# Sun shadow localization based on affine invariants 

Du Jian, Xiao Wang<br>School of Mechanical Engineering, Southwest Petroleum University, Chengdu 610500, China<br>*Corresponding Author: Jian Du


#### Abstract

By studying the laws of earth revolution and rotation, this paper establishes the shadow model of the earth object in the sunlight, and obtains the variation law of the shadow of the erect long pole. Based on this law, this paper establishes a corresponding inversion model, which obtains the location, date and length data of video or image shooting based on data inversion through the change of shadow of the erect long rod in video or image. Firstly, based on Kepler's third law, the geocentric position coordinates at any time are obtained by studying the earth's revolution, and the translation vector of the local coordinate system to the global coordinate system is obtained. Then, the local coordinate system is obtained by the rotation angle of the earth and the ecliptic angle. The rotation matrix to the global coordinate system. Thus, the global coordinates of the sole, the top of the pole and the center of the earth at any time are obtained. Then, the straight line equation connecting the top of the pole with the sun and the tangent plane equation of the bottom of the pole obtain the projection point of the rod. Finally, the distance formula is obtained by the distance formula between the two points, and the length variation curve of the shadow is obtained.


## Keywords

Rigid body motion, least squares, projection inversion, genetic algorithm.

## 1. Introduction

Taking the sun as the coordinate origin, the revolution plane of the earth's sphere is the X0Y plane (that is, the connection of the summer solstice and the winter solstice is the X axis, and the connection of the vernal equinox and the autumnal equinox is the Y axis), and the positive direction of the Z axis and the movement direction of the center of the earth satisfy the right hand. Guidelines to establish a global coordinate system. According to Kepler's first law, both the Earth and the planet move in an elliptical orbit with the sun as a focal point [1]. The earth's orbit is elliptical, as shown in the following figure:
Therefore, the earth's orbital equation is:

$$
\begin{equation*}
\frac{X^{2}}{a^{2}}+\frac{Y^{2}}{b^{2}}=1 \tag{1}
\end{equation*}
$$

By referring to the data, the long semi-axis and the short half-axis of the orbit are respectively 14.958 million kilometers and 149 million kilometers, that is, the earth's orbit equation is:

$$
\begin{equation*}
\frac{X^{2}}{1.4958 \times 10^{11}}+\frac{Y^{2}}{1.49 \times 10^{11}}=1 \tag{2}
\end{equation*}
$$

Due to the relationship between the date and the earth's revolution, the 2014 winter solstice (0:00:00 GMT) on December 22, 2014 is used as the starting time, and the time and time are based on GMT. Convert to seconds for calculation [2].


Figure 1 Earth's revolution diagram

## 2. Analysis

### 2.1 Establishment of local Cartesian coordinate system

Taking the center of the earth as the origin, the connection between the center of the earth and $(0 \mathrm{E}, 0 \mathrm{~N})$ is the positive $x$-axis, the connection between the center of the earth and ( $90 \mathrm{E}, 0 \mathrm{~N}$ ) is the positive y axis, and the connection between the center of the earth and $(90 \mathrm{E}, 90 \mathrm{~N})$ The line is the z -axis positive direction, and the local Cartesian coordinate system and the corresponding spherical coordinate system as shown in the following figure are established [3]:


Figure 2. The relationship between Cartesian coordinate system and spherical coordinate system

### 2.2 Mutual transformation between local spherical coordinate system and local Cartesian coordinate system

By analyzing the geometry we can convert the spherical coordinates and the spatial Cartesian coordinate system. $\gamma$ is the radius of the earth, $\theta$ is the longitude corresponding to the location, and $\phi$ is the angle between the latitude and the radius [4].

The spherical coordinate system $(r, \theta, \varphi)$ has a conversion relationship with the Cartesian coordinate $\operatorname{system}(x, y, z)$, and the spherical coordinate system is converted into a Cartesian coordinate system as follows:

$$
\left\{\begin{array}{cc}
x=r \sin \theta \cos \phi & \phi \in[0,2 \pi]  \tag{3}\\
y=r \sin \theta \sin \phi & \theta \in[0, \pi] \\
z=r \cos \theta & r \in[0,+\infty]
\end{array}\right.
$$

The formula for converting a Cartesian coordinate system to a spherical coordinate system is as follows:

$$
\left\{\begin{array}{c}
r=\sqrt{x^{2}+y^{2}+z^{2}}  \tag{4}\\
\phi=\arccos \frac{z}{\sqrt{x^{2}+y^{2}+z^{2}}} \\
\theta=\arctan \frac{x}{y}
\end{array}\right.
$$

The latitude and longitude of the earth is established by finding the spherical coordinate system. Looking at the relevant information, the radius of the earth is $r=6378.137 \mathrm{~km}$.

### 2.3 Conversion between local Cartesian coordinate system and local spherical coordinate system

The Z axis is a general position straight line, which can be placed in a general position straight line after being rotated twice through the initial position (the plumb line). The two rotations are: first turn the $\tau$ angle around the Y axis, then turn the $\omega$ angle around the Z axis [5].


Figure 3. Local coordinate system and global coordinate system


Figure 4.The relationship between the Zaxis direction and the angles of $\omega$ and $\varphi$

Through the above analysis, the coordinate transformation from the local coordinate system to the global coordinate system is actually a graphic combination transformation. After the combination of the graphics and the matrix inversion, the mutual conversion relationship between the two can be derived. The matrices are:

$$
T_{\omega}=\left[\begin{array}{cccc}
\cos \omega & 0 & -\sin \omega & 0 \\
0 & 1 & 0 & 0 \\
\sin \omega & 0 & \cos \omega & 0 \\
0 & 0 & 0 & 1
\end{array}\right] T_{\tau}=\left[\begin{array}{cccc}
\cos \tau & \sin \tau & 0 & 0 \\
-\sin \tau & \cos \tau & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] T_{d}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
x_{0} & y_{0} & z_{0} & 1
\end{array}\right]
$$

Combined transformation matrix:

$$
T=T_{\omega} \cdot T_{\tau} \cdot T_{d}=\left[\begin{array}{cccc}
\cos \omega \cos \tau & \cos \omega \sin \tau & -\sin \omega & 0  \tag{5}\\
-\sin \tau & \cos \tau & 0 & 0 \\
\sin \omega \cos \tau & \sin \omega \sin \tau & \cos \omega & 0 \\
x_{0} & y_{0} & z_{0} & 1
\end{array}\right]
$$

$\omega, \varphi$ can be derived from Figure 4, the direction cosine of the Z axis is:

$$
\left\{\begin{array}{l}
R_{1}=\cos \alpha  \tag{6}\\
R_{2}=\cos \beta \\
R_{3}=\cos \gamma
\end{array}\right.
$$

Substituting $\cos \omega, \sin \omega, \cos \varphi, \sin \varphi$ into the above formula:

$$
T=\left[\begin{array}{cccc}
\frac{\cos \alpha \cos \beta}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & \frac{\cos \beta \cos \gamma}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & -\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2} & 0 \\
-\frac{\cos \beta}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & \frac{\cos \alpha}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & 0 & 0 \\
\cos \alpha & \cos \beta & \cos \gamma & 0 \\
x_{0} & y_{0} & z_{0} & 1
\end{array}\right]
$$

The $\mathrm{x}, \mathrm{y}$, and z coordinates of the corresponding points can be derived to convert the local coordinate system to the global coordinate system.

### 2.4 Convert global coordinate system to local coordinate system

It can be seen from the above analysis that the P point coordinates satisfy the following relationship

$$
\left\{\begin{array}{llll}
{[x} & y & z & 1
\end{array}\right] \cdot T=\left[\begin{array}{llll}
X & Y & Z & 1
\end{array}\right],\left[\begin{array}{llll}
{\left[\begin{array}{llll}
x & y & z & 1
\end{array}\right]=\left[\begin{array}{llll}
X & Y & Z & 1
\end{array}\right] \cdot T^{-1}} \tag{8}
\end{array}\right.
$$

That is, only the inverse matrix $T^{-1}$ of the T matrix is required, and then the local coordinate system is obtained according to the global coordinate system, so that the conversion from the global coordinate system to the local coordinate system can be realized. For the inverse matrix of (5), we get:

$$
|T|=1
$$

$$
T^{-1}=\left[\begin{array}{cccc}
\cos \omega \cos \tau & -\sin \tau & \sin \omega \cos \tau & 0  \tag{9}\\
\cos \omega \sin \tau & \cos \omega & \sin \omega \sin \tau & 0 \\
-\sin \omega & 0 & \cos \omega & 0 \\
A & B & C & 1
\end{array}\right]
$$

Substituting $\cos \omega, \sin \omega, \cos \varphi, \sin \varphi$ into the above formula:

$$
\begin{gather*}
T^{-1}=\left[\begin{array}{cccc}
\frac{\cos \alpha \cos \gamma}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & -\frac{\cos \beta}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & \cos \alpha & 0 \\
-\frac{\cos \beta \cos \gamma}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & \frac{\cos \alpha}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} & \cos \beta & 0 \\
-\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2} & 0 & \cos \gamma & 0 \\
A & C & 1
\end{array}\right]  \tag{10}\\
\left\{\begin{array}{ccc}
A=-x_{0} \cdot \frac{\cos \alpha \cos \gamma}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}}-y_{0} \cdot \frac{\cos \beta \cos \gamma}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}}+z_{0} \cdot\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2} \\
B=x_{0} \cdot \frac{\cos \beta}{\left(\cos ^{2} \alpha+\cos \beta\right)^{2}}-y_{0} \cdot \frac{\cos \alpha}{\left(\cos ^{2} \alpha+\cos ^{2} \beta\right)^{1 / 2}} \\
C=-x_{0} \cdot \cos \alpha-y_{0} \cdot \cos \beta-z_{0} \cdot \cos \gamma
\end{array}\right. \tag{11}
\end{gather*}
$$

By substituting (11) into equation (9), the coordinates in the local coordinate system can be obtained, that is, the conversion from the global coordinate system to the local coordinate system is realized. $\tau=23.5^{\circ}$.

$$
\begin{equation*}
\omega=t_{1} \cdot \omega_{1}=0.0131 \cdot t_{1} \tag{12}
\end{equation*}
$$

### 2.5 The relationship between date and time and the revolution of the earth

The schematic diagram of the earth's revolution is shown in the above figure. This article is based on the stellar year. The 2014 winter solstice (0:00:00 GMT) on December 22, 2014 is used as the starting time. Through the relationship of the month, day and second, the shadow length of the pole of Tiananmen Square between 9:00 and 15:00 Beijing time on October 22, 2015 can be obtained [6].

$$
\begin{equation*}
h=h_{1}-8 \tag{13}
\end{equation*}
$$

Among them, h is Green Day time, $\mathrm{h}_{1}$ is Beijing time, that is, converted to Greenwich time from October 22, 2015 1:00-7:00. Therefore, it can be found that the time of the earth revolution is (25355600-26377200) seconds from the start time.

### 2.6 Determination of the position of the earth

Through calculation, the linear velocity of the earth around the sun is obtained as:

$$
\begin{equation*}
V=\sqrt{\frac{2 a-d}{a r} G M} \tag{14}
\end{equation*}
$$

Where 2 a is the long axis of the Earth's motion orbit, $d=F P$ is the distance from point p to the Earth's trajectory focus F, M is the mass of the sun, and G is the universal gravitational constant. Through calculation, the orbital circumference of the earth is obtained as $9.4 \times 10^{8} \mathrm{~km}$. Integrate the line speed to get the distance traveled by the Earth:

$$
\begin{align*}
S & =\int_{0}^{t} v d t \\
& =\int_{0}^{t} \sqrt{\frac{2 a-d}{a r} G M d s} \tag{15}
\end{align*}
$$

## 3. Conclusion

Based on the known information of the topic, Figure 5 is derived. Where D is the position of the sun, $\left(X_{s}, 0,0\right)$ is the position of the sun in the global coordinate system; B is the position of the top of the pole, $\left(X_{t}, Y_{t}, Z_{t}\right)$ is the coordinate of the top of the pole in the global coordinates; A is the position of the sole, and $\left(X_{t 1}, Y_{t 1}, Z_{t 1}\right)$ is the global coordinate of the sole The coordinates in the middle; C is the position of the shadow fixed point, and $\left(X_{t}, Y_{t}, Z_{t}\right)$ is the coordinate of the shadow fixed point in the global coordinate.


Figure 5. Sun pole top shadow schematic
The equation of the line between the sun and the top of the pole is

$$
\left\{\begin{array}{l}
X=X_{t}+t \cdot\left(X_{s}-X_{t}\right)  \tag{18}\\
Y=Y_{t}-t \cdot Y_{t} \\
Z=Z_{t}-t \cdot Z_{t}
\end{array}\right.
$$

Further, the equation for finding the tangent plane of the sole is:

$$
\begin{equation*}
\left(X_{t}-X_{b}\right) \cdot\left(X-X_{b}\right)+\left(Y_{t}-Y_{b}\right) \cdot\left(Y-Y_{b}\right)+\left(Z_{t}-Z_{b}\right) \cdot\left(Z-Z_{b}\right)=0 \tag{19}
\end{equation*}
$$

Combine the above two equations to solve:

$$
\begin{equation*}
t=\frac{-\left(\left(X_{b}-X_{t}\right) *\left(X_{o}-X_{t}\right)+\left(Y_{b}-Y_{t}\right) *\left(Y_{o}-Y_{t}\right)+\left(Y_{b}-Y_{t}\right) *\left(Z_{o}-Z_{t}\right)\right)}{\left(Y_{t} *\left(Y_{o}-Y_{t}\right)+Z_{t} *\left(Z_{o}-Z_{t}\right)-\left(X_{o}-X_{t}\right) *\left(X_{\text {sunt }}-X_{t}\right)\right)} \tag{20}
\end{equation*}
$$

Then use the following formula to solve the coordinates of the shadow

$$
\left\{\begin{array}{c}
X_{S}=X_{t}+t *\left(X_{s u n}-X_{t}\right)  \tag{21}\\
Y_{s}=Y_{t}-t * Y_{t} \\
Z_{s}=Z_{t}-t * Z_{t}
\end{array}\right.
$$

The eccentricity of the earth's orbit is $\mathrm{e}=0.016722$. Then pass the distance formula between two points:

$$
\begin{equation*}
d=\sqrt{\left(X_{t}-X_{t 1}\right)^{2}+\left(Y_{t}-Y_{t 1}\right)^{2}+\left(Z_{t}-Z_{t 1}\right)^{2}} \tag{22}
\end{equation*}
$$

Find the shadow length, use MATLAB programming, and then make the following shadow length curve:


Figure 6. Shadow length change function image
It can be seen from Fig. 6 that the curve of the length of the sun shadow of the straight bar 3 meters high in Tiananmen Square between October 9th, 2015 and between 9:00-15:00 Beijing time is parabolic. At 9:00 am, the shadow has the longest length of 7.0436 meters; at 12:00, the shadow length is the shortest, about 3.7518 meters; at 15:00, the shadow length is 6.5436 meters.


Figure 7. A schematic view of the shadow end space
From Fig. 7, the direction of the shadow can be obtained, and the direction of the shadow has a certain relationship with the rotation and revolution of the earth.


Figure 8. Schematic diagram of the shadow in the global coordinate system
Figure 8 is a three-dimensional perspective view from which the spatial representation of the shadow in the global coordinate system can be clearly seen.


Figure 9. Shadow position transformation in global coordinates
From Figure 9, the transformation of the shadow position in the global coordinates can be obtained, thereby obtaining the global coordinates of the shadow points, which is convenient for calculating the shadow length of the straight rod.

## References

[1] Jiang Qiyuan, Xie Jinxing. Mathematical Model Third Edition. Higher Education Press, (2003)
[2] Su Jinming, Yan Shenyong, Wang Yongli, MATLAB Engineering Mathematics, Publishing House of Electronics Industry, (2005)
[3] Information
on:http://wenku.baidu.com/link?url=ahBCO04XSHzbK2ZBBmRAsp4FEaUL0S9Q5_3rs34gNb4f4U-X0L18YxuLGMIogCcaExiWOF-O18HhfeodfkaHV0-HfkCIFJag7jjQd7pW
[4] Information on: http:// wenku.baidu.com/link?url=7d4SdPn276liKqgkFjTC_ILNBwoZvtUSCx8-fmb6 ds O-Dk7CP_umbExhVz4NzHJI_ob-LV236x1AisN2f36Gtc2s6bWsEHpgVMUHZ-scqKm\&qq-pf-to= pcqq.c2c
[5] Xu Shejiao, coordinate transformation between two coordinate systems in three-dimensional graphics system, Journal of Xidian University, (1996)
[6] Xiong Wei, Wang Ailing, et al. The concept and definition of equivalent earth radius, Journal of ElectroPotical Science, (1997)

