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# Design and Simulation of Intelligent Trolleybus Path Tracking Controller Based on Preview Control

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## Abstract

Intelligent trolleybus is a complex dynamic system, this paper establishes a nonlinear seven-degree-of-freedom trolleybus model, it includes longitudinal, lateral, yaw motion and the force status of the four tires. Taking the trolleybus model as the object, its motion characteristics and path tracking are studied. This paper chooses a path tracking method similar to human driving behavior: preview control, the widely used and robust PID controller is used as the controller for the front wheel steering of the trolleybus. Finally, in MATLAB, the serpentine line is given as the desired path, and the path tracking of the trolleybus is simulated. The simulation results show that the trolleybus can track the serpentine line quickly, accurately and steadily.

## Keywords

Intelligent trolleybus, Path tracking, PID controller, Preview control.

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## 1. Introduction

Driverless driving is an important trend in the development of intelligent vehicles, a large number of domestic and foreign vehicle manufacturers and Internet technology companies jointly develop driverless technology, such as Baidu, Google, Tesla, BMW and so on. China Baidu won the first batch of automatic driving road test number plates in Beijing, the US Google driverless car has successfully traveled more than 800,000 kilometers and has zero accidents.[1,2]

The subject of this paper is the design of intelligent trolleybus path tracking controller. In order to study the sport characteristics of trolleybus, this paper establishes a seven-degree-of-freedom dynamic model including the DUGOFF tire model and adopts a path tracking algorithm similar to people's driving style. Establish a trackless track path tracking model in MATLAB simulation environment, and give a typical experiment to simulate the path tracking effect of trolleybus. The simulation results show that the trolleybus can track the desired path quickly, accurately and steadily within the allowable error range.[3,4]

## 2. Intelligent Trolleybus Mathematical Modeling

### 2.1 Intelligent trolleybus dynamics modeling

For vehicles traveling at low speeds, a kinematic model can be used to analyze the motion of the vehicle. When the vehicle is driving at medium and high speeds, it is not enough to study the motion of the vehicle only through the kinematic model. It is also necessary to analyze the essence of the vehicle movement - force. In order to better study the motion characteristics of vehicles, this paper

establishes a nonlinear vehicle dynamics model in the local coordinate system. As shown in Figure 1.

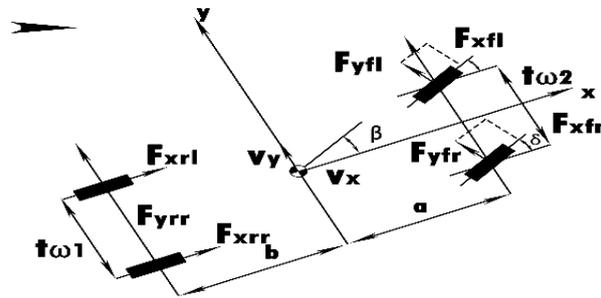


Fig. 1 Vehicle dynamics model

In order to simplify the model, the up and down displacement of the vehicle along the z-axis direction of the local coordinate system, the roll about the x-axis rotation, and the pitch of the y-axis rotation are ignored. The model includes three degrees of freedom of longitudinal, lateral and yaw motion of the vehicle. According to Figure 2.1, the yaw motion balance equation is

$$I_z \cdot \dot{r} = [(F_{xfl} + F_{xfr}) \sin \delta + (F_{yfl} + F_{yfr}) \cos \delta] a + [(F_{xfr} - F_{xfl}) \cos \delta + (F_{yfl} - F_{yfr}) \sin \delta] \frac{t\omega_1}{2} + (F_{xrr} - F_{xrl}) \frac{t\omega_2}{2} - (F_{yrl} - F_{yrr}) b \tag{1}$$

Considering that the motion of the vehicle is ultimately based on the force of the four tires of the vehicle, the force characteristics of the vehicle tire are studied to establish a Dugoff nonlinear tire model:[5,6]

$$F_{xij} = \frac{C_{xij} S_{xij}}{1 - S_{xij}} f(\lambda_{ij}) \tag{2}$$

$$F_{yij} = \frac{C_{yij} \tan \alpha_{xij}}{1 - S_{xij}} f(\lambda_{ij}) \tag{3}$$

$$f(\lambda_{ij}) = \begin{cases} \lambda_{ij} (2 - \lambda_{ij}) & \text{for } \lambda_{ij} < 1 \\ 1 & \text{for } \lambda_{ij} \geq 1 \end{cases} \tag{4}$$

$$\lambda_{ij} = \frac{\mu F_{zij} (1 - S_{xij})}{2 \sqrt{C_{xij}^2 S_{xij}^2 + C_{yij}^2 \tan^2 \alpha_{xij}}} \tag{5}$$

$$\mu = \mu_0 \left( 1 - A_s V_{t-ij} \sqrt{S_{xij}^2 + \tan^2 \alpha_{xij}} \right) \tag{6}$$

### 2.2 Intelligent Trolleybus Kinematics Modeling

Create a global coordinate system and a local coordinate system as shown.

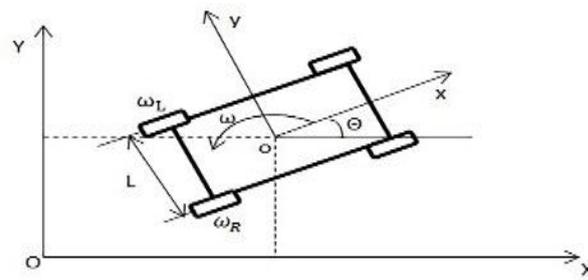


Fig.2 Local coordinate system and global coordinate system

The vehicle motion state in the global coordinate system can be described as

$$\zeta_g = \begin{bmatrix} \dot{X}_c \\ \dot{Y}_c \\ \dot{\theta}_c \end{bmatrix} \tag{7}$$

The motion state in the local coordinate system can be described as

$$\zeta_l = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} \tag{8}$$

Converting the motion state in the local coordinate system to the motion state in the global coordinate system through the transformation matrix.

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Then there is

$$\zeta_R = R(\theta) \cdot \zeta_l \tag{10}$$

The same goes:

$$\zeta_l = R(\theta)^{-1} \cdot \zeta_R \tag{11}$$

After mapping and transformation, the motion state vector in the global coordinate system is obtained. Through the kinematic equations (12) to (14), it can be seen that the vehicle is displaced along the X-axis, the Y-axis, and the yaw angle in the global coordinate system, that is, the posture of each moment in the vehicle motion, connect the vehicle position coordinates at each moment to get the actual movement track of the vehicle.

$$X = \dot{X}_c t \tag{12}$$

$$Y = \dot{Y}_c t \tag{13}$$

$$\theta = \omega t \tag{14}$$

### 2.3 Vehicle Pose Error Model

Let the pose of the desired point be

$$P_d = [X_d \ Y_d \ \theta_d]^T \tag{15}$$

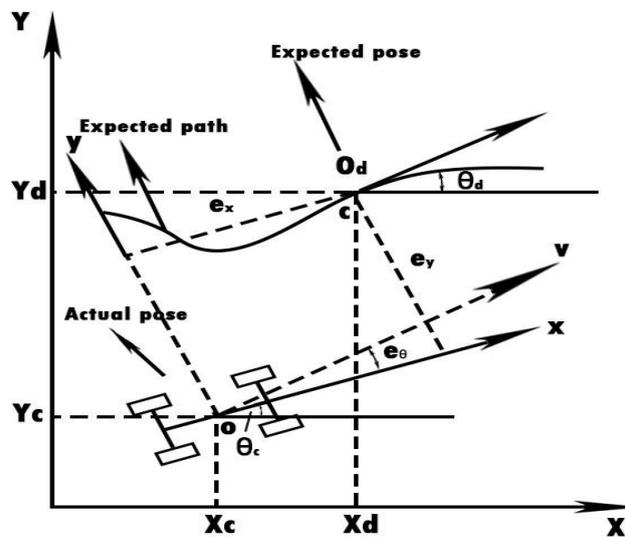


Fig.3 Vehicle pose error model

As shown in Figure 3, the relative pose error of the vehicle and the desired point in the global coordinate system is:

$$e = [X_d - X_c \ Y_d - Y_c \ \theta_d - \theta_c] \tag{16}$$

Through the transformation matrix, the pose error in the local coordinate system can be obtained:

$$e_p = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_d - X_c \\ Y_d - Y_c \\ \theta_d - \theta_c \end{bmatrix} \tag{17}$$

## 3. Intelligent Trolleybus Path Tracking Algorithm

### 3.1 Preview Distance and Preview Point

This paper uses the pre-shoot control method to achieve vehicle tracking. Imitate the range of the eyes when people drive the vehicle. When the vehicle travels at a higher speed, the preview distance is larger, and when the vehicle travels at a lower speed, the preview distance should be reduced. According to the above-mentioned idea of determining the preview distance, the formula for calculating the preview distance is given:

$$L_a = \begin{cases} L_{\min}, & |v| < |v|_{change} \\ K_v |v|, & |v| \geq |v|_{change} \end{cases} \quad (18)$$

In the above formula,  $L_{\min}$  represents the minimum pre-sight distance of the intelligent trolleybus, which is determined by the minimum turning radius. This paper takes 4m. The product of  $K_v$  and speed is the pre-attack distance in the case of faster vehicle speed, where  $K_v$  is the gain of the preview distance, and its size directly determines the extent to which the pre-attack distance is affected by the speed of the vehicle. The larger the  $K_v$ , the greater the influence of the preview distance by the speed. According to the parameters of the trolleybus,  $K_v$  is taken as 1. The relationship between the speed and the length of the preview distance is a fold line, as shown in Figure 4.

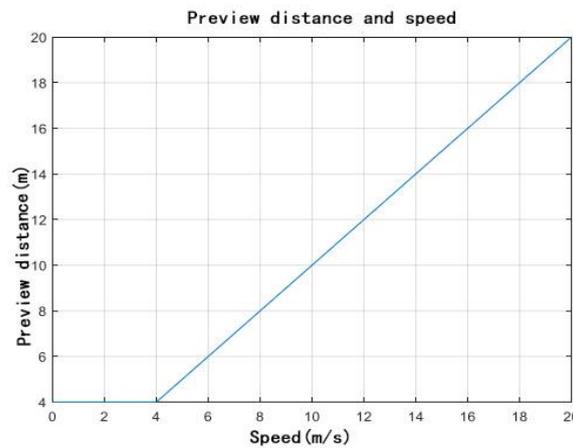


Fig. 4 Preview distance function image about speed

The process of determining the preview point is shown in Figure 5. After the vehicle photographs the road image through the camera and processes the image, the information of the desired path ahead is acquired. According to the algorithm of the preview distance, the preview distance at the current speed is obtained, and finally the preview point is obtained from the desired path  $O_d$ .

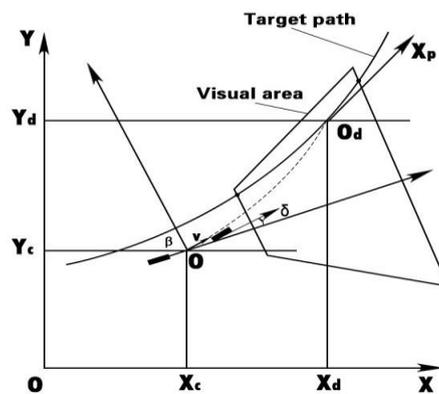


Fig. 5 Determination of preview point

### 3.2 Path Planning Curve

After the pre-pick point is obtained, a virtual optimal path is planned in real time from the vehicle centroid to the pre-pick point. In the planning process, considering the continuity of the vehicle pose change and the pose of the preview point, the path is planned as a cubic curve. As shown in Figure 6.

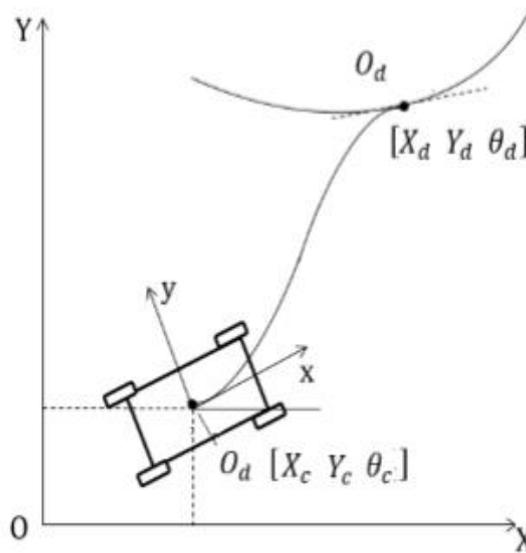


Fig.6 Path of vehicle centroid to preview point

The function expression of the planned path set in the local coordinate system is:

$$y(x) = A + Bx + Cx^2 + Dx^3 \tag{19}$$

The function expression for the planned path is:

$$y(x) = \frac{\omega}{2v} x^2 + \frac{y_e - \frac{\omega}{2v} x_e^2}{x_e^3} x^3 \tag{20}$$

Get the desired yaw rate acceleration is:

$$\dot{\omega}_d = \dot{v}\rho + \frac{v^2[\ddot{y}(1 + \dot{y}^2) - 3\dot{y}\ddot{y}^2]}{[(1 + \dot{y}^2)]^3} \tag{21}$$

The expected yaw rate acceleration of the vehicle in the current pose and motion state is:

$$\dot{\omega}_d = \frac{\dot{v}\omega}{v} + 6v^2 \frac{y_e - \frac{\omega}{2v} x_e^2}{x_e^3} \tag{22}$$

The difference between the desired yaw rate acceleration and the actual yaw rate acceleration calculated by the above equation will be used as the input to the controller. The controller outputs a

control amount so that the vehicle travels as close as possible to the desired yaw angular acceleration to achieve the purpose of path tracking.

### 3.3 Expectation Path Selection

In order to verify the effectiveness of the intelligent trolleybus path tracking algorithm and controller, this paper chooses to track the serpentine line. The model of the serpentine line and its parameters are shown in Figure 7.

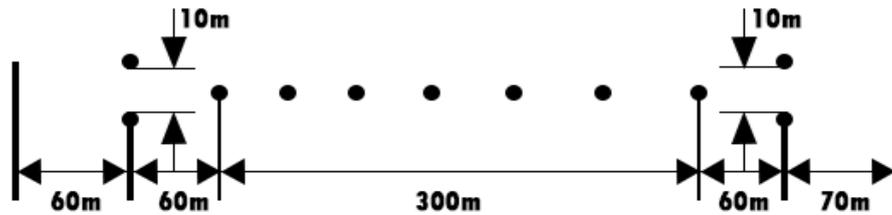


Fig.7 The road model of the snake line

Considering that the trolley and the yaw rate do not change suddenly during the actual running of the trolley, it is necessary to round the corner of the serpentine line. The serpentine line after rounding is shown in Figure 8.

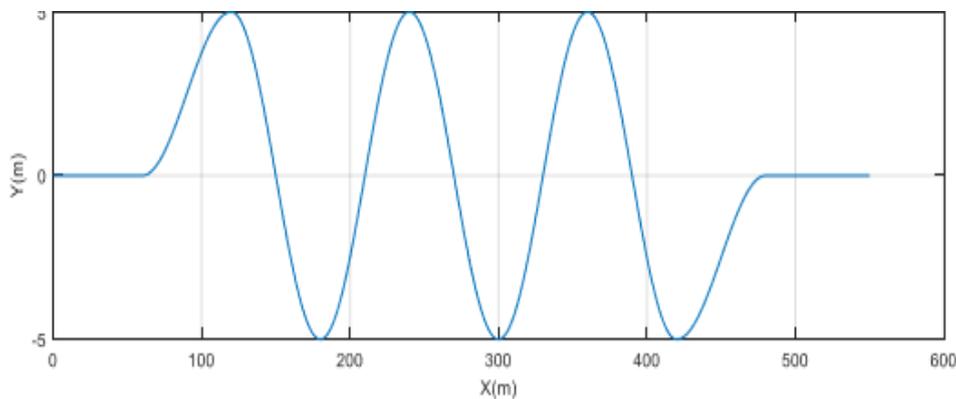


Fig.8 Road model map of snake shaped line after rounded treatment

The function expression of the serpentine line is :

$$f(x) = \begin{cases} 0 & x \in [0, 40) \\ \frac{1.25 \left( 1.0 + \sin \left( \frac{\pi \left( \frac{x}{20} + 1 \right)}{2} \right) \right)}{2.0} & x \in [40, 80) \\ 1.25 \cos \left( \frac{\pi x}{20} \right) & x \in [80, 280) \\ \frac{1.25 \left( -1.0 + \sin \left( \frac{\pi \left( \frac{x}{20} + 1 \right)}{2} \right) \right)}{2.0} & x \in [280, 320) \\ 0 & x \in [320, 400] \end{cases} \quad (23)$$

### 4. PID Controller Design

For the design of the PID controller for intelligent trolleybus path tracking, The input of the controller is the error between the desired yaw rate and the actual yaw rate, and the output is the front wheel angle. Selecting a PID controller with reasonable parameters can make the control error tend to zero, and make the actual yaw rate as close as possible to the desired yaw rate to achieve the purpose of path tracking. Tuning the parameters of the PID controller with a sinusoidal signal. First use a proportional controller with a sinusoidal signal as input, set the proportional parameter  $K_p$  to 10 to control the output of the system. The simulation results are shown in Figure 9.

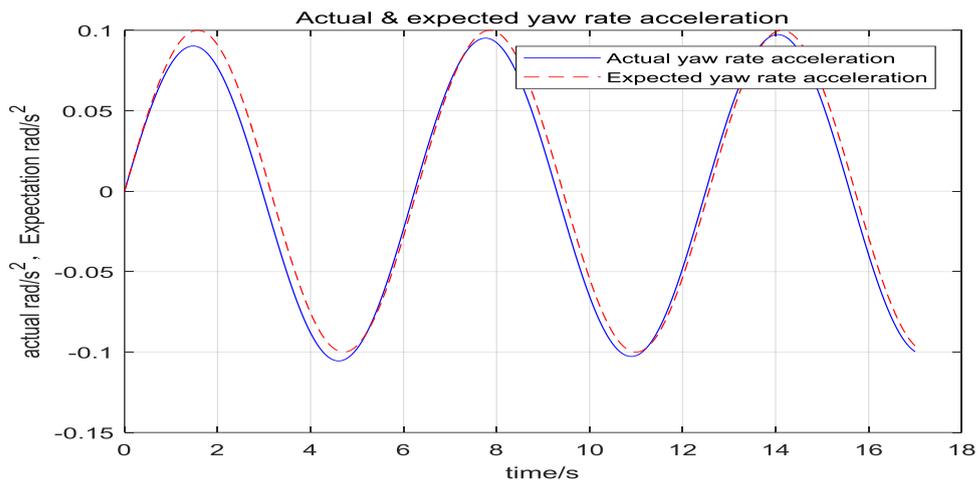


Fig.9 Control effect of P controller

As can be seen from Figure 10, the output signal always leads the input signal and cannot reach the desired amplitude, but the response speed of the system is good. Therefore, the output of the system is controlled by PI control, wherein the proportional parameter  $K_p$  is set to 28, the integral parameter  $K_i$  is 10, and the simulation result is shown in Fig. 10.

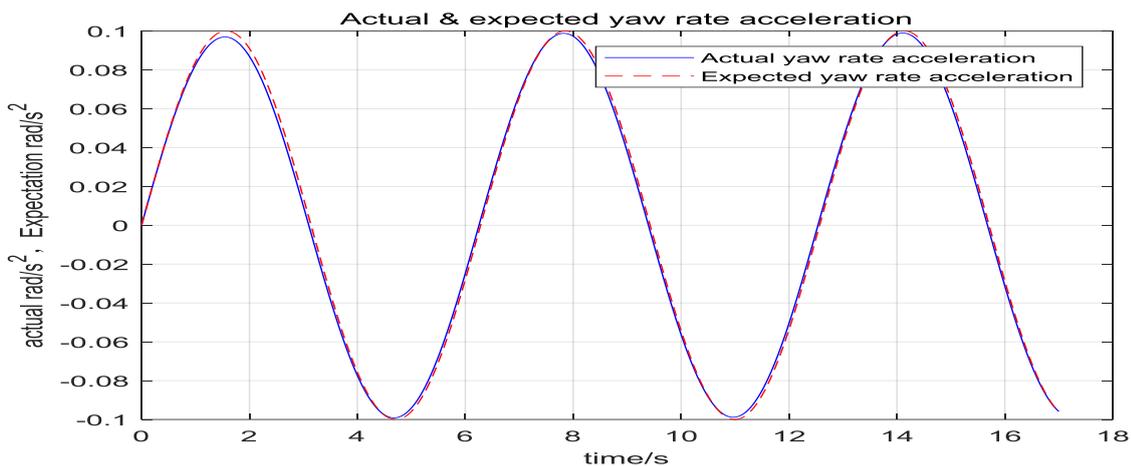


Fig.10 Control effect of PI controller

As can be seen from Figure 10, the output signal and the input signal are basically coincident, indicating that the controller can achieve the desired control effect, so the PI controller is selected as the controller for the front wheel steering.

### 5. Simulation Experiment of Intelligent Trolleybus Path Tracking

#### 5.1 Simulation of Path Tracking Control Algorithm

In the selection of parameters such as length, mass and moment of inertia of intelligent trolleybus, this paper refers to the parameters of ordinary trams. The specific parameters are shown in Table 1.

Table 1 Part of the system parameters

parameter	Numerical value
Vehicle quality m/kg	6300
Moment of inertia of the whole vehicle around the z-axis Iz/(kg.m <sup>2</sup> )	1350
Centroid to front and rear axle distance (a,b)/m	3.5,3.5
Distance between front and rear wheels(tw1,tw2)/m	2.55
Front and rear wheel corner stiffness Cayif/(N·rad-1)	23164,38345
Front and rear wheel vertical stiffness Cs/(KN·m-1)	43164,58345
Centroid height h/m	1.5
Tire rolling radius R/m	0.44
Expected speed m/s	10

Establish a trolleybus dynamics model in Simulink as shown in Figure 11, enter the front wheel steering angle and the desired longitudinal speed, outputs the longitudinal, lateral and yaw motion components of the trolleybus.

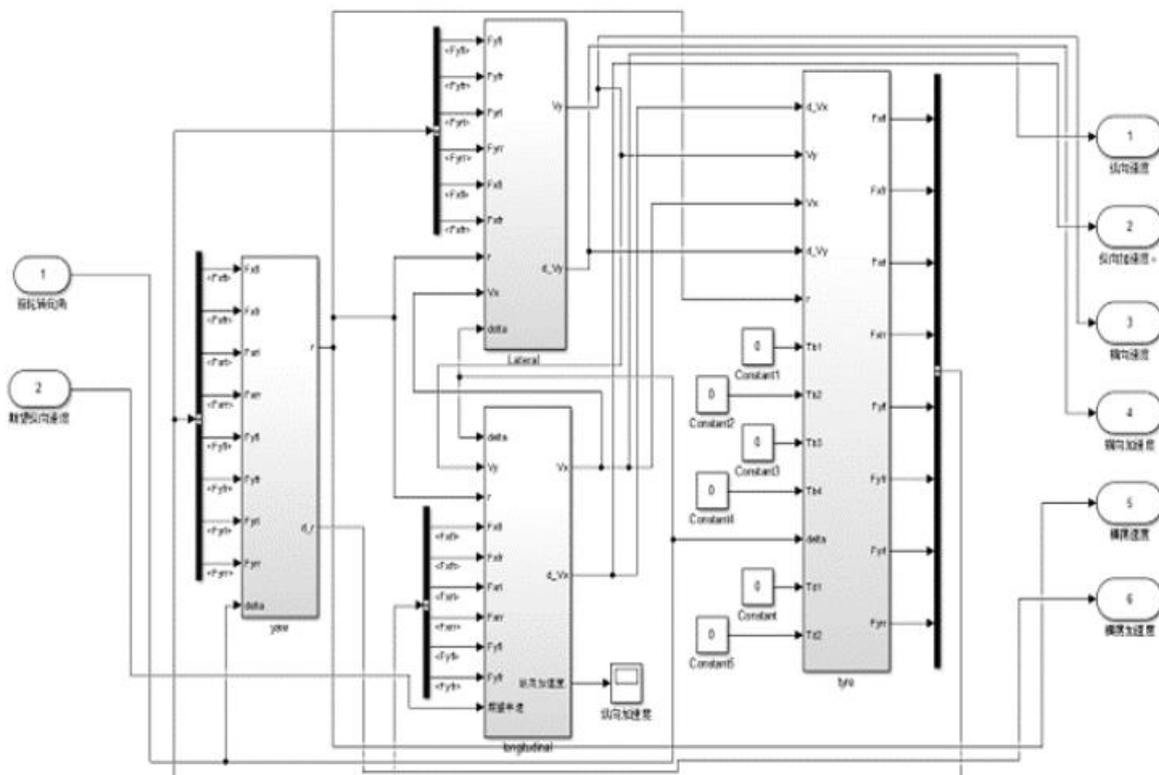


Fig.11 Trolley vehicle dynamics model

The Dugoff nonlinear tire model is built as shown in Figure 12. Input  $\delta$ ,  $v_x$ ,  $v_y$ ,  $a_x$ ,  $a_y$ ,  $\omega$ , and output the longitudinal and lateral forces of each tire.

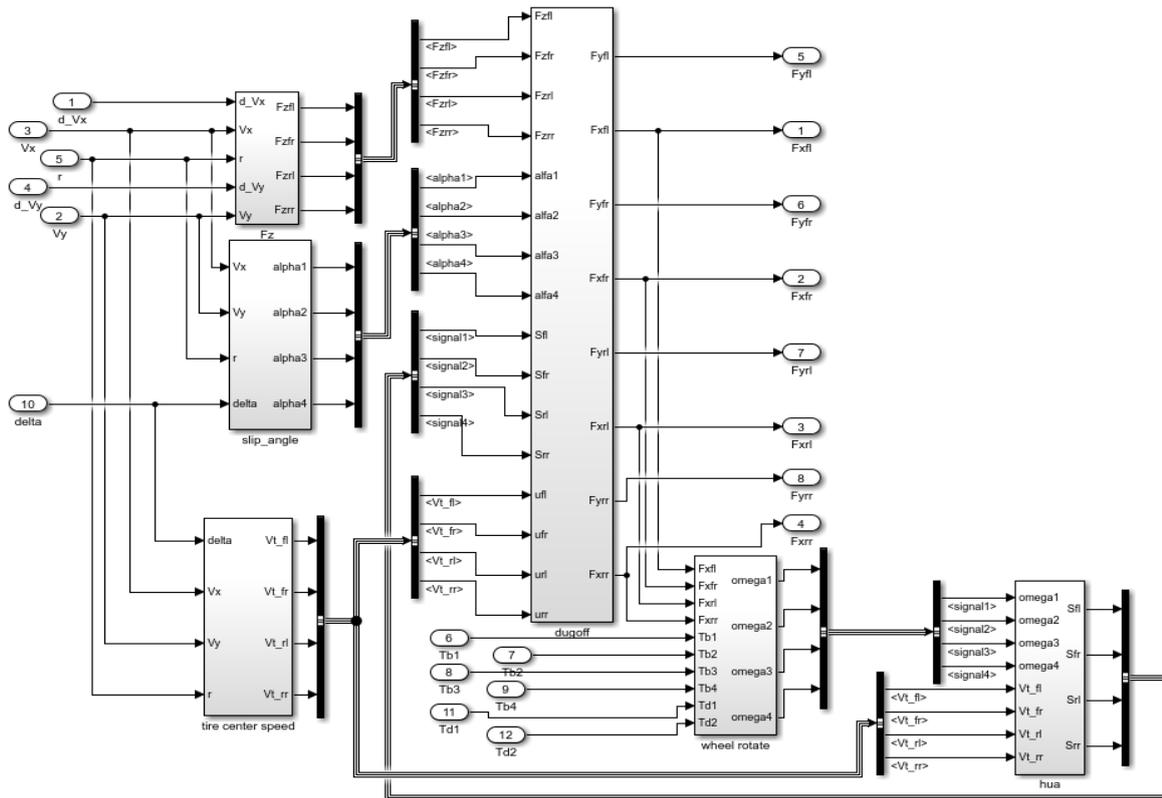


Fig.12 Dugoff nonlinear tire model

In the above figure, the Fz module is the vertical load model of each tire; the slip angle module is the lateral slip ratio model of each tire; the tire center speed module is the wheel wheel speed model; the wheel rotate module is the rotational angular velocity model of each wheel; the hua model is each tire longitudinal slip rate; the dugoff module is the longitudinal and lateral force model of each tire.

The trolleybus kinematics model is shown in Figure 13. Enter the vertical, horizontal and yaw speeds in the local coordinate system, and output the speed, yaw rate and pose in the global coordinate system.

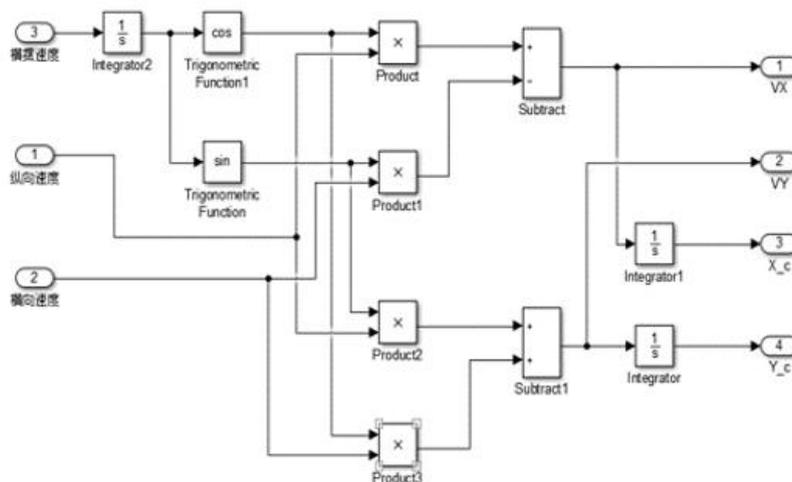


Fig.13 Trolley vehicle kinematics model

The desired yaw rate acceleration generator model is shown in Figure 14. Enter the speed and yaw rate in the global coordinate system and output the desired yaw acceleration.

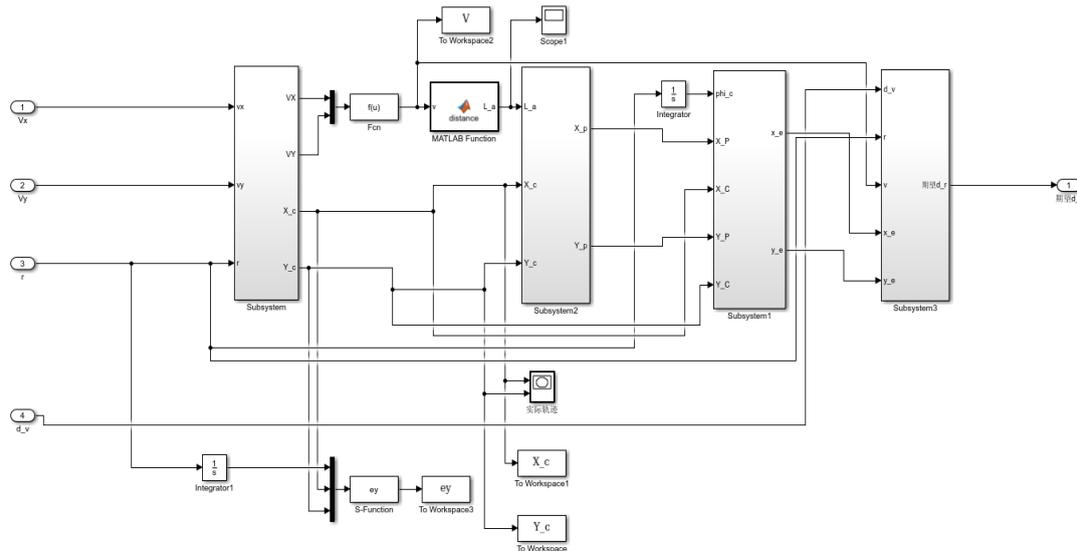


Fig.14 Model of expected yaw acceleration producer

According to the overall modeling schematic, the overall model of the trolleybus path tracking in Simulink is shown in Figure 15. The input amount of the overall model is the front wheel angle, and the information such as the motion posture of the trolley bus is output. Transfer data such as coordinates, actual speed and yaw rate of the vehicle in the global coordinate system to the work area at any time, then use the M file to write the drawing program, and you can get the simulation result graph.

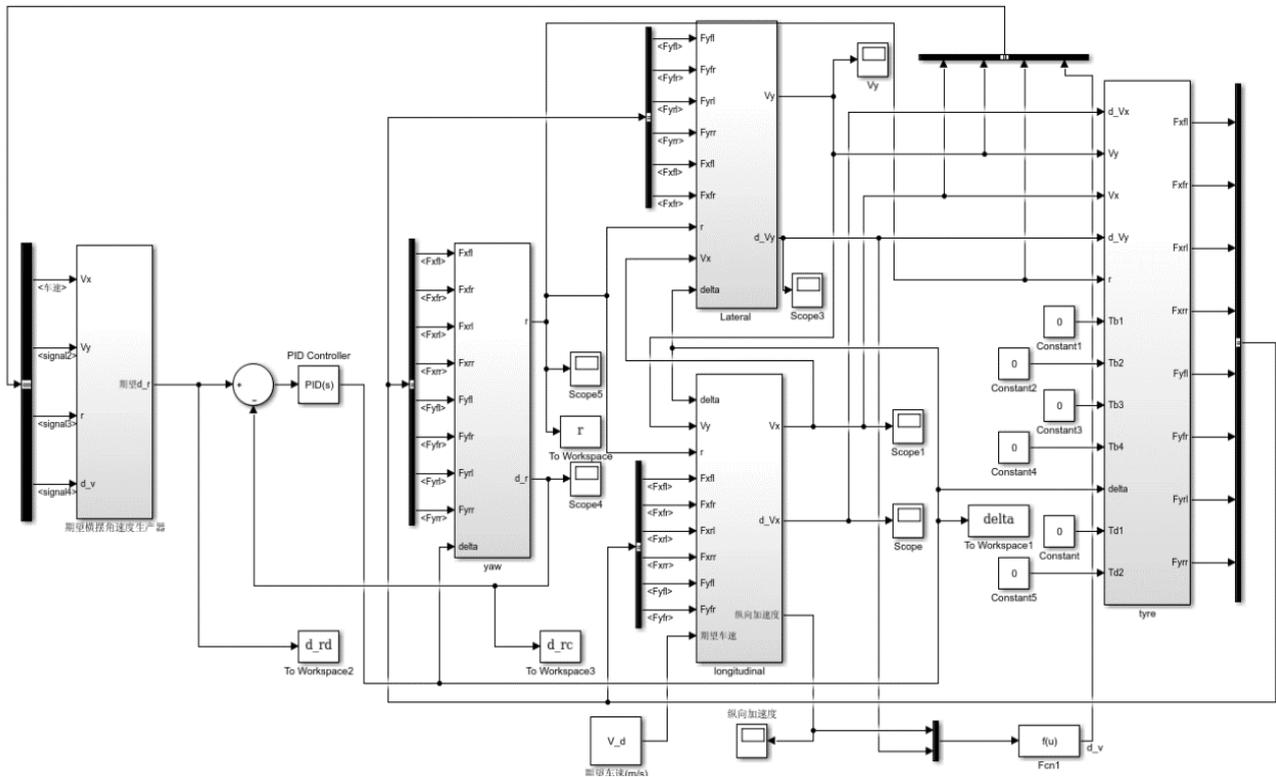


Fig.15 The overall structure diagram of the vehicle path tracking model

### 5.2 Simulation Results and Analysis

The expected and actual yaw rate acceleration curve when tracking the serpentine line, as shown in Figure 16.

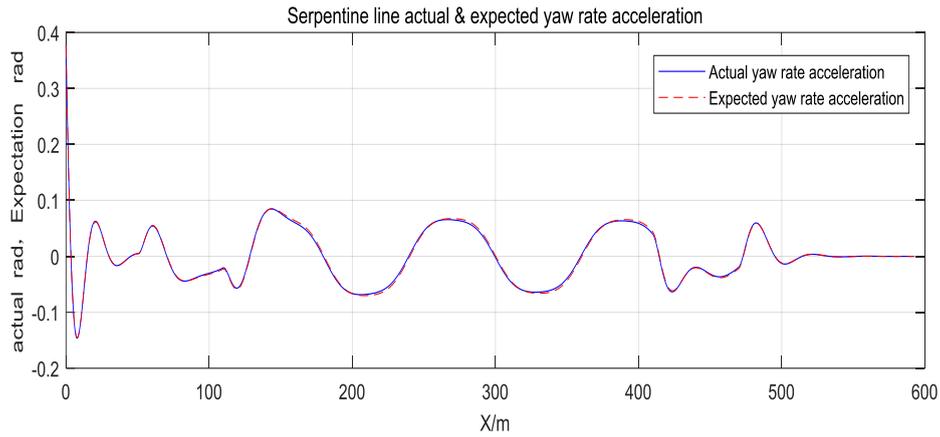


Fig.16 Expected and actual yaw rate acceleration curve - serpentine line

The simulation result of the serpentine line is shown in Figure 17.

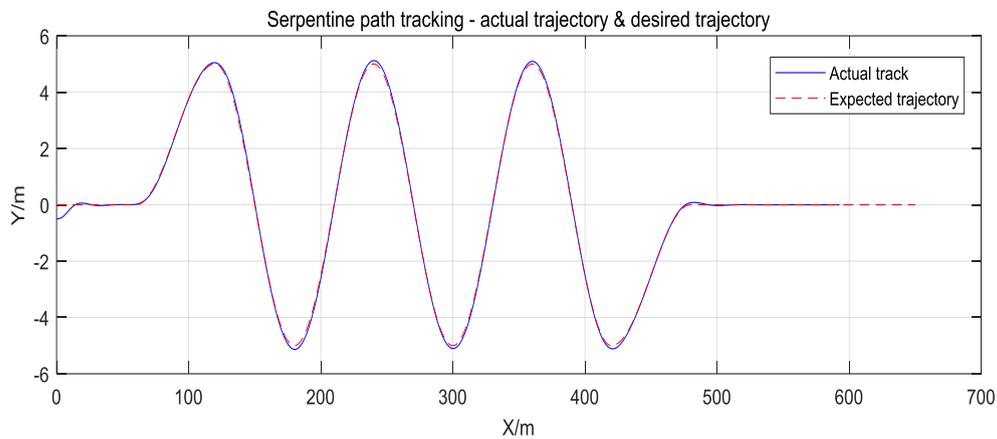


Fig.17 Serpentine path tracking simulation results

The serpentine line tracks the lateral error as shown in Figure 18.

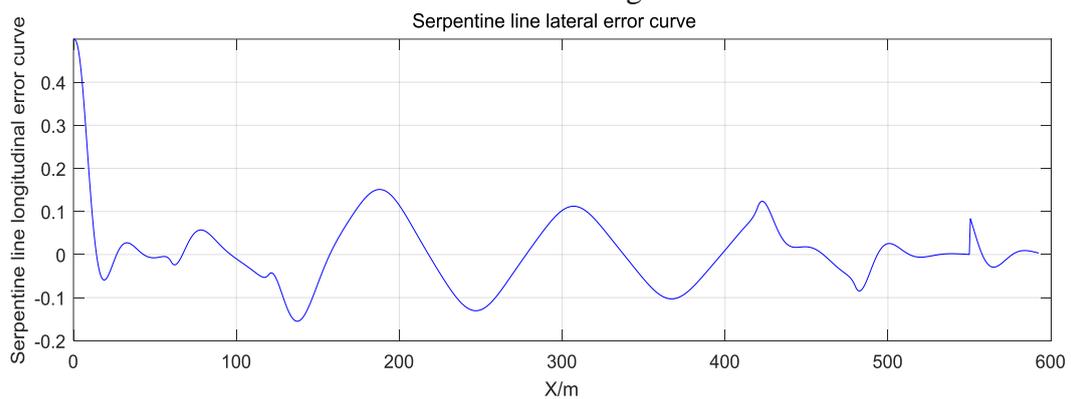


Fig.18 Maximum error of serpentine path tracking

It can be seen from Fig. 16 and Fig. 17 that when the trolley bus faces complex roads, it can still track the path accurately, quickly and stably, and the maximum error is about 0.15m, which is 15cm. When starting to travel away from the desired path, the path can be reached faster and stabilized. Tracking straight line segments without error, for the turn, the actual path and the desired path will have a certain deviation, but the error is within the acceptable range.

## 6. Conclusion

This article uses a PID controller that is widely used in industry. It can be seen from the simulation experiment that the PID controller can control the front wheel angle so that the trolley bus can reach the desired yaw angular acceleration faster. The downside is that when the trolley bus travels on a complex line, the actual yaw acceleration has a certain time lag relative to the desired yaw rate acceleration. It is necessary to further tune the parameters of the PID control or use other control methods to obtain better dynamic characteristics.

From the results of the path tracking simulation experiment of the serpentine line, we can see that: Trolleybus can reduce initial error at a faster rate; When tracking straight line segments, after a period of adjustment, it can travel without errors; On the whole, the trolleybus can track the desired path quickly, accurately and steadily with less error, and has better tracking effect on common roads and complex roads. It can be seen that the path tracking algorithm and the controller can adapt to most roads, so the effectiveness of the path tracking control algorithm and the PID controller is verified.

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