Calculation of Equivalent Tensile Stiffness of Sucker Rod Based on Equal Deformation Method

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Abstract

The tensile stiffness of sucker rod is one of important physical properties of sucker rod, which is of great significance for strength checking, failure analysis and dynamics research of sucker rod string. This paper presents a new method for calculating the tensile stiffness of sucker rods, i. e. the equal deformation method. The formula for calculating the axial tension deformation of the sucker rod is derived. Based on the principle of equal deformation, the formula for calculating the equivalent tension stiffness of the sucker rod is obtained, and the variation rule of the equivalent tension stiffness is obtained.

Keywords

Equal deformation; sucker rod; tensile; stiffness.

1. Introduction

Tensile stiffness of sucker rod is a key parameter in studying the dynamics of sucker rod string [1]. The methods to determine the tensile stiffness of sucker rod are usually test method and theoretical calculation method [2,3]. The theoretical calculation method is simple and effective, which is calculated by elastic modulus and cross-section area of materials. The test method is to make the object to be measured into the test label, and then carry out the actual calculation with the help of the tensile testing machine. The test method is accurate and intuitive, but it can not test the special-shaped structural parts and the cost is high. Therefore, considering comprehensively, theoretical calculation method is the best way to determine the tensile stiffness of sucker rod [4]. A transient system is discussed by Xu Chunhui The concept of equivalent tensile stiffness is introduced according to the result that the vertical load is proportional to the cubic of the dimensionless vertical displace- ment [5].

In calculating the stiffness of sucker rod, the influence of coupling and thread is not taken into account in the conventional calculation method. In practice, the axial force of sucker rod is larger, and the thread will produce certain deformation after the force, which can not be ignored. Therefore, considering the influence of coupling and thread, a calculation method of equivalent tensile stiffness of sucker rod based on the principle of equal deformation is proposed in this paper.

2. Axial tensile deformation of sucker rod

To ensure the accuracy and feasibility of calculation, the sucker rod is simplified. It is assumed that the material of the sucker rod is the same and the density is uniform; the sucker rod body and the coupling are cylindrical. When calculating, the X direction is downward positive, the top position of the sucker rod body is the origin of the coordinate system, L_0 is the length of the sucker rod body, L_1 is the total length of the coupling, L_2 is the length of the thread end; A_0 is the cross-section area of the sucker rod body, A_1 is the cross-section area of the coupling, A_2 is the projection area of the thread connection in the axial direction; the sucker rod is affected by gravity and external force F. As shown in Fig 1.

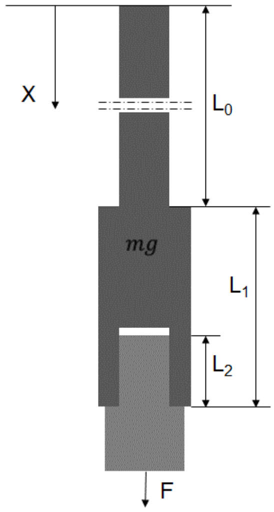

Fig 1. Simplified schematic diagram of sucker rod

Basic equation of tensile stiffness:

$$K = EA \tag{1}$$

Where E is modulus of elasticity of materials; A is section area of parts.

The axial tension deformation of sucker rod can be expressed by line integral as follows:

$$\varepsilon = \int_l \frac{F}{k} dx \tag{2}$$

The axial tension deformation of sucker rod body, coupling and thread under gravity and external load can be calculated by Eqs (2). The deformation of sucker rod body can be expressed as:

$$\varepsilon_1 = \frac{F + mg}{K_0} L_0 - \frac{\rho g A_0}{2K_0} L_0^2 \tag{3}$$

m is the sucker rod quality. The deformation of sucker rod coupling can be expressed as:

$$\varepsilon_2 = \frac{F + mg - \rho A_0 L_0 g + \rho A_1 L_0 g}{K_1} \left(L_1 - L_2\right) - \frac{\rho A_1 g \left[\left(L_0 + L_1 - L_2\right)^2 - L_0^2\right]}{2K_1} \tag{4}$$

Thread deformation of sucker rod can be expressed as:

$$\varepsilon_3 = \frac{F + mg - \rho A_0 L_0 g - \rho A_1 g \left(L_1 - L_2\right) - \rho g A_1 \left(L_2 - L_0 - L_1\right)}{K_2} L_2 - \frac{\rho A_1 g \left[\left(L_0 + L_1\right)^2 - \left(L_0 + L_1 - L_2\right)^2\right]}{2K_2} \tag{5}$$

Total deformation of sucker rod:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{6}$$

3. Equivalent stiffness calculation of sucker rod based on equal deformation method

Equivalent tensile stiffness:

$$K=\frac{\int_0^L F+\rho gA\left(L-x\right)dx}{\varepsilon} \tag{7}$$

Where, A is the cross-sectional area of the sucker rod. Since the actual sucker rod consists of rod body and coupling, the cross-sectional area A of the sucker rod is a variable related to coordinate X. When calculating equivalent stiffness, in order to simplify the calculation, it is necessary to simplify the calculation model. Reference Eqs (7) simplifies the molecule and the sucker rod cross-section area A is simplified to the average cross-section area of the sucker rod $\bar{A}$.

$$\bar{A}=\frac{A_0L_0}{L_0+L_1}+\frac{A_1L_1}{L_0+L_1} \tag{8}$$

From Eqs (11) and (12), the formula for calculating the equivalent tensile stiffness of sucker rods can be obtained by integral.

$$K=\frac{\left(F+mg\right)\left(L_0+L_1\right)-\frac{1}{2}\rho g\bar{A}\left(L_0+L_1\right)^2}{\varepsilon} \tag{9}$$

$$\begin{aligned}\varepsilon&=\varepsilon_1+\varepsilon_2+\varepsilon_3\\&=\frac{F+mg}{K_0}L_0-\frac{\rho gA_0}{2K_0}L_0^2+\frac{F+mg-\rho A_0L_0g+\rho A_1L_0g}{K_1}\left(L_1-L_2\right)\\&-\frac{\rho A_1g}{2K_1}\left[\left(L_0+L_1-L_2\right)^2-L_0^2\right]\\&+\frac{F+mg-\rho A_0L_0g-\rho A_1g\left(L_1-L_2\right)-\rho A_1g\left(L_2-L_0-L_1\right)}{K_2}L_2\\&-\frac{\rho A_1g}{2K_2}\left[\left(L_0+L_1\right)^2-\left(L_0+L_1-L_2\right)^2\right]\end{aligned} \tag{10}$$

4. Sensitivity analysis of equivalent stiffness of sucker rod

Considering self-weight, combining Eqs (9) and (10), the equivalent stiffness of sucker rod is related to the axial external force, the length of sucker rod body, the length of coupling, the length of thread section, the elastic modulus of sucker rod material, the density of sucker rod material and the cross-sectional area of each section of sucker rod.

When the sucker rod gravity is not taken into account, the theoretical formulas of equivalent stiffness of sucker rod can be simplified by combining Eqs (9) and (10).

$$\begin{aligned}K&=\frac{F\left(L_0+L_1\right)}{\frac{FL_0}{K_0}+\frac{F\left(L_1-L_2\right)}{K_1}+\frac{FL_2}{K_2}}\\&=\frac{\left(L_0+L_1\right)K_0K_1K_2}{L_0K_1K_2+\left(L_1-L_2\right)K_0K_2+L_2K_0K_1}\end{aligned} \tag{11}$$

Eqs (11) shows that the equivalent stiffness of the sucker rod is independent of the axial external force acting on the sucker rod without considering the sucker rod gravity. The factors affecting the equivalent stiffness of the sucker rod are the material elastic modulus of the sucker rod, the length of the sucker rod body, the length of the coupling and the cross-sectional area of each section of the sucker rod.

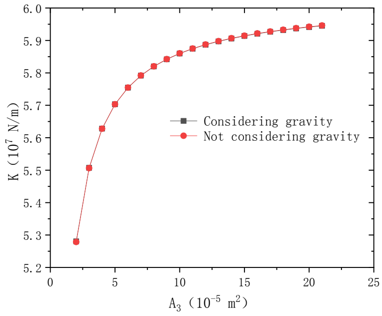

Fig 2. Relation curve between equivalent tensile stiffness and projected area of thread connection in the axis of sucker rod

Considering the rod gravity and not considering the rod gravity, the equivalent tensile stiffness considering the rod gravity is less than the equivalent tensile stiffness without the rod gravity, and the difference is about 0.013%, which can be neglected. With the increase of projection area of threaded connection in the axial direction of sucker rod, the equivalent tensile stiffness increases gradually, the curve becomes smooth gradually, and the increase rate decreases gradually, with an increase of 12.6%.

5. Conclusion

(1) Equivalent tensile stiffness is proportional to the elastic modulus of sucker rod material. The bigger the elastic modulus is, the bigger the equivalent tensile stiffness is.

(2) Equivalent tensile stiffness is more sensitive to the projection area of threaded connection in the axial direction of sucker rod, and the influence range is 12.6% in a certain working condition. When calculating the equivalent tensile stiffness of sucker rod, the influence of thread connection can not be neglected. The influence of thread should be taken into account in calculating the equivalent tensile stiffness of sucker rod.

References

[1] Sztefek P, Olsson R. Tensile stiffness distribution in impacted composite laminates determined by an inverse method[J]. Composites Part A Applied Science & Manufacturing, 2008, 39(8):1282-1293.

[2] Volino P, Magnenat-Thalmann N, Faure F. A simple approach to nonlinear tensile stiffness for accurate cloth simulation[J]. Acm Transactions on Graphics, 2009, 28(4):1-16.

[3] Su S L. Finite Element Calculation for the Tensile Elastic Modulus of Honeycomb Materials with Different Pore Shape[J]. Applied Mechanics & Materials, 2016, 847:141-145.

[4] Weidmann G W, Ogorkiewicz R M. Time-dependence of the tensile stiffness and anisotropy of a reinforced thermoplastic[J]. Journal of Materials Science, 1974, 9(2):193-200.

[5] Chunhui X U , Linnan Z , Taiyan Q , et al. DISCUSSION OF A TRANSIENT SYSTEM[J]. Mechanics in Engineering, 2014, 36(2):212-215.