Widening Estimation Range of Pareto Clutter Parameters Based on Fractional Moments

Juntao Fu [1, a], Guoan Zhou [2, b, *] and Hong Chen [3, c]

[1] Air Force Engineering University Graduate College, Xi'an Shaanxi, China

[2] Air Force Engineering University Air and Missile Defence College, Xi'an Shaanxi, China

[3] Unit 61769, Lvliang Shanxi, China

[a]821823977@qq.com, [*, b] zhouguoan@sina.cn, [c]578920855@qq.com

Abstract

One improved expression of parameter estimator for Pareto distribution is derived. The proposed estimator has the closed-form expression and widens the valid estimation range of shape parameter. In order to achieve that, positive fractional order is used. The improved estimator solves the problems that the previous estimators cannot get the closed-form expression or estimate the valid value of shape parameter in the small range. Simulation results show that the proposed estimator can estimate the valid value of shape parameter in the small range and have better estimation performance especially in the small range of shape parameter.

Keywords

Performance especially; closed-form expression; previous estimators.

1. Introduction

Compared with K distribution model, the Pareto distribution is a more dominant statistical clutter model when the radar works in the rough sea or against the wind [1-5]. Therefore, in order to improve the detection performance in the rough sea, we must improve the accuracy of parameter estimation for the sea clutter [6].

Besides traditional methods which include maximum likelihood estimator (MLE) [7] and method of moment (MoM) [4], there are many research findings of the parameter estimation for Pareto distribution in recent years [8-9], such as more advanced methods which include method of fractional moment (MoFM) [10] and x log x-based estimator [4] etc. However, there are still many problems to solve. For example, MoFM cannot obtain a closed form expression because Pareto distribution model has two parameters: shape parameter and scale parameter. It is inefficient to solve the problem with numerical computation. Due to the properties of Digamma function, x log x-based estimator does not estimate correctly in the range of shape parameter less than 1, which limits the range of parameter estimation.

In this correspondence, one closed form estimator which uses fractional moments $< x^r >$ is proposed. According to the deduction, the estimation expression that has only a unique solution is proved. The estimator can estimate the shape parameter which values in $(j+s,+\infty)$. It is tested by MATLAB to compare performance of different estimators. The results of simulation experiment show that the performance of proposed estimator is better than other estimators, especially in the range of shape parameter which values in $(j+s,1)$, such as MoM and x log x-based estimators.

2. Problem Description

the probability density function (pdf) of a Pareto distributed random variable X, is given by [11]

$$< x^r >= \int_b^\infty x^r \frac{ab^a}{x^{a+1}} dx = \frac{ab^r}{a-r}, \quad a > r \tag{1}$$

Where a is the shape parameter and b is the scale parameter.

The estimator of MoM is given by [11]

$$< X >< X^{-1} >= \left(1 - \frac{1}{a^2}\right)^{-1} \tag{2}$$

The x log x-based estimator is given by [4]

$$\frac{< x\ln(x) >}{< x >} - < \ln(x) >= \frac{1}{a-1} - \frac{1}{a} \tag{3}$$

Note that both of MoM and x log x-based estimators only work well for $a > 1$ [4,11]. Therefore, if both of the previous estimators continue estimating the true value of shape parameter a which values in $(0,1)$, estimation value is incorrect and meaningless.

Proposed estimator: the assemble average $< x^r >$ is given by [12]

$$< x^r >= \int_b^\infty x^r \frac{ab^a}{x^{a+1}} dx = \frac{ab^r}{a-r}, \quad a > r \tag{4}$$

We define the following statistics

$$V_{j,s} = \frac{< x^{j+s} >}{< x^j >< x^s >}, j > 0, \ s > 0 \tag{5}$$

Calculating (5) and we obtain

$$V_{j,s} = \frac{(a+j)(a+s)}{(a+j+s)a}, \ a > j+s \tag{6}$$

(6) can be written as quadratic equation

$$(1-V_{j,s})a^2 - (j+s)(1-V_{j,s})a + js = 0 \tag{7}$$

Now, the question is whether there was a solution. If it exists, we can obtain

$$\Delta = (j+s)^2(1-V_{j,s})^2 - 4(1-V_{j,s})js \geq 0 \tag{8}$$

Solving (8), we obtain

$$V_{j,s} \geq 1 \text{ or } V_{j,s} \leq \frac{(j-s)^2}{(j+s)^2} \tag{9}$$

Discussing both of the assumptions. If $V_{j,s} \geq 1$ is tenable, we can obtain

$$\frac{(a-j)(a-s)}{(a-j-s)a} \geq 1 \tag{10}$$

Solving (10), we obtain

$$a < 0 \ or \ a > j+s \tag{11}$$

$a < 0$ has not any practical significance because a must be in the range $(0, +\infty)$ by the definition of shape parameter. We can eliminate it.

If $V_{j,s} \leq \frac{(j-s)^2}{(j+s)^2}$ is tenable, we obtain by the same method

$$0 < a < j+s \tag{12}$$

It also hasn't any practical significance and should be eliminated.

By both of the assumptions, it proves existence and uniqueness of estimation value for a when $a > j+s$. Then, the solution for a is given from (7) in a closed form when $a > j+s$ as follow

$$\hat{a} = \frac{1}{2}\left(\sqrt{(j+s)^2 - \frac{4js}{1-\hat{V}_{j,s}}} - j - s\right) \tag{13}$$

As can be seen from (13), the expression is closed and can be used to estimate shape parameter a effectively which values in $(j+s,1)$ when $j+s$ is much less than 1.

3. Simulation Results

In the following part, we use MATLAB to assess the performances of the proposed estimator and compare it with different estimators. The MATLAB routines that simulate samples of random variable for Pareto distribution is as follow

$$\boldsymbol{x} = \text{exprnd}\left(\frac{1}{\text{gamrnd}(a,1/b,1,R)}\right) \tag{14}$$

In this routine, R is the quantity of samples. The simulation range of a is from 0.01 to 3 with step 0.01. The experiments of every shape parameter value are repeated one thousand times. The relative bias $(\hat{a}-a)/a$ is used to evaluate their estimation performances. Furthermore, it can also be evaluated by standard deviation of a.

As can be seen from Fig. 1, the relative bias values of two previous estimators increase faster and faster as a declines, while the relative bias values of the proposed estimator is close to zero all the time in $(j+s,+\infty)$. As can be seen from Fig. 2, when a values in $(j+s,1)$, the standard deviation of two previous estimators is much bigger and bigger than that of the proposed estimator as a declines. The simulation results show that the proposed estimator has better performance than other estimators especially when a values in $(j+s,1)$.

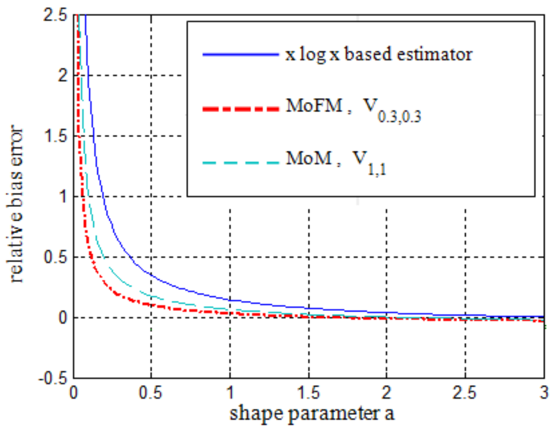

Fig. 1 relative bias values of four estimators

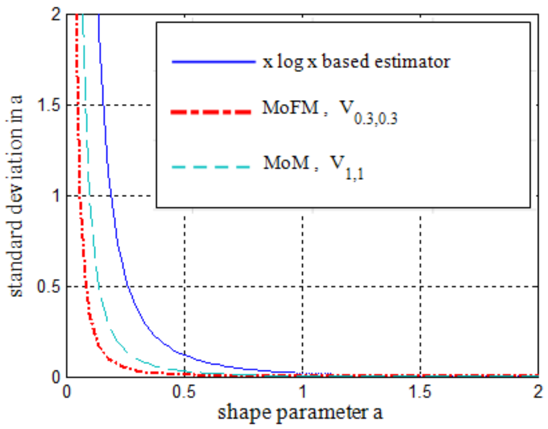

Fig. 2 standard deviation of four estimators

4. Conclusion

In order to get the closed-form expression and solve the problem that can't estimate the true value of a which values in $(j+s,1)$ for the Pareto distribution, one improved estimator is proposed. The proposed estimator can estimate the valid value of a in $(j+s,+\infty)$ and has better estimation performance than three previous estimators especially when a values in $(j+s,1)$. The results are tested and verified with experiments.

References

[1] Bocquet, S.: 'Closed-form parameter estimators for Pareto distributed clutter with noise', Electron. Lett, 2015, 51, pp. 1924–1926, doi: 10.1049/el.2015.2565

[2] Bocquet, S.: 'Parameter estimation for Pareto and K distributed clutter with noise', Radar Sonar Navig, 2015, 9, pp. 104–113, doi: 10.1049 / iet-rsn.2014.0148

[3] Bocquet, S.: 'Simulation of correlated Pareto distributed sea clutter'. IEEE Int. Conf. on Radar, Adelaide, September 2013, pp. 258–261, doi: 10.1109 /RADAR. 2013.6651995

[4] Weinberg, G.V.: 'Constant false alarm rate detectors for Pareto clutter models', Radar Sonar Navig., 2013, 7, pp. 153–163, doi: 10.1049 / iet-rsn.2011.0374

[5] Weinberg, G.V.: 'Validity of whitening-matched filter approximation to the Pareto coherent detector', Signal Process., 2012, 6, pp. 546–550, doi: 10.1049 /iet -spr.2011.0304

[6] Weinberg, G.V.: 'Assessing Pareto fit to high-resolution high-grazing-angle sea clutter', Electron. Lett, 2011, 47, pp. 516–517, doi: 10.1049/el.2011.0518

[7] Mezache, A., Chalabi, I., Soltani, F., and Sahed, M.: 'Estimating the Pareto plus noise distribution parameters using non-integer order moments and [zlog(z)] approaches', Radar Sonar Navig., 2016, 10, pp. 192–204, doi: 10.1049 /iet -rsn.2015.0170

[8] Balleri, A., Nehorai, A., and Wang, J.: 'Maximum likelihood estimation for compound-Gaussian clutter with inverse gamma texture', Trans. Aerosp. Electron. Syst., 2007, 43, (2), pp. 775–780, doi: 10.1109 /TAES.2007.4285370

[9] Rosenberg, L., and Bocquet, S.: 'Application of the Pareto plus noise distribution to medium grazing angle sea-clutter', J. Sel. Top. Appl. Earth Obs. Remote Sens., 2015, 8, pp. 255–261, doi: 10.1109 / JSTARS.2014.2347957

[10] Weinberg, G.V.: 'Estimation of Pareto clutter parameters using order statistics and linear regression', Electron. Lett., 2013, 49, pp. 845–846, doi: 10.1049/el.2013.0916

[11] Mohamed S., Amar M. and Faouzi S.: 'Closed-form estimators for the Pareto clutter plus noise parameters based on non-integer positive and negative order moments', IET Radar, Sonar & Navigation., 2017,11(2), pp.359-369, doi: 10.1049/iet-rsn.2016.0323

[12] Ali M., Toufik L. and Amar M.: 'New Pareto clutter parameters estimators based on log-moments and fractional negative-moments', Detection Systems Architectures and Technologies, Algiers, Algeria, February 2017, pp. 1-5, doi: 10.1109/DAT.2017.7889175

[13] [Amar Mezache, et al.: 'K-clutter Plus Noise Parameter Estimation Using Fractional Positive and Negative Moments', IEEE Trans. Aerospace and Electronic Systems, 2016, 52(2), pp.960-967, doi: 10.1109/TAES.2015.150153