate response

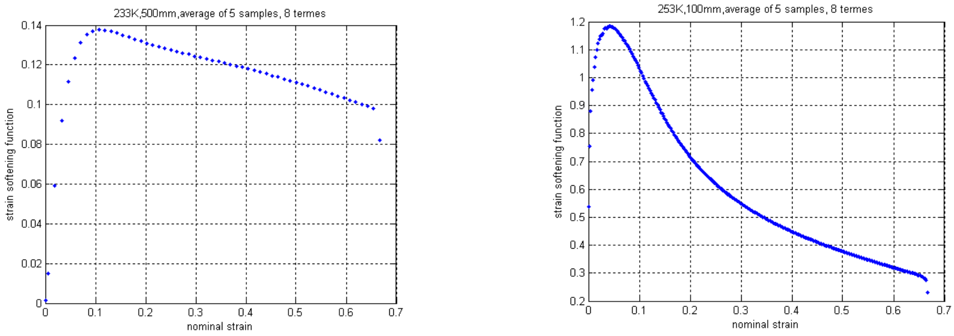
$$\begin{aligned}I_{n,i}&=\overline{R}_E\int_0^{\psi_n}G_i\exp[-\alpha_i(t_n-\tau)]d\tau\\&=\overline{R}_E G_i\frac{[1-\exp(-\alpha_i\psi_n)]}{\alpha_i}\end{aligned} \tag{11}$$

3. Experiments and analysis

In this paper, the uniaxial tensile experiments of composite solid propellants with the constant strain rate are carried out at different temperatures using different tensile rates. At the same time a set of loading/unloading tests have been done.

3.1 Strain softening function of composite solid propellant

According to the uniaxial tensile experiment data about composite solid propellant and the previously mentioned theory, the strain-softening functions with different temperature and tensile rate could be calculated. Fig.1 shows the function with the temperature of 233K, the tensile rate of 500mm/min and the temperature of 253K , the tensile rate of 100mm/min.



（a）233k, 500mm/min （b）253K, 100mm/min

Fig.1 Strain softening function with different temperature and tensile rate

3.2 Model prediction of loading and unloading for composite solid propellant

Fig.2 is the comparison between the calculated results of stress-strain response for loading and unloading of composite solid propellant with 293K, 2mm/min tensile rate and the experimental data. The agreement between the calculated results and the experimental data is better than Swanson's[6].

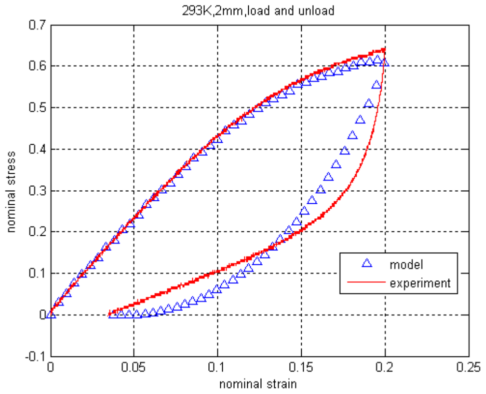


Fig.2 Model prediction results of stress-strain response for loading-unloading comparison with experimental data

A delicate model is adopted here for the strain softening function in the unloading experiment. The coefficient θ is set as zero in the unloading experiment, namely the second invariant of the strain partial factor $\sqrt{IIE'}$ is decreasing now, and the strain softening function g could be defined as the function of $\sqrt{IIE'}$. Most loading/unloading lagging in the model due to g . When $\sqrt{IIE'}$ is less than the maximum achieved in loading procedure ever, g would yield the different value too. The maximum of the second invariant of strain deviation is written as $\sqrt{IIE'_{\max}}$, and the function g could be defined as

$$g = g(\sqrt{IIE'_{\max}})\left\{1 - C[1 - \frac{\sqrt{IIE'}}{\sqrt{IIE'_{\max}}}]\right\} \tag{12}$$

In which

$$IIE' = -(e_1e_2 + e_2e_3 + e_3e_1) + \frac{1}{4}(\gamma_{12}^2 + \gamma_{23}^2 + \gamma_{31}^2) \tag{13}$$

For uniaxial tensile test

$$\varepsilon_1 = \varepsilon, \quad \varepsilon_2 = \varepsilon_3 = -u\varepsilon \tag{14}$$

Where u is Poisson's ratio, and the corresponding component of strain deviation is

$$e_1 = \frac{2}{3}\varepsilon(1+u), \quad e_2 = e_3 = -\frac{1}{3}\varepsilon(1+u) \tag{15}$$

The propellant is regarded as an approximate uncompressed object, and setting $u = \frac{1}{2}$, thus

$$IIE' = \frac{3}{4}\varepsilon^2 \tag{16}$$

4. Conclusion

The strain softening function model presented in this paper is modified considering its lagging effect during cyclic loading and unloading, which makes the model more delicate and agrees well with the loading and unloading test data. However, the damage effect is not taken into account in the model, which is also the next step to improve the model.

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