Minimization Power Flow Algorithm Based on Nonlinear Optimization and The Comparison of Its Two Typical Implementations

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Abstract

Power flow calculation is one of the most basic problems of the power system. In this paper, the parameters of power system steady-state operation are computed with a special algorithm - minimization power flow based on nonlinear optimization. Minimization power flow algorithm is aimed to find the minimum value of the objective function under a hidden equality constraint. To obtain the steady-state solution, the paper adopted two different nonlinear optimization methods including interior-point method and optimal multiplier Newton method. The effectiveness of the two implementations was verified by an example of a typical two-machine five-node system. As the results indicated, the algorithm can solve the power flow problem in rectangular coordinate system accurately, and it also has significant superiority in providing judgment of the existence of valid solutions in ill-conditioned systems.

Keywords

Minimization power flow, nonlinear optimization, interior- point method, optimal multiplier Newton method, MATLAB.

1. Introduction

The power flow calculation is a significant problem in the analysis of steady-state operation of the power system. The aim of power flow algorithm is to obtain the operating state of the entire system according to the given operating conditions and network structure. State of system refers to variables such as the voltage (amplitude and phase angle) on each bus, the power distribution in the network, and the power loss [1, 2]. The result of system's power flow calculation is the basis of power system stability calculation and fault analysis.

The computer provides great convenience for the power flow calculation of the system. The main concerns are shown the following aspects: the convergence of the calculation method, the reliability, running speed of program, requirement for computer storage capacity, the convenience as well as flexibility of the algorithms and so forth [3].

2. Minimization Power Flow

2.1 Problem Finding

Although various methods had been proposed to solve the steady-state power flow, such as Newton-Ralfsnn's method and P-Q decoupling method, difficulties occurred when dealing with the calculation of power flow in ill-conditioned systems [4]; for instance, heavy load systems. Those conditions could result in the oscillation of calculation process and even non-convergence.

Under those conditions, it can be very difficult to determine whether the nonlinear equations have no solutions, or it is the imperfection of power flow algorithm that leads to the failure of calculation.

2.2 Significances

It was discovered that: the calculation of power flow can be transferred to an optimization problem of obtaining the minimum of an objective function, which derives from the power flow equations. This gave birth to a new method called minimization power flow, whose algorithm principle is entirely different from methods mentioned above.

It utilizes nonlinear programming and minimization techniques, making sure that computational process will never end up in divergence [5]. As long as the power flow problem has solutions, the objective function of minimization power flow would become zero rapidly. Otherwise, the objective function would be reduced to a positive value instead of zero, under the circumstance that power flow equations have no solutions.

This characteristic of minimization power flow has solved the problem of ill-conditioned system effectively, and it provides a sharp judgment of the existence of solutions under the given conditions.

2.3 Modeling

The power flow equations can be expressed as:

$$f_i(x) = g_i(x) - b_i = 0 \qquad (i = 1, 2, \ldots, n) \tag{1}$$

Or

$$f(x) = 0$$

In the second equation, refers to an n-dimensional vector consisting of unknown state variables such as the amplitude and phase angle of voltage vector on each bus, or the real and imaginary part of voltage vector on each bus; $f(x)$ refers to the difference between actual power flow and the given values, which is supposed to be zero in the steady-state operation of power system.

Considering $f(x)$ can be either positive or negative, the objective function can be deduced:

$$F(x) = \sum_{i=1}^{n} f_i(x)^2 = \sum_{i=1}^{n} (g_i(x) - b_i)^2 \tag{2}$$

Or

$$F(x) = [f(x)]^T f(x)$$

If solutions of power flow equations exist, $F(x)$ in the last equation should be zero. So the objective function can be written as:

$$\min F(x) \tag{3}$$

As the objective function indicates, power flow has been turned into an optimization problem.

3. Implementations of Minimization Power Flow

Nonlinear optimization is a very common problem in the process of design and dispatch of power system. As mentioned above, minimization power flow can be solved by obtaining the minimum of objective function.

Note that the problem seems to be an unconstrained optimization problem, but in the actual programming process there is a hidden constraint which should not be ignored. The constraint is: the voltage vector of slack bus should be 1.0 (p.u.). That is to say, in rectangular coordinate system, the real and imaginary part of slack bus voltage should be 1.0 and 0 respectively.

So finally, the problem becomes a typical nonlinear optimal problem, which is aimed to find the minimum value of the objective function under an equality constraint. To obtain this optimal solution, the paper adopted two different methods including interior-point method and optimal multiplier Newton method. Both of them were realized in the MATLAB software, and the former was incorporated in the MATLAB fmincon function [6].

3.1 Interior-point Method

Introduced into power system optimization in the 1990s, the interior-point method (also referred to as the barrier method) is a certain class of algorithms that solve linear and nonlinear convex optimization problems [7].

As it is essentially a combination of Lagrange function, Newton method and logarithmic barrier function, it can not only handle inequality constraints very well, but also adequately inherit the advantages of Newton method in dealing with power flow calculation.

Considering the programming implementation of interior-point method, the paper used fmincon function to complete this algorithm. As one of those very useful functions integrated in MATLAB software, this function specializes in finding a constrained minimum of a function of several variables. Therefore, this function is suitable for minimization power flow problem.

3.2 Optimal Multiplier Newton Method

Optimal multiplier Newton method puts the objective function as:

$$F^{(k+1)} = F(x^{(k+1)}) = F(x^{(k)} + \mu^{*(k)}\Delta x^{(k)}) \tag{4}$$

In this formula, $\Delta x^{(k)}$ refers to the search direction of the kth iteration, which can be expressed according to Newton method:

$$\Delta x^{(k)} = -J(x^{(k)})^{-1} f(x^{(k)}) \tag{5}$$

$\mu^{*(k)}$ refers to the optimal multiplier of the kth iteration, which can be figured out according to steps in the block diagram [8, 9], see Fig. 1.

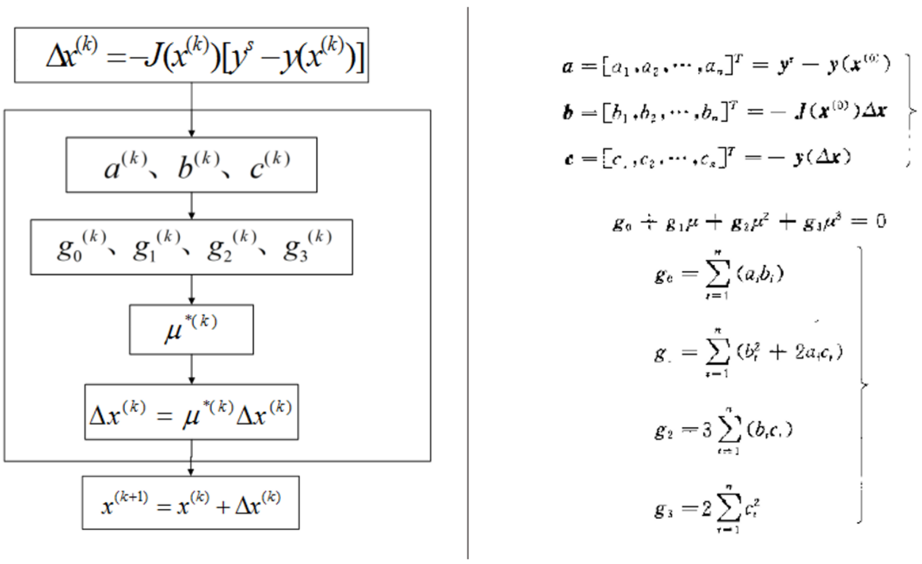

Fig 1. Block diagram of optimal multiplier Newton method

4. Simulation and Results

To verify the accuracy and rapidity of the two minimization power flow algorithms, a typical two-machine five-node system was chosen as an example to carry out simulation, see Fig. 2.

Assume that the generator voltage of G1 and G2 are 1.0 (p. u.). Both the active power and reactive power that G1 produces can be adjusted, but the active power that G2 produces is fixed at 0.5 (p. u.).

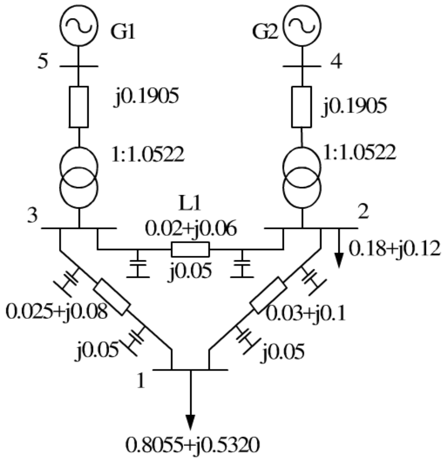

Fig 2. A typical two-machine five-node system

The simulation was conducted with the above two typical implementations respectively. From the results listed below, the two implementations have consistent power flow solutions. And it is quite reasonable to conclude that both algorithms can converge to the actual steady-state solution accurately.

Node voltage magnitude:

0.9916 1.0175 1.0229 1.0000 1.0000

Node voltage phase angle (°):

-7.4745 -5.8545 -5.5862 -0.2018 0

Node active power:

-0.8055 -0.1800 0 0.5000 0.4967

Node reactive power:

-0.5320 -0.1200 0 0.1977 0.1706

Branch losses:

0.0053 - 0.1244i 0.0057 - 0.1636i 0.0003 - 0.1033i 0.0000 + 0.0551i 0.0000 + 0.0526i

Fig 3. Results of interior-point method

Node voltage magnitude:

0.9916 1.0175 1.0229 1.0000 1.0000

Node voltage phase angle (°):

-7.4524 -5.8323 -5.5666 -0.1784 0

Node active power:

-0.8055 -0.1800 0 0.5000 0.4950

Node reactive power:

-0.5320 -0.1200 0 0.1976 0.1704

Branch losses:

0.0053 - 0.1244i 0.0057 - 0.1636i 0.0003 - 0.1033i 0.0000 + 0.0551i 0.0000 + 0.0522i

Fig 4. Results of optimal multiplier method

As simulation process indicated, the interior-point algorithm has some different features compared to optimal multiplier Newton Method although they converged to the identical optimal solution.

1) Different code size: Using interior-point algorithm was apparently simpler. Because it is the integrated function of MATLAB and complex algorithm has been incorporated inside the function, the lengthy code does not need to be written any more, thus saving the time for writing a program by reducing its code size;

2) Different cost: Overall, the number of iteration of method using interior-point method was much less than that of optimal multiplier Newton method. However, the time cost of interior-point method was longer. Table 1 shows the differences between the two algorithms in terms of their cost. Note that: The recorded number and time are around the average time cost of multiple runs.

Table 1. Cost comparison between two implementations

Implementations	Number of iterations	Running time
interior-point method	20	0.0930
optimal multiplier Newton method	104	0.0200

5. Conclusion

Minimization power flow provides a valid solution to obtain the steady-state parameters of power system, and it also has significant superiority in providing judgment of the existence of valid solutions in ill-conditioned systems.

What these facts suggest is that, on the premise of convergence achieved, optimal multiplier Newton method performs better when minimization power flow programming requires higher speed. However, interior-point algorithm would be the first option when considering the efficiency of programming and algorithm running. To increase the efficiency of optimal multiplier Newton method, it may be feasible to update Jacobi Matrix less frequently.

Based on principles mentioned above, different methods can be chosen to achieve better practicability according to different application occasions in minimization power flow.

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