A Multi-scale Descriptor for 3D Shapes Based on Feature Structural Graph

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Abstract

In this paper, we propose a novel multi-scale descriptor for 3D shapes, which represents the properties within a local neighborhood restricted by a range of structure-aware distance. The descriptor consists of two main parts, the first is to express the local features of certain point using traditional features, and the second is to further encode the structure information among these features using an undirected graph. Our descriptor is isometric/symmetric invariant and robust under noise, which can be used for various shape analysis applications, such as registration, correspondence, segmentation.

Keywords

Multi-scale, heat kernel, structural graph, shape analysis.

1. Introduction

With the rapid development of acquisition hardware and 3D modeling techniques, tasks like obtaining, reconstructing and manufacturing 3D shapes are becoming easier and easier. 3D shapes now are among the common forms of multimedia with others like text and images, and researchers tend to conduct 3D shape analysis and processing in various applications, such as shape segmentation that decomposes 3D shapes into several individual parts [1, 2], shape correspondence that builds 1-to-1 correspondence between different shapes [3, 4], and shape retrieval [5, 6]. However, the key to all shape analysis tasks mentioned above is how to define an appropriate descriptor that distinguish shapes or points on the shape from each other. Even though many descriptors have been proposed for different specific applications, it still has challenge to define a more feasible and flexible descriptor that applies to various applications.

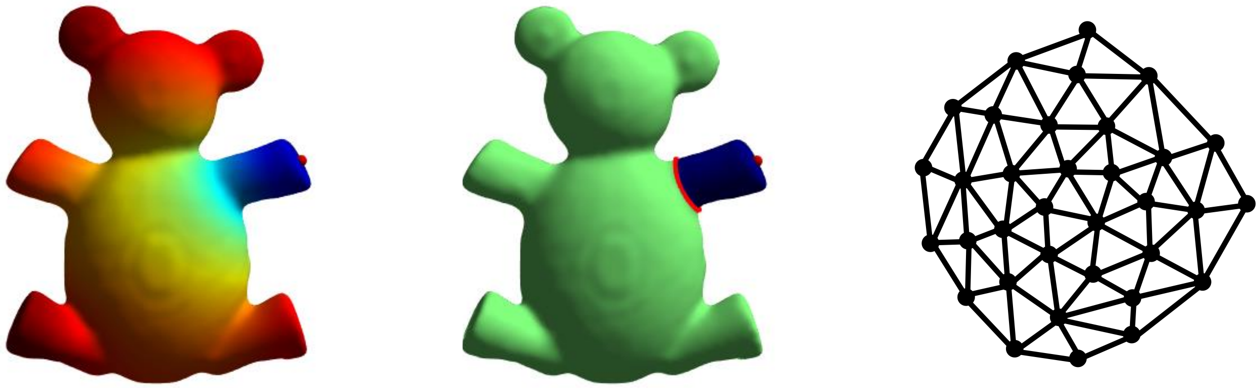
Conventional shape descriptors mainly focus on a local neighborhood of certain point on the shape, and use the statistics of some signals defined on the shape as the description of that point, such as Spin Images [7], histogram of curvature [8] or Shape Diameter Function (SDF) [9]. The histogram of certain signal can well describe the local or global feature of a 3D shape, and signals like curvature only characterize local geometric properties. Thus, the histogram descriptors usually are isometry-invariant, and can be used to distinguish different points based on their local neighborhood even though there exist non-rigid deformations between shapes. This kind of descriptors is one of the most popular methods, and has been widely used in 3D shape segmentation and retrieval because of its simplicity and effectiveness. However, these descriptors simply rely on the statistics of signals or features within certain local region, and ignore the structure of features in that region which heavily restricts their capability of description.

In this paper, to further exploit the structure of features on 3D shapes, we propose a totally new and simple multi-scale point descriptor based on feature structural graph defined on the surface of shapes. Firstly, to alleviate the influence of triangulation, we apply Poisson disk sampling [10] with certain point's neighborhood to generate uniformly distributed sample points, and then use Heat Kernel Signature (HKS) [11] as original low-level sample feature that measures the local geometric properties of certain small neighborhood. Secondly, we build a structural graph of features that describes the similarity or connection between any two sample points, and the structural graph is represented as an affinity matrix. Finally, any two affinity matrices are compared by their ordered leading eigenvalues, which is robust under noise or different sampling results. With this novel descriptor, we can achieve multi-scale shape registration or correspondence, shape segmentation and shape retrieval as well. In particular, our contributions of this paper can be summarize as follows,

We propose a new shape description method based on feature structural graph, which deeply exploit the intra-structure of features and make the capability of description more powerful;

We use the ordered eigenvalues of affinity matrix to compare between structural graphs instead of complex graph matching, which is simple but robust;

We achieve various shape analysis and processing tasks with our new descriptor, such as shape registration, segmentation, and correspondence.



(a) Bi-harmonic Distance (b) Local region (c) Structure graph

Figure 1. Illustration of our multi-scale descriptor definition. (a) Color coded bi-harmonic distance to the red point on the teddy's left hand, red corresponds to long distance and blue corresponds to short distance; (b) the selected local neighborhood of the red point restricted by a distance range indicated with a red circle; (c) the structure graph built from uniformly random sample points.

2. Multi-scale Descriptor Definition

In this section, we will detail our multi-scale descriptor definition, which mainly includes: bi-harmonic distance-based neighborhood selection, uniformly Poisson disk sampling and local description, and structural graph built from random features. In the following, we assume a 3D shape is represented as a triangular mesh $M = (V, F)$, where $V = \{v_1, v_2, \dots, v_n\}$ contains n vertices.

2.1 Bi-harmonic Distance Based Neighborhood Selection

As we discussed in the beginning, most point descriptors are defined based on a local neighborhood of the point, so the first thing before defining a point descriptor is to select an appropriate neighborhood. In this paper, we adopt the bi-harmonic distance [12] defined on the surface of a shape that measure the distances between pairs of points on the surface. The bi-harmonic distance between vertex v_i and v_j can be expressed as,

$$D(i,j)^2 = \sum_{k=1}^{m} \frac{\left(\phi_k(i) - \phi_k(j)\right)^2}{\lambda_k^2}, \tag{1}$$

where $\{\lambda_k\}$ and $\{\phi_k(\cdot)\}$ are, respectively, the first m non-zero eigenvalues and the corresponding eigenvectors of the Laplace-Beltrami operator with "cotangent formula" discretization [13]. Fig. 1(a) shows an example of bi-harmonic distance, where the distances to the red point on the teddy's left hand are color coded with red corresponding to long distances and blue corresponding to short distances. The bi-harmonic distance has many favorable advantages, such as locally isotropic, globally structure aware, isometry-invariant and insensitive to noise and small topology changes, so it is very suitable for defining local neighborhood of certain point. Given a point v_i on the surface, its neighborhood is defined as,

$$N_s(i) = \{v_j \mid D(i,j) \leq s\}, \quad (2)$$

where s is the radius of the neighborhood, which controls the spatial scale of our descriptor, namely the range of features taken into consideration. Fig. 1(b) shows an example of selected local neighborhood according to a center point v_i and the distance range s, indicated with red point and red circle respectively.

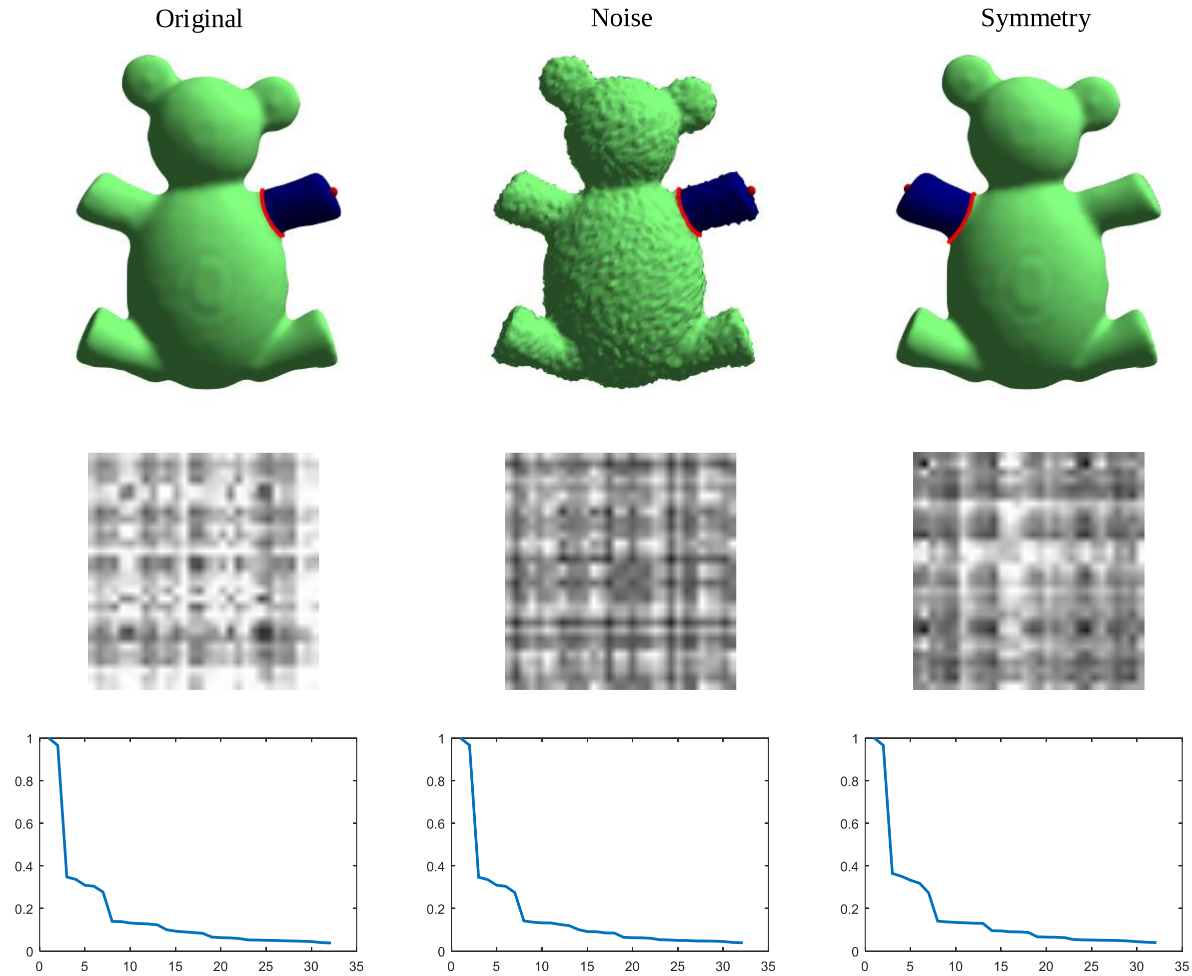


Figure 2. Illustration of the robustness of the feature structure descriptor under noise and symmetry. Top row: red points on the teddy shape with certain local neighborhood colored in blue. Middle row: affinity matrices of feature structure graphs of these selected regions. Bottom row: the leading eigenvalues of the affinity matrices, serving as the descriptors.

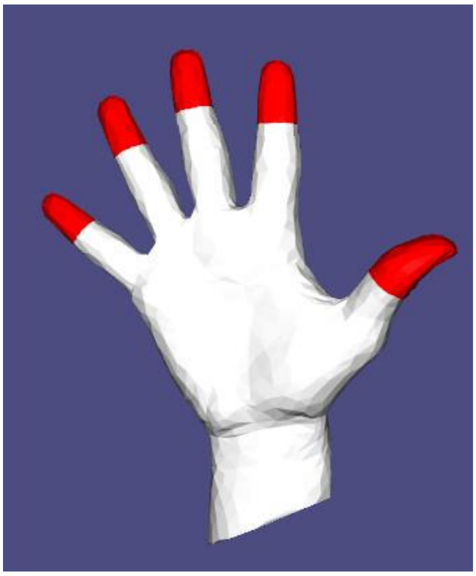
2.2 Uniform Poisson Disk Sampling and Local Description

Usually a 3D shape is discretized as a triangular mesh, which represents the surface via many connected small triangles. However, the triangulation is commonly not uniform because of some technical restriction or to keep level of details sometimes. In other words, the sizes of triangles are not equal to each other and the vertices are not evenly distributed. To alleviate the influence of this irregular triangulation, one intuitive way is to apply a uniform sampling to obtain a set of regularly spaced sample points. In this paper, we adopt the Poisson disk sampling [10] in the local neighborhood of certain point defined above, which generates a uniformly random distribution where the minimum distance between each sample is $2r$, therefore a disk of radius r centered on each sample does not overlap any other disk. Fig. 1(c) shows an example of Poisson disk sampling in 2D, where black color indicated sample points are roughly uniformly distributed.

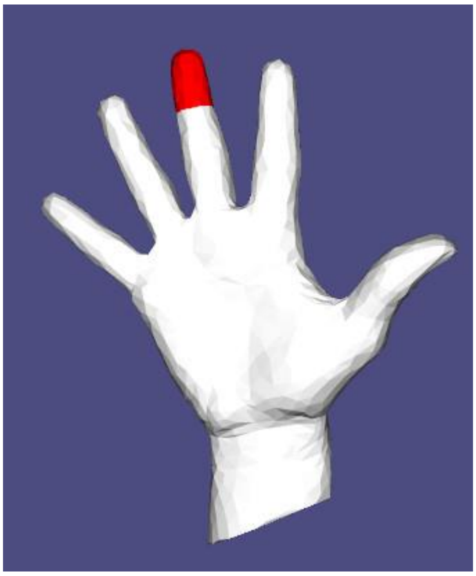
After obtain several uniformly distributed sample points, then we use Heat Kernel Signature (HKS) [HKS] to characterize their local geometric properties namely features, which restricts the heat kernel only to the temporal domain, and the HKS at vertex v_i is expressed as a discrete sequence of $\{h_{t_1}(i), h_{t_2}(i), \ldots, h_{t_n}(i)\}$ values sampled at times $t_1, t_2, \ldots t_n$, where $h_t(i)$ can be expressed as

$$(h_t(i) = \sum_{k=0}^{n} exp(-\lambda_k t)\, \phi_k^2(i), \tag{3}$$

where $\{\lambda_k\}$ and $\{\phi_k(\cdot)\}$ are the eigenvalues and eigenvectors of the Laplace-Beltrami operator as those in Eq. 1. In practical implementation, we do not need to compute all eigenvalues and eigenvector, only several tens of smallest non-zero eigenvalues and eigenvectors are enough to compute both bi-harmonic distance and HKS, because both Eq. 1 and Eq. 3 decays exponentially as eigenvalues increases, which means the influence of big eigenvalues is very small, and these eigenvalues and eigenvectors can be ignored accordingly. Besides, the time t is usually sample logarithmically in temporal domain, which gives a more faithful approximation to HKS at small scales, when the signature changes more rapidly.



Registration with small scale



Registration with large

Figure 3. Illustration of the multi-scale ability of our shape descriptor. The pictures show the points (red) on the hand with similar properties comparing to a point at the tip of the middle finger with small scale (left) and large scale (right).

2.3 Structural Graph Built From Random Samples

So far, for each vertex on a 3D shape, we have several uniformly distributed random samples within a local neighborhood defined by structure-aware bi-harmonic distance, and each random sample's local property is characterized by its HKS. The aim in this paper is to define a high-level multi-scale descriptor based on these low-level features. Different from traditional descriptor simply relies on statistics of certain geometry signal, the descriptor we define here is built from a structural graph, which not only indicates the feature of the vertex itself but also the mutual relations among features that lies in a small neighborhood.

As we know, the descriptor for one vertex we propose in this paper is defined by the low-level features of random samples around this vertex, which consists of two main parts, a descriptor that characters the vertex's local feature directly based on HKS and a descriptor that characters the feature structure of the random samples. The first descriptor is defined as a concatenation of the average HKS values of all random samples resulting in a vector of averages (HKS_{μ}) and the variance of these signatures resulting in a vector of variances (HKS_{σ^2}), which is similar to that in [14] and can be expressed as

$$(D\_s\hat{}1\ (v\_i\ ) = \left\{\mathrm{HKS}_{\mu}\big(N_s(i)\big)_{[0,1]}, \mathrm{HKS}_{\sigma^2}\big(N_s(i)\big)_{[0,1]}\right\} \tag{4}$$

where $N_s(i)$ is a set of vertices in which each vertex's hi-harmonic distance is smaller than s, and the notation $[0,1]$ indicates that each part is normalized separately to the range $[0,1]$ by setting the L_{∞}-norm to 1. The second part of the descriptor, which is the core of this paper, is to describe the structure of features within the local neighborhood, and it is expressed as an undirected graph wherein the nodes represent different local features and the edges build the connections among these local features, as shown in Fig 1(c).

However, comparing between two graphs is not easy, one popular way is graph matching [15], and this algorithm is not that easy to implement and usually not efficient enough as well. Here in this paper, we adopt a new but simple way to encode the structure graph into quantitative numbers which makes it much easier to compare between graphs. Firstly, we represent the structure graph as an affinity matrix, whose entries are defined as follows,

$$\mathbf{A}_{ij} = \begin{cases} d_{ij} & if\ i = j \\ -\sum_{i} d_{ij} & if\ i \neq j \end{cases}, \tag{5}$$

where d_{ij} is the distance between the local features of sample points v_i and v_j, and can be simply computed as the Euclidean distance of the two local descriptors. Fig. 2 shows 3 examples of the affinity matrices, but these 3 matrices greatly differ from each other despite they represent similar points with same range of neighborhood, which is mainly due to the inconsistent order of sample points. Then, we compute the first k leading eigenvalues of the affinity matrix, shown in the bottom row of Fig. 2, which obviously resolves the affects introducing by inconsistent order of sample points and can be used to describe the feature graph in a quantitative manner.

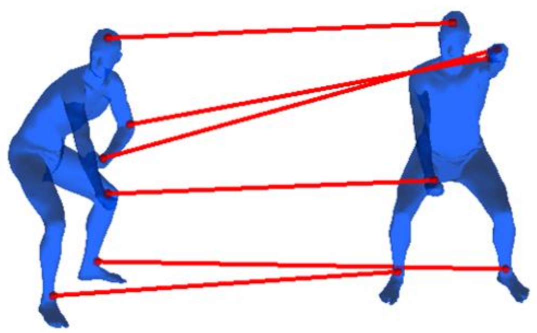


Figure 4. Our descripor applies to shape correspondence

3. Various Applications

So far, we have detailed our novel descriptors which consists of a description based on traditional HKS-like local features and a new description that encodes the structure information of features. In this section we will show different applications of our newly proposed descriptors.

3.1 Multi-scale Registration

The first application of our descriptor is shape registration. Shape registration is to find similar points of certain given point on the same or another shape. As we know, our descriptor is defined to describe the properties within a local neighborhood restricted by a distance range, so it is very suitable for shape registration, or multi-scale registration more particularly. Fig. 3 shows an example of multi-scale registration of our descriptor, the red regions on the fingers are those points with similar properties to a point at the tip of the middle finger. The left picture shows the registration result of our descriptor in a small scale, namely using small distance range to select local neighborhood, and we can see that all of the five finger tips are highlighted as similar regions of the point at middle finger, which means these five finger tips are treated similar to each other in a local sense. The picture on the right side shows the registration result of our descriptor in a large scale, or saying using large distance range. In this case, only the points on the middle finger are highlighted as similar regions because tips of other 4 fingers are different from that of the middle one in a global sense.

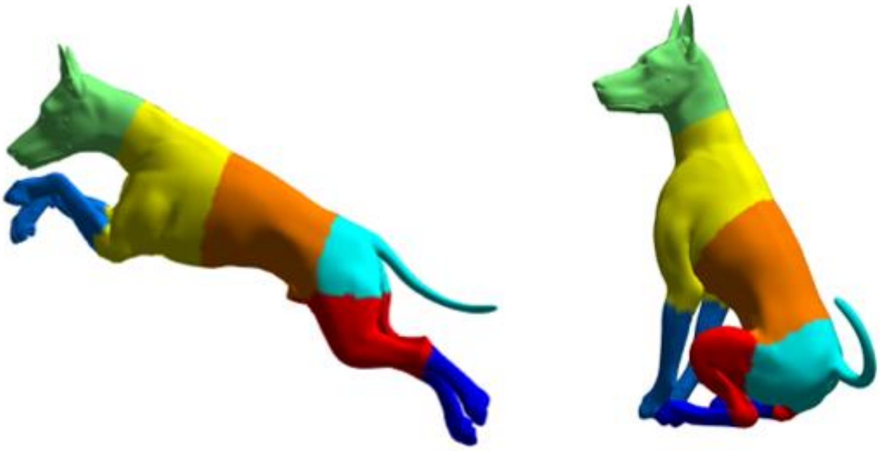


Figure 5. Our descriptor applies to shape segmentation.

3.2 Sparse Shape Correspondence

Similar to shape registration, our descriptor can also be used to build 1-to-1 correspondence across different shapes. Benefited from the isometric invariance and robustness of HKS and structure graph definition, our descriptor keeps invariant under isometric deformations even with noise perturbation on the vertices, as shown in Fig. 2. With this isometric invariance, we can build sparse 1-to-1 correspondence between shapes with isometric or near-isometric deformations, simply by connecting one point to its most similar point on another shape, as shown in Fig. 4. Note that, here the ambiguity of symmetry between left and right side of the human shape is manually resolved, because our descriptor is also symmetric insensitive and cannot distinguish left and right sides. With his sparse correspondence, we can further apply a coarse-to-fine strategy [16] to obtain dense correspondence. However, this is out of scope of this paper, readers interested in this can refer to [16] for more details.

3.3 Shape Segmentation

From the last two applications, we can find points with similar properties using our proposed descriptors. If we cluster points with similar properties together, we will naturally obtain shape segmentation result. Fig. 5 shows two examples of the shape segmentation results when using our descriptor. Here, we concatenate the descriptors and spatial coordinates of the vertices and apply a simple k-means clustering algorithm which ensembles the points into 7 different segments. We can see that the segments are consistent between the two dog shapes, which once again proves the robustness of our descriptor under deformations.

Beyond these 3 applications, our descriptor can also be used for shape retrieval by using bag-of-words algorithm that converts a local point descriptor to a global shape descriptor, or used for holes filling when searching for similar patches, and many other descriptor-related shape analysis applications.

4. Conclusion

In this paper, we propose a novel point descriptor that consists of two main parts, the first part is based on traditional HKS-like local descriptor that express the local features, and the second part is based on an undirected graph of these local features that encodes the structural information. Our descriptor has many favorable properties, such as isometric or symmetric invariance and robustness under noise, and can be used for many shape analysis applications, such as shape correspondence, multi-shape registration, shape segmentation, shape retrieval, etc.

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