

# Research on Liner Schedule Model Based on Slow Steaming

Shuwen Zhang

School of Transportation, Shanghai Maritime University, Shanghai 201306, China.

719144941@qq.com

---

## Abstract

**With the effects of uncertainties in port operation on waiting berth time, berthing time, cross-port inventory cost and multi-berth time window taken into account, a model of liner ship route schedule is developed, which determines the number of ships on route, the estimated berthing time and the speed of every leg of a voyage. In order to further reduce fuel costs and carbon dioxide emissions, the robust optimization model of liner schedule is improved. The congestion waiting time is incorporated into the leg time to construct the congestion buffer time, and the slow steaming is adopted to reduce fuel consumption. The results show that the impact of cross-port inventory cost on total cost increases with the increase of unit inventory cost, and decreases with the increase of uncertain budget. The cost-effectiveness of adding congestion buffer time becomes more significant with the increase of berthing time.**

## Keywords

**Berth time window; uncertainties in port operation; variable inventory cost; slow steaming.**

---

## 1. Introduction

In the context of the weakening of global economic growth, the international liner shipping industry is under tremendous market pressure. In order to reduce costs, improve efficiency and enhance the scientific decision-making, liner companies are constantly seeking new optimization tools and methods in liner shipping management. According to the statistics of Shanghai International Shipping Research Center, from January to September 2018, large international liner companies have a total of 27 route adjustments. The liner company's optimization of port of call and capacity allocation will inevitably lead to the adjustment of liner schedule. Therefore, the stable, reliable, scientific and reasonable container liner schedule has become an important issue for liner companies. Because the ideal design environment of classical schedules is quite different from the actual operation environment, the reasonable design of container liner schedules is of great significance to reduce the operation cost of shipping companies.

At present, many scholars have made deep research on the optimization of liner schedule. ALHBRBI et al.[1] studied the design of liner schedule for container supply chain network with port time window. A mixed integer non-linear model based on port availability was established to minimize ship cost and fuel cost. According to Notteboom [2], 93.6% of the delays are caused by the prolongation of the ship's wait time of the berth and uncertain container handling time. In addition, compared with the arrival time, the shipowner is more concerned about when the container will berth and operate. It is more practical to replace the expected arrival time with the expected berthing time. Therefore, this paper proposes a multi-berth time window constraint to ensure the feasibility of the schedule.

In order to ensure the economic and environmental protection of operation, Xu Huan et al.[3] sought the optimal speed of ships in each leg to minimize fuel consumption and carbon emissions of voyages.

At present, the study of liner schedules for uncertainties in port time caused by port congestion is mostly focused on extending the time of ships in port to cope with the waiting time caused by port congestion, which is different from the actual operation mode. In the face of port congestion, shipowners prefer to slow down their arrival at the port in the last section to reduce fuel costs and carbon and sulfur emissions. For example, the port may inform the ship of the estimated congestion time in advance. Considering the congestion time, the ship will reduce the original speed of 13 knots to 11 knots, so that it start handling immediately when it arrives at the port.

Secondly, the existing research is limited to the impact of fixed inventory cost on the total cost of voyage when the port time is fixed, and does not take into account the impact of variable cross-port inventory cost caused by the uncertainties of port operation. BAKSHI et al. [4] estimated the inventory cost of containerized goods to be 0.5% of the value of the container per day. Considering the uncertainties of port operation time, the cost of container inventory is also variable.

In this paper, a multi-berth time window constraint is proposed. Considering the influence of uncertainties in port operation on waiting time, berthing time and variable cross-port cargo inventory cost, a robust optimization model of container liner schedule is established. Combining with market operation mechanism, the uncertain waiting time of congested port is transformed into the congestion buffer time of the previous leg, which prolongs the sailing time of the leg and reduces the speed. Improve the robust optimization model of liner schedule, distribute uncertain budget according to the he uncertainties of port operation by weight, and seek the best balance between ship cost, fuel cost and variable cross-port inventory cost on the premise of ensuring that port berths can provide services for ships within the estimated date.

## 2. Mathematical model and algorithm

### 2.1 Robust optimization method

Robust optimization method can effectively deal with the uncertainty of model parameters without affecting the objective function, and subjectively control the conservativeness of the model through uncertain budget  $\Gamma$ , thus avoiding the limitation of the effectiveness of the optimization results due to excessive constraints[5]. We can let  $t_i^{in}$  be the estimated time from entry to berthing,  $P_i$  be the estimated berth time period of ships at port i, and both of them are uncertain parameters. Two uncertain budget parameters are used to reflect the uncertainty of the whole voyage, and the uncertain budget is allocated according to the weight according to the uncertainties of port operation, and the impact of the uncertainty on the liner schedule is more accurately considered.

Robust deterministic ship schedule model, the range of two uncertain parameters as follows:

$$t_i^{in} = \left[ \overline{t_i^{in}} - \overline{t_i^{in}}, \overline{t_i^{in}} + \overline{t_i^{in}} \right], P_i = \left[ \overline{P_i} - \overline{P_i}, \overline{P_i} + \overline{P_i} \right], \left( \overline{t_i^{in}} > 0, \overline{P_i} > 0 \right) \tag{1}$$

Define the deviation of two parameters in proportion to theirs nominal value as follows:

$$z_{it} = \left( t_i^{in} - \overline{t_i^{in}} \right) / \overline{t_i^{in}}, \quad [-1, 1] \tag{2}$$

$$z_{ip} = \left( P_i - \overline{P_i} \right) / \overline{P_i}, \quad [-1, 1] \tag{3}$$

Assuming that the number of legs in a route is n, let  $\Gamma_m$  and  $\Gamma_{pm}$  be the uncertain budget corresponding to two uncertain parameters[6].

$$\sum_{i=1}^n |z_{it}| = \Gamma_m, 0 \leq \Gamma_m \leq n \tag{4}$$

$$\sum_{i=1}^n |z_{ip}| = \Gamma_{pm}, 0 \leq \Gamma_{pm} \leq n \tag{5}$$

When  $\Gamma_m = \Gamma_{pn} = 0$ , the model degenerates into a deterministic problem without considering the uncertainty of parameters. When  $\Gamma_m = \Gamma_{pn} = n$ , the model considers the worst case, that is, maximizes the parameter uncertainty.

### 2.2 Robust optimization model

The essence of liner schedule design is to configure suitable ships to operate on the route at reasonable speed, so that the liner company's total operating cost is the lowest to obtain the best economic and environmental benefits. Before presenting the model, we list the notation below.

Let  $C^{op}$  represents fixed operating cost of a ship deployed on the ship route,  $L_i$  represents the distance of ship in leg  $i$ ,  $s_i$  defines the reserved time for sailing in leg  $i$ ,  $k$  be the functional Coefficient of ship,  $k_1$  be the diesel oil consumption of ships during sailing,  $k_2$  be the diesel oil consumption of ships at berth,  $f$  be the ship fuel oil price,  $f'$  be the ship diesel price,  $\beta$  be the hourly inventory cost per container,  $Q_i$  be the quantity of cargo carried by at port  $i$ ,  $Q'$  be the quantity of cargo discharged at port  $i$ ,  $b_{n+1}^{arr}$  defines the instant when the ship returned to berth at the first port after a round trip,  $b_i^{dep}$  defines the instant when the ship is expected to leave berth at port  $i$ ,  $t_i^{out}$  be the estimated time from berthing to departure,  $W$  be any instant in a week,  $W_i^d$  be the available time for service of berth  $d$  in a week,  $D_i$  defines a set of all berths at port  $i$ ,  $\Pi_i^d$  represents only if the vessel arrives at the  $i$  port of call at  $w$  and berth  $d$  is used, it will be 1, otherwise it will be 0,  $\Pi_i^{dw}$  represents only if the vessel arrives at the  $i$  port of call and berth  $d$  is used, it will be 1, otherwise it will be 0,  $\sigma_i^{dw}$  represents only if the berth  $d$  of the  $i$  port of call has a time window at  $w$ , it will be 1, otherwise it will be 0,  $V^{max}$  be the maximum speed of ship,  $V^{min}$  be the minimum speed of ship,  $Z^+$  be a set of nonnegative integer,  $t_i^{jam}$  be the congestion buffer time in a leg,  $m$  be the number of ships deployed on the ship route,  $b_i^{arr}$  be the estimated arrival instant at the  $i$ th berth,  $V_i$  be the sailing speed on leg  $i$ .

The robust optimization model is expressed as follows:

$$\min Z = C^{op} \cdot m + \sum_{i \in I} \frac{L_i}{V_i} \cdot \frac{1}{24} (fkv_i^3 + fk_1) + f' \sum_{i \in I} \frac{1}{24} P_i k_2 + \beta \sum_{i \in I} [Q_i (\frac{L_i}{V_i} + s_i + t_i^{out} + \bar{t}_{i+1}^{in} + \sigma_{(i+1)t}) + \theta_m \Gamma_m \frac{\bar{t}_{i+1}^{in} + \sigma_{(i+1)t}}{\sum_{j=1}^n (\bar{t}_{j+1}^{in} + \sigma_{(j+1)t})} + (Q_i - Q') \cdot \left( \frac{\bar{p}_i + \sigma_{ip}}{p_i + \sigma_{ip}} + \theta_{pn} \Gamma_{pn} \frac{\bar{p}_i + \sigma_{ip}}{\sum_{j=1}^n (\bar{p}_j + \sigma_{jp})} \right)] \tag{6}$$

s.t.

$$0 \leq b_1^{arr} \leq 168 \tag{7}$$

$$b_j^{arr} = b_1^{arr} + \sum_{i=1}^{j-1} \left( \frac{\bar{p}_i + t_i^{out} + \frac{L_i}{V_i} + s_i}{\bar{t}_{i+1}^{in} + \sigma_{(i+1)t} + \sigma_{ip}} \right) + \theta_m \Gamma_m \frac{\sum_{i=1}^{j-1} (\bar{t}_{i+1}^{in} + \sigma_{(i+1)t})}{\sum_{i=1}^n (\bar{t}_{i+1}^{in} + \sigma_{(i+1)t})} + \theta_{pn} \Gamma_{pn} \frac{\sum_{i=1}^{j-1} (\bar{p}_i + \sigma_{ip})}{\sum_{i=1}^n (\bar{p}_i + \sigma_{ip})}, j > 1 \tag{8}$$

$$\theta_m + \sigma_{(i+1)t} \geq \bar{t}_{i+1}^{in}, \theta_{pn} + \sigma_{ip} \geq \bar{p}_i, \forall i \leq n \tag{9}$$

$$\theta_m \geq 0, \theta_{pn} \geq 0, \sigma_{(i+1)t} \geq 0, \sigma_{ip} \geq 0, \forall i \leq n \tag{10}$$

$$b_{n+1}^{arr} - b_1^{arr} = 168m \tag{11}$$

$$\sum_{d \in D_i} \Pi_i^d = 1, i \in I \tag{12}$$

$$\Pi_i^{dw} \leq \sigma_i^{dw}, i \in I, d \in D_i, w \in W \tag{13}$$

$$b_i^{arr} \bmod 168 \in W_i^d, i \in I, d \in D_i \tag{14}$$

$$V^{\min} \leq V \leq V^{\max} \tag{15}$$

$$1 \leq m \leq m^{\max} \tag{16}$$

$$m, b_i^{arr} \in Z^+, \forall i \in I \tag{17}$$

The objective function (6) minimizes the expected weekly total costs include ship costs, bunker cost and inventory cost. Eq. (7) eliminates symmetric solutions. Eqs. (8)–(10) define the moment at which the ship is expected to berth at each port of call. Eq. (11) states the relationship between the time required for a ship to complete a voyage and the number of ships required to be placed on the route. Eq. (12) states ships need only one berth at port *i* each time. Eq. (13) states if berth *d* of port *i* has no time window at *w* moment, the ship can not reach the berth at that moment. Eq. (14) states the berth time window. Eq. (15) restricts the range of speed change. Eq. (16) limits the upper and lower limits of the number of ships on the route. Eq. (17) states the range of variable values.

### 2.3 Robust optimization model considering port congestion time

Because of the approximate cubic relationship between ship speed and fuel consumption, the change of ship speed has an inseparable impact on ship operation cost and carbon and sulfur emissions. Because of unnecessary waiting time caused by port congestion, the diesel oil price consumed by ships in port is relatively high [7]. The fuel consumption in port is proportional to the stay time of ships in port.

The uncertain berth waiting time due to port congestion in the model is transformed into the congestion buffer time in the leg, so as to prolong the sailing time in the last leg of the ship and slow down the sailing of the ship, then the improved predicted berthing time constraint is as Eq. (18).

$$b_i^{arr} = b_{i-1}^{dep} + t_{i-1}^{out} + \frac{L_{i-1}}{V_{i-1}} + s_{i-1} + t_i^{jam} + t_i^{in} \tag{18}$$

$s_{i-1}$  is the reserve time of the last leg to cope with the influence of wind, current and wave on the speed of the ship. When the ship is delayed at the last port, the reserve time of the last leg can be used to speed up the schedule. When congestion occurs in the next port, the port notifies the vessel of

anticipated congestion time  $t_i^{jam}$ , and the sailing time of the leg is extended to  $\left(\frac{L_{i-1}}{V_{i-1}} + s_{i-1} + t_i^{jam}\right)$ , so

the ship can also reach berth at the same time when it slows down. Eq.(6) is rewritten to Eq. (19), Eq. (15) is rewritten to Eq. (21) and Eq. (20) is added to express the actual speed at which the ship slows down after considering the congestion buffer time.

$$\min Z = C^{op} \cdot m + \sum_{i \in I} \frac{L_i}{V_i} \cdot \frac{1}{24} (fkv_i^3 + fk_1) + f' \sum_{i \in I} \frac{1}{24} P_i k_2 + \beta \sum_{i \in I} [Q_i \left(\frac{L_i}{V_i'} + s_i + t_i^{out} + \bar{t}_{i+1}^{in} + t_i^{jam}\right) + (Q_i - Q_i') \cdot \left( \frac{\bar{p}_i + \sigma_{ip}}{\bar{p}_i + \sigma_{ip} + \theta_{pn} \Gamma_{pn} \frac{\bar{p}_i + \sigma_{ip}}{\sum_{j=1}^n (\bar{p}_j + \sigma_{jp})}} \right) ] \tag{19}$$

$$V_i = \frac{L_i}{\frac{L_i}{V_i'} + t_{i+1}^{jam}} \tag{20}$$

$$V^{\min} \leq V_i, V_i' \leq V^{\max} \tag{21}$$

### 3. Numerical experiments

We carry out numerical experiments to validate the effectiveness of the proposed approach. The robust optimization models are solved by Lingo. Consider a ship route such as the FEE, The port rotation of the ship route has a total of *n* ports of call. Define a set  $I = \{1, 2, \dots, n\}$ . The order of port of

calls on FEE routes is as follows: Pusan,Ningbo,Shanghai, Suez Canal, Rotterdam, Hamburg, Antwerp, Southampton, Pusan.

According to the ship type demonstration, the liner company selected the ship with a capacity of 14000 TEU to be put into operation on the route. The fixed cost per week is 380,000 USD, the unit inventory cost is 0.3 USD, the maximum number of ships is 20, the minimum speed is 14 kn, and the maximum speed is 26 kn. The functional Coefficient of ship is 0.0164, the diesel consumption of the ship during sailing is 2.2 (t/h), the diesel consumption of the ship at the time of berthing is 2.6 (t/h), the price of the ship's fuel oil is 550 USD/t, and the price of the ship's diesel is 650 USD/t. The nominal value (h) of the arrival time is 2, 10, 25, 9, 1, 7, 8, 4, and the arrival interval radius (h) is 1, 5, 19, 0, 3, 4, 1, 1. The nominal value (h) of the berth time is 37, 32, 20, 9, 47, 54, 32, 31, and the radius (h) of the berth time interval is 7, 5, 5, 0, 12, 14, 5, 3.

Table 1 Model parameters

Leg	Distance (n mile)	Reserved time of leg(h)	Container volumes (TEU)	Constant container volumes(TEU)
1	503	4	11000	7000
2	110	1	11500	7500
3	7113	24	12000	8500
4	3336	10	12500	9000
5	297	2	8000	6200
6	365	3	9000	6800
7	236	2	8500	6000

According to the above related data, the Lingo software is used to solve the robust optimization model. When the uncertainty budget is taken as  $\Gamma_m = \Gamma_p = 0.2n$ , the results are shown in Table 2:

Table 2 Optimum Results

Number	Port	Actual berth time	Optimized berth time
1	Pusan	9	9
2	Ningbo	103	94
3	Shanghai	174	178
4	Suez Canal	622	647
5	Rotterdam	831	859
6	Hamburg	906	939
7	Antwerp	998	1034
8	Southampton	1062	1090
9	Pusan	1857	1857

According to the calculation results, the total operating cost of the liner company is 11,747,760 USD per week. A total of 11 vessels are required to be deployed on this route, with a schedule of 77 days, which is the same as the actual vessel schedule of the liner company.

### 3.1 Consider the impact of inventory cost changes in the port

Analyze the impact of unit inventory costs and uncertain budgets on cross-port inventory costs. Table 3 shows the proportion of inventory cost in total cost under the influence of different unit inventory cost and different value of uncertain budget when  $\beta = 0.3$ . It can be seen from the table that with the increase of unit inventory cost, the proportion of inventory cost in total cost increases greatly. Therefore, when the unit inventory cost is high, the inventory cost has a great influence on the service level of the liner company.

Table 3 Proportion of Inventory Cost in Total Cost

Unit inventory cost	Ratio	Uncertain budget	Ratio
0.3	21.66%	0	45.34%
0.1	45.34%	1.6	44.69%
0.5	58.02%	4.8	44.17%
0.7	65.42%	8	43.97%

Figure 1 shows the changes in fuel costs and inventory costs under different uncertain budgets. With the increase of the uncertain budget, the port time of ship increases, the delay caused by the port increases. In order to catch up with the vessel schedule, vessel should increase the speed of the leg accordingly. When there are delays such as congestion in port, liner companies can determine the optimal speed based on the compromise between fuel cost and inventory cost, and make a trade-off between the total cost of a week and liner service level.

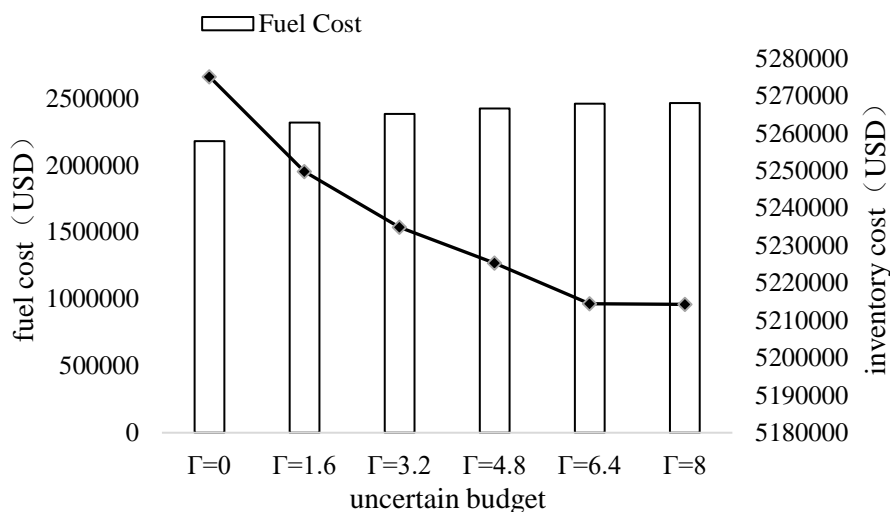


Fig. 1 The impact of uncertain budgets on costs

### 3.2 Optimization results of two models

On the basis of model 1, model 2 is optimized to incorporate the waiting berth time caused by port congestion into the sailing time of the previous leg, so as to reduce the sailing speed of the leg. The optimization results are shown in table 4.

Table 4 Model optimization results with uncertain budget is 1.6n

Model	Total cost (USD)	Fuel cost (USD)	Inventory cost(USD)	Ship cost (USD)	Ship number
1	11747630	2317912	5249721	4180000	11
2	11648330	2238950	5229380	4180000	11

As it shown in table 4, when the sailing time of a certain leg is prolonged to  $\left(\frac{L_{i-1}}{V_{i-1}} + s_{i-1} + t_i^{jam}\right)$ , the ship slows down, and the fuel cost and inventory cost will be reduced. As for shipping companies, increasing the congestion buffer time is a win-win strategy to reduce the total cost and improve the service level of liner transportation.

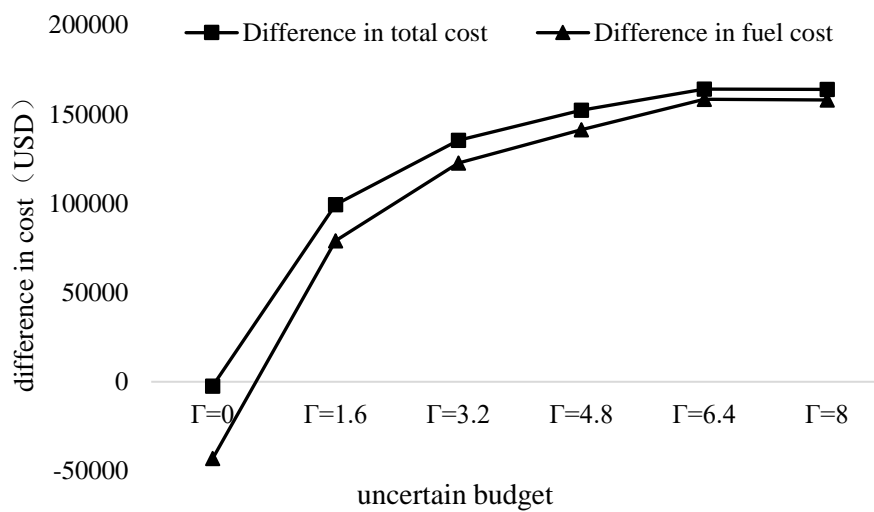


Fig. 2 Cost gap between Model 1 and Model 2

Figure 2 shows the cost difference between Model 1 and Model 2. It can be seen from Fig. 2 that the total cost of the model 1 and the fuel cost gradually increase with the increase of the uncertain budget compared with the model 2, and the cost advantage of the model 2 becomes more and more obvious. In the face of port congestion, the ship's choice to slow down the sailing to save fuel costs is more substantial than the on-time arrival of the port, model 2 is more in line with actual operational needs.

#### 4. Conclusion

Under the background of low carbon and low sulfur, liner companies are influenced by environmental factors, and tend to choose operation solutions with dual economic and environmental benefits. This paper proposes a multi-berth time window constraint, comprehensively considers the impact of uncertainties in port operation on the waiting berth time, the berthing time and the variable inventory cost, and establish a robust optimization model considering port congestion time. The results show that when the unit inventory level is high, the proportion of inventory cost is large, which has a significant impact on the service level of liner companies. The optimization results of model 1 and model 2 show that with the increase of uncertain budget, that is, the impact of port congestion and uncertainties in port operation on ship schedule increases, model 2 will have more obvious economic and environmental protection effect to reduce the total cost.

#### References

[1] Alharbi A, Wang S, Davy P. Schedule design for sustainable container supply chain networks with port time windows [J]. Advanced Engineering Informatics, 2015, 29(3):322-331.

- [2] Notteboom T E . The Time Factor in Liner Shipping Services[J]. Maritime Economics & Logistics, 2006,8(1):19-39.
- [3] Xu Huan, Liu Wei, Liu Junyu. Compilation of Shipping Schedule for Low Carbon Economy Off-duty Ships [J]. Journal of Chongqing Jiaotong University (Natural Science Edition), 2014, 33 (1): 129-132. (In Chinese)
- [4] Bakshi N, Gans N. Securing the Containerized Supply Chain: Analysis of Government Incentives for Private Investment[J]. Management Science, 2010, 56(2):219-233.
- [5] Research on Vehicle Routing Problem in Emergency Management Based on Robust Optimization [D]. Henan University, 2007. (In Chinese)
- [6] Bertsimas D, Sim M. The Price of Robust-ness[J]. Operations Research,2004,52(1):35-53.
- [7] Yang Chen, Li Haobo, Chen Jihong and others. Container transit port selection model considering carbon and sulfur costs [J]. Mathematical practice and understanding, 2017 (17): 61-69. (In Chinese)